Representations of Solutions in Generalized Theory of Micropolar Thermoelastic Diffusion with Triple Porosity

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Abstract: The goal of this paper is to define the basic governing equations for a medium with anisotropic micropolar thermoelastic properties, including mass diffusion and triple porosity. Additionally, this paper also aims to develop the fundamental solutions for the system of equations under different conditions, such as steady, pseudo-, quasi-static oscillations, and equilibrium.

Key words: thermoelastic diffusion, triple porosity, pores, steady oscillations.

I. Introduction

The micropolar theory of elasticity is a conceptual framework that considers the micro-structural motion of different materials, including polycrystalline materials and materials with fibrous structures etc. This theory incorporates both displacement and microrotation vectors to describe the motion of solids. Several researchers (referenced as [1–6]) have extended this theory by incorporating thermal effects, resulting in the development of the micropolar theory of thermoelasticity. This expanded theory takes into account the coupling between mechanical deformation and temperature changes in materials.

Diffusion is indeed the process by which mass or particles move from regions of higher concentration to regions of lower concentration. It is a fundamental mechanism of transport that occurs in various systems, such as gases, liquids, and solids. The driving force behind diffusion is the concentration gradient, which represents the difference in concentration between two regions. As particles move down the concentration gradient, they tend to distribute themselves more evenly, resulting in a net movement from areas of high concentration to areas of low concentration. Diffusion plays a crucial role in numerous natural and industrial processes, including chemical reactions, biological systems, and materials science. The authors Nowacki [7–10], Sherief et al. [11], Aouadi [12], and Kansal and Kumar [13] have contributed to the development of various theories of thermoelastic diffusion. These theories aim to describe the coupled mechanical behavior of temperature, concentration, and strain fields in elastic solids. Aouadi [14] specifically derived the theory of generalized micropolar thermoelastic diffusion, building upon the Lord-Shulman theory with the inclusion of one relaxation time. This theory provides a framework for analyzing the simultaneous effects of thermal, diffusive, and mechanical phenomena in materials.

In a triple porosity elastic material, the body exhibits three distinct levels of pore structures. Each level represents a different scale of porosity within the material. The first level is referred to as macro porosity, which represents the largest visible pores within the material. These pores are typically observable to the naked eye or through macroscopic imaging techniques. The second level is known as meso porosity, which corresponds to an intermediate scale of porosity. The pores at this level are smaller in size compared to macro porosity but larger than the micro porosity. The final level is referred to as micro porosity, which represents the smallest scale of pores within the material. These pores are characterized by their microscopic dimensions and may not be directly observable without the aid of high-resolution imaging techniques or advanced microscopic analysis. The presence of these three levels of porosity allows for a more comprehensive characterization of the material's permeability, transport properties, and mechanical behavior, as it considers variations in pore sizes and their distribution throughout the material.

The works of Svanadze [15], Straughan [16] and Kansal [17] have contributed to the establishment of governing equations in the field of elasticity and thermoelasticity, specifically addressing the concept of triple porosity. Svanadze [18–23] further investigated boundary value problems related to elastic solids and thermoelastic solids with triple porosity. The motivation behind developing theories for multiple porosity elasticity lies in the wide range of applications that require such frameworks, which continue to emerge. One notable application area is in mathematical biology and health sciences. The replacement of damaged long bones in humans poses a significant challenge for surgeons, as the porosity of bone can vary significantly, ranging from 14% in the outer layer to 42% in the inner layer. To accurately model long bones, a multi-porosity theory that accounts for graded porosity materials may be necessary. Another crucial field where multiple porosity elasticity finds application is geophysics. For instance, a comprehensive description of landslides may require the utilization of double porosity theory to account for the complex behavior of porous materials in such scenarios. Straughan [24] has discussed various applications of multiple porosity in his book, shedding light on the practical significance of these theories across different disciplines.

Indeed, fundamental solutions play a crucial role in solving boundary value problems, particularly in the context of elasticity and thermoelasticity. The use of fundamental solutions allows for an integral representation of the solution to a boundary value problem, which is often more amenable to numerical methods than directly solving a differential equation with specified boundary and initial conditions. In the investigation of boundary value problems within the theories of elasticity and thermoelasticity using the potential method, it becomes necessary to construct fundamental solutions for the corresponding systems of partial differential equations and establish their basic properties. These fundamental solutions serve as building blocks for solving more complex problems and provide insights into the behavior of the underlying physical phenomena. Svanadze [15, 25] has made contributions in this regard by constructing fundamental solutions using elementary functions in the theory of elasticity and thermoelasticity with triple porosity. By utilizing elementary functions, the solutions become more accessible and amenable to analysis and numerical techniques, facilitating the study and solution of boundary value problems within these theories.

This paper focuses on deriving the constitutive relations and field equations for anisotropic generalized micropolar thermoelastic diffusion with triple porosity. In Section 2, derivation of these equations and relations and simplification of the anisotropic system of equations into an isotropic system of equations are given. Sections 3 and 4 entail the construction of the fundamental solution for the case of steady oscillations. This fundamental solution has been expressed in terms of elementary functions so that it becomes more accessible and easier to analyze. Moving on to Section 5, the author constructs the fundamental solutions for cases of pseudo-static, quasi-static oscillations, and equilibrium. This allows for a comprehensive understanding of the system's behavior under different conditions. In Section 6, the paper establishes some basic properties of the fundamental matrix. These properties help in further analyzing and understanding the solutions obtained and their implications. Overall, the paper covers the derivation of constitutive relations and field equations, reduction of the system to an isotropic form, construction of fundamental solutions for various types of oscillations, and establishing fundamental matrix properties. These contributions provide a foundation for studying the behavior and properties of anisotropic generalized micropolar thermoelastic diffusion with triple porosity.

II. Basic Equations

Following [1–5], the equations of motions of a linear micropolar elasticity are

$$\sigma_{ji,j} + \rho\tilde{F}_i = \rho\ddot{u}_i, \tag{1}$$

$$\varepsilon_{ijk}\sigma_{jk} + \mu_{ji,j} + \rho\tilde{G}_i = \rho J_{ij}\ddot{\varphi}_j, \tag{2}$$

where σ_{ji} are the components of stress tensor, μ_{ji} are the components of moment of couple stress tensor, ρ is the density, u_i are the components of the displacement vector $\mathbf{u}$, $\tilde{F}_i$ are the components of the external forces per unit mass, $\tilde{G}_i$ are the components of the external applied couple per unit mass, ε_{ijk} is the alternating tensor, J_{ij} are the components of microinertia tensor, φ_j are the components of microrotation vector $\boldsymbol{\varphi}$.

The law of conservation of energy for an arbitrary material volume V bounded by a surface A at time t can be written as

$$\begin{aligned}\int_V \rho[\dot{u}_i\ddot{u}_i + J_{ij}\dot{\varphi}_i\ddot{\varphi}_j + \kappa_1\dot{\nu}_1\ddot{\nu}_1 + \kappa_2\dot{\nu}_2\ddot{\nu}_2 + \kappa_3\dot{\nu}_3\ddot{\nu}_3 + \dot{U}]\mathrm{d}V = \\ \int_V \rho[\tilde{F}_i\dot{u}_i + \tilde{G}_i\dot{\varphi}_i + \Lambda_i\,\dot{\nu}_i]\mathrm{d}V + \int_A [f_i\dot{u}_i + \mu_i\dot{\varphi}_i + +\Omega_{ij}\varpi_j\dot{\nu}_i - q_i\varpi_i]\mathrm{d}A,\end{aligned} \tag{3}$$

where U is the internal energy per unit mass, q_i are the components of heat flux vector $\mathbf{q}$, f_i are the components of surface traction vector $\mathbf{f}$ occurring on the surface A, μ_i are the components of couple stress vector $\boldsymbol{\mu}$ occurring on the surface A, ν_i are the volume fraction fields corresponding to macro-, meso-, micro-pores respectively, κ_i are the coefficients of equilibrated inertia, Λ_i are extrinsic equilibrated body forces per unit mass associated to macro-, meso-, micro-pores respectively, Ω_{ij} are the components of equilibrated stress vectors corresponding to ν_i measured per unit area of surface A respectively, ϖ_i are the components of outward unit normal vector $\boldsymbol{\varpi}$ to the surface A.

The components f_i and μ_i are, respectively, connected to stress and couple stress vectors by the relations

$$f_i = \sigma_{ji}\varpi_j, \quad \mu_i = \mu_{ji}\varpi_j. \tag{4}$$

Using equation (4) in equation (3) and applying divergence theorem, we get

$$\begin{aligned}\int_V \rho[\dot{u}_i\ddot{u}_i + J_{ij}\dot{\varphi}_i\ddot{\varphi}_j + \kappa_1\dot{\nu}_1\ddot{\nu}_1 + \kappa_2\dot{\nu}_2\ddot{\nu}_2 + \kappa_3\dot{\nu}_3\ddot{\nu}_3 + \dot{U}]\mathrm{d}V = \int_V \rho[\tilde{F}_i\dot{u}_i + \tilde{G}_i\dot{\varphi}_i + \Lambda_i\,\dot{\nu}_i]\mathrm{d}V + \\ \int_V [\sigma_{ji,j}\dot{u}_i + \sigma_{ji}\dot{u}_{i,j} + \mu_{ji,j}\dot{\varphi}_i + \mu_{ji}\dot{\varphi}_{i,j} + \Omega_{ij,j}\dot{\nu}_i + \Omega_{ij}\dot{\nu}_{i,j} - q_{i,i}]\mathrm{d}V.\end{aligned} \tag{5}$$

Since equation (5) is valid for every part of the body, therefore local form of conservation of energy is obtained as

$$\begin{aligned}\rho[\dot{u}_i\ddot{u}_i + J_{ij}\dot{\varphi}_i\ddot{\varphi}_j + \kappa_1\dot{\nu}_1\ddot{\nu}_1 + \kappa_2\dot{\nu}_2\ddot{\nu}_2 + \kappa_3\dot{\nu}_3\ddot{\nu}_3 + \dot{U}] = \rho[\tilde{F}_i\dot{u}_i + \tilde{G}_i\dot{\varphi}_i + \Lambda_i\dot{\nu}_i] + \\ \sigma_{ji,j}\dot{u}_i + \sigma_{ji}\dot{u}_{i,j} + \mu_{ji,j}\dot{\varphi}_i + \mu_{ji}\dot{\varphi}_{i,j} + \Omega_{ij,j}\dot{\nu}_i + \Omega_{ij}\dot{\nu}_{i,j} - q_{i,i}.\end{aligned} \tag{6}$$

Equation (6) with the assistance of equations (1) and (2) yields a simplified form of conservation of energy

$$\rho\dot{U} = \sigma_{ji}\dot{\varepsilon}_{ji} + \mu_{ji}\dot{\varphi}_{i,j} + \Omega_{ij}\dot{\nu}_{i,j} - q_{i,i} - \Upsilon_i\dot{\nu}_i, \tag{7}$$

where ε_{ji} and $\Upsilon_i,\ i = 1,2,3$ satisfy the relations

$$\begin{aligned}\varepsilon_{ji} = u_{i,j} - \varepsilon_{kji}\varphi_k, \Omega_{1j,j} + \Upsilon_1 + \rho\Lambda_1 = \rho\kappa_1\ddot{\nu}_1, \\ \Omega_{2j,j} + \Upsilon_2 + \rho\Lambda_2 = \rho\kappa_2\ddot{\nu}_2, \Omega_{3j,j} + \Upsilon_3 + \rho\Lambda_3 = \rho\kappa_3\ddot{\nu}_3.\end{aligned} \tag{8}$$

Following Nowacki [26], the balance of entropy can be composed as

$$\int_V \rho\dot{S}\mathrm{d}V + \int_A \left(\frac{q_i}{T}\right)\varpi_i\mathrm{d}A - \int_A \left(\frac{P\eta_i}{T}\right)\varpi_i\mathrm{d}A = \int_V \left[-\frac{q_i}{T^2}T_{,i} - \frac{P_{,i}}{T}\eta_i + \frac{P}{T^2}\eta_i T_{,i}\right]\mathrm{d}V. \tag{9}$$

where S, P, are entropy and chemical potential per unit mass respectively, η_i are the components of mass diffusion flux vector $\boldsymbol{\eta}$, T is absolute temperature.

The equation (9) can be written in the local form

$$\rho\dot{S} + \left(\frac{q_i}{T}\right)_{,i} - \left(\frac{P\eta_i}{T}\right)_{,i} = -\frac{q_i}{T^2}T_{,i} - \frac{P_{,i}}{T}\eta_i + \frac{P}{T^2}\eta_i T_{,i}. \tag{10}$$

The right hand side of equation (10) is the entropy source

$$\Re = -\frac{q_i}{T^2}T_{,i} - \frac{P_{,i}}{T}\eta_i + \frac{P}{T^2}\eta_i T_{,i} \geq 0.$$

On the basis of above relation, equation (10) can be represented in the form of an inequality called Clausius-Duhem inequality

$$\rho\dot{S} + \frac{q_{i,i}}{T} - \frac{q_i}{T^2}T_{,i} - \frac{P}{T}\eta_{i,i} - \frac{P_{,i}}{T}\eta_i + \frac{P}{T^2}\eta_i T_{,i} \geq 0. \tag{11}$$

The equation of conservation of mass is

$$\eta_{j,j} = -\dot{C}, \tag{12}$$

where C is the concentration of the diffusion material in the elastic body.

Inequality (11) with the help of equations (7) and (12) becomes

$$\rho T\dot{S}-\rho\dot{U}+\sigma_{ji}\dot{\varepsilon}_{ji}+\mu_{ji}\dot{\varphi}_{i,j}+\Omega_{ij}\dot{\nu}_{i,j}-\Upsilon_i\dot{\nu}_i-\frac{q_i}{T}T_{,i}+P\dot{C}-P_{,i}\eta_i+\frac{P}{T}\eta_i T_{,i}\ \geq\ 0, \tag{13}$$

Helmholtz free energy function Γ is stated as

$$\Gamma = U-TS. \tag{14}$$

Then, inequality (13) in the context of linear theory becomes

$$-\rho[\dot{\Gamma}+\dot{T}S]+\sigma_{ji}\dot{\varepsilon}_{ji}+\mu_{ji}\dot{\varphi}_{i,j}+\Omega_{ij}\dot{\nu}_{i,j}-\Upsilon_i\dot{\nu}_i-\frac{q_i}{T}T_{,i}+P\dot{C}-P_{,i}\eta_i\ \geq\ 0. \tag{15}$$

The function Γ can be expressed in terms of independent variables $\varepsilon_{ji},\varphi_{i,j},\nu_i,\nu_{i,j}$,$T,T_{,i},C$ and $C_{,i}$. Therefore, we have

$$\dot{\Gamma}=\frac{\partial\Gamma}{\partial\varepsilon_{ji}}\dot{\varepsilon}_{ji}+\frac{\partial\Gamma}{\partial\varphi_{i,j}}\dot{\varphi}_{i,j}+\frac{\partial\Gamma}{\partial\nu_i}\dot{\nu}_i+\frac{\partial\Gamma}{\partial\nu_{i,j}}\dot{\nu}_{i,j}+\frac{\partial\Gamma}{\partial T}\dot{T}+\frac{\partial\Gamma}{\partial T_{,i}}\dot{T}_{,i}+\frac{\partial\Gamma}{\partial C}\dot{C}+\frac{\partial\Gamma}{\partial C_{,i}}\dot{C}_{,i}. \tag{16}$$

Inequality (15) with the help of equation (16) becomes

$$\left[\sigma_{ji}-\rho\frac{\partial\Gamma}{\partial\varepsilon_{ji}}\right]\dot{\varepsilon}_{ji}+\left[\mu_{ji}-\rho\frac{\partial\Gamma}{\partial\varphi_{i,j}}\right]\dot{\varphi}_{i,j}+\left[\Omega_{ij}-\rho\frac{\partial\Gamma}{\partial\nu_{i,j}}\right]\dot{\nu}_{i,j}-\left[\Upsilon_i+\rho\frac{\partial\Gamma}{\partial\nu_i}\right]\dot{\nu}_i$$

$$-\rho\left[S+\frac{\partial\Gamma}{\partial T}\right]\dot{T}+\left[P-\rho\frac{\partial\Gamma}{\partial C}\right]\dot{C}-\rho\frac{\partial\Gamma}{\partial T_{,i}}\dot{T}_{,i}-\rho\frac{\partial\Gamma}{\partial C_{,i}}\dot{C}_{,i}$$

$$-\frac{q_i}{T}T_{,i}-P_{,i}\eta_i\ \geq\ 0.$$

Let us introduce the notations

$$\boldsymbol{\phi}=\boldsymbol{\nu}-\boldsymbol{\nu}_0,\quad \theta=T-T_0,$$

where $\boldsymbol{\phi}=(\phi_1,\phi_2,\phi_3)$, T_0 is the reference temperature of the body chosen such that $|\frac{\theta}{T_0}|\ll 1$, $\boldsymbol{\nu}_0$ are the volume fraction fields in reference configuration.
In the linear theory, the independent variables are $\varepsilon_{ji},\varphi_{i,j},\phi_i,\phi_{i,j},\theta$ and C. The above inequality should be convinced for all rates $\dot{\varepsilon}_{ji},\dot{\varphi}_{i,j},\dot{\phi}_i,\dot{\phi}_{i,j},\dot{\theta},\dot{\theta}_{,i},\dot{C}$ and $\dot{C}_{,i}$. Hence the coefficients of above variables must vanish, that is,

$$\sigma_{ji}=\rho\frac{\partial\Gamma}{\partial\varepsilon_{ji}},\quad \mu_{ji}=\rho\frac{\partial\Gamma}{\partial\varphi_{i,j}},\quad \Omega_{ij}=\rho\frac{\partial\Gamma}{\partial\phi_{i,j}},\quad \Upsilon_i=-\rho\frac{\partial\Gamma}{\partial\phi_i},\quad S=-\frac{\partial\Gamma}{\partial\theta},\quad P=\rho\frac{\partial\Gamma}{\partial C},\frac{\partial\Gamma}{\partial\theta_{,i}}=\frac{\partial\Gamma}{\partial C_{,i}}=0,$$

$$q_i\theta_{,i}+P_{,i}\theta\eta_i\ \leq\ 0. \tag{17}$$

It is assumed that the undeformed body is free from stresses and has zero intrinsic equilibrated body forces and entropy. If the body has a centre of symmetry, then we have

$$\begin{aligned}2\rho\Gamma=&c_{jikl}\varepsilon_{ji}\varepsilon_{kl}+d_{jikl}\varphi_{j,i}\varphi_{k,l}-2a_{ji}\varepsilon_{ji}\theta-2b_{ji}\varepsilon_{ji}C+2c_{ji}\varepsilon_{ji}\phi_1+2d_{ji}\varepsilon_{ji}\phi_2\\&+2f_{ji}\varepsilon_{ji}\phi_3+\alpha_i\phi_i^2+2\alpha_4\phi_1\phi_2+2\alpha_5\phi_2\phi_3+2\alpha_6\phi_3\phi_1\\&+A_{ij}\phi_{1,i}\phi_{1,j}+B_{ij}\phi_{2,i}\phi_{2,j}+C_{ij}\phi_{3,i}\phi_{3,j}+2D_{ij}\phi_{1,i}\phi_{2,j}\\&+2E_{ij}\phi_{2,i}\phi_{3,j}+2F_{ij}\phi_{3,i}\phi_{1,j}-2\ell_i\phi_i\theta-2\varepsilon_i\phi_iC-\frac{\rho C_e\theta^2}{T_0}-2a\theta C+bC^2.\end{aligned}$$

Here c_{jikl} is the tensor of elastic constants, d_{jikl} is the tensor of microrotation constants, a_{ji}, b_{ji} are, respectively the tensors of thermal and diffusion expansions, $c_{ji}, d_{ji}, f_{ji}, A_{ij}, B_{ij}, C_{ij}, D_{ij}, E_{ij}, F_{ij}$ and α_i, $i = 1, \ldots, 6$ are functions which are typical in porous theories and ℓ_i, ε_i are coupling constants. The constitutive coefficients have the following symmetries

$$\begin{gathered} a_{ji} = a_{ij}, \quad c_{jikl} = c_{ijkl} = c_{klji}, \\ b_{ji} = b_{ij}, \quad d_{jikl} = d_{ijkl} = d_{klji}, \\ c_{ji} = c_{ij}, \quad d_{ji} = d_{ij}, \quad f_{ji} = f_{ij}, \\ A_{ij} = A_{ji}, \quad B_{ij} = B_{ji}, \quad C_{ij} = C_{ji}, \\ D_{ij} = D_{ji}, \quad E_{ij} = E_{ji}, \quad F_{ij} = F_{ji}. \end{gathered}$$

Using the previous equation in the system of equations (17), the following constitutive equations are obtained:

$$\sigma_{ji} = c_{jikl}\varepsilon_{kl} + c_{ji}\phi_1 + d_{ji}\phi_2 + f_{ji}\phi_3 - a_{ji}\theta - b_{ji}C, \tag{18}$$

$$\mu_{ji} = d_{ijkl}\varphi_{k,l}, \tag{19}$$

$$\begin{aligned} \Omega_{1j} &= A_{ij}\phi_{1,i} + D_{ij}\phi_{2,i} + F_{ij}\phi_{3,i}, \\ \Omega_{2j} &= D_{ij}\phi_{1,i} + B_{ij}\phi_{2,i} + E_{ij}\phi_{3,i}, \\ \Omega_{3j} &= F_{ij}\phi_{1,i} + E_{ij}\phi_{2,i} + C_{ij}\phi_{3,i}, \end{aligned} \tag{20}$$

$$\begin{aligned} \Upsilon_1 &= -c_{ji}\varepsilon_{ji} - \alpha_1\phi_1 - \alpha_4\phi_2 - \alpha_6\phi_3 + \ell_1\theta + \varepsilon_1 C, \\ \Upsilon_2 &= -d_{ji}\varepsilon_{ji} - \alpha_4\phi_1 - \alpha_2\phi_2 - \alpha_5\phi_3 + \ell_2\theta + \varepsilon_2 C, \\ \Upsilon_3 &= -f_{ji}\varepsilon_{ji} - \alpha_6\phi_1 - \alpha_5\phi_2 - \alpha_3\phi_3 + \ell_3\theta + \varepsilon_3 C, \end{aligned} \tag{21}$$

$$\rho S = a_{ji}\varepsilon_{ji} + \ell_i\phi_i + \frac{\rho C_e\theta}{T_0} + aC, \tag{22}$$

$$P = -b_{ji}\varepsilon_{ji} - \varepsilon_i\phi_i - a\theta + bC. \tag{23}$$

Equations (1), (2) and (8) with the aid of equations (18)-(21) become

$$c_{jikl}\varepsilon_{kl,j} + c_{ji}\phi_{1,j} + d_{ji}\phi_{2,j} + f_{ji}\phi_{3,j} - a_{ji}\theta_{,j} - b_{ji}C_{,j} + \rho\tilde{F}_i = \rho\ddot{u}_i, \tag{24}$$

$$\varepsilon_{ijk}(c_{jkpl}\varepsilon_{pl} + c_{jk}\phi_1 + d_{jk}\phi_2 + f_{jk}\phi_3 - a_{jk}\theta - b_{jk}C) + d_{ijkl}\varphi_{k,lj} + \rho\tilde{G}_i = \rho J_{ij}\ddot{\varphi}_j, \tag{25}$$

$$\begin{aligned} -c_{ji}\varepsilon_{ji} + A_{ij}\phi_{1,ij} + D_{ij}\phi_{2,ij} + F_{ij}\phi_{3,ij} - \alpha_1\phi_1 - \alpha_4\phi_2 - \alpha_6\phi_3 + \ell_1\theta + \varepsilon_1 C + \rho\Lambda_1 &= \rho\kappa_1\ddot{\phi}_1, \\ -d_{ji}\varepsilon_{ji} + D_{ij}\phi_{1,ij} + B_{ij}\phi_{2,ij} + E_{ij}\phi_{3,ij} - \alpha_4\phi_1 - \alpha_2\phi_2 - \alpha_5\phi_3 + \ell_2\theta + \varepsilon_2 C + \rho\Lambda_2 &= \rho\kappa_2\ddot{\phi}_2, \\ -f_{ji}\varepsilon_{ji} + F_{ij}\phi_{1,ij} + E_{ij}\phi_{2,ij} + C_{ij}\phi_{3,ij} - \alpha_6\phi_1 - \alpha_5\phi_2 - \alpha_3\phi_3 + \ell_3\theta + \varepsilon_3 C + \rho\Lambda_3 &= \rho\kappa_3\ddot{\phi}_3. \end{aligned} \tag{26}$$

The linearized form of inequality (11) is

$$\rho T_0\dot{S} = -q_{i,i}. \tag{27}$$

Using equation (22) in equation (27), we get

$$T_0\left[a_{ji}\dot{\varepsilon}_{ij} + \ell_i\dot{\phi}_i + a\dot{C}\right] + \rho C_e\dot{\theta} = -q_{i,i}. \tag{28}$$

Generalized Fourier's law of heat conduction equation is

$$q_i + \tau_0\dot{q}_i = -K_{ij}\theta_{,j}, \tag{29}$$

where $K_{ij} = K_{ji}$ are coefficients of thermal conductivity tensor, τ_0 is the thermal relaxation time which will ensure that the heat conduction equation will predict finite speeds of heat propagation.
Equation (29) with the help of equation (28) becomes

$$\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left[T_0(a_{ji}\varepsilon_{ji} + \ell_i\phi_i + aC) + \rho C_e\theta\right] = K_{ij}\theta_{,ij}. \tag{30}$$

Similar to equation (29), generalized Fick's law of mass diffusion is

$$\eta_i + \tau^0\dot{\eta}_i = -l_{ij}P_{,j}, \tag{31}$$

where $l_{ij} = l_{ji}$ are coefficients of diffusion tensor, τ^0 is the diffusion relaxation time which ensures that the equation satisfied by the concentration will also predict finite speeds of propagation of matter from one medium to the other.
Using equations (12) and (23) in equation (31), we get

$$-l_{ij}[b_{kl}\varepsilon_{kl,ij} + \varepsilon_k\phi_{k,ij} + a\theta_{,ij} - bC_{,ij}] = \dot{C} + \tau^0\ddot{C}. \tag{32}$$

If we take

$$\begin{gathered} c_{jikl} = \lambda\delta_{ji}\delta_{kl} + (\mu + K^*)\delta_{jk}\delta_{il} + \mu\delta_{jl}\delta_{ik}, \\ K_{ij} = K\delta_{ij}, \quad l_{ij} = D\delta_{ij}, \quad J_{ij} = J\delta_{ij}, \end{gathered}$$

$$d_{jikl} = \alpha\delta_{ji}\delta_{kl} + \gamma\delta_{jk}\delta_{il} + \beta\delta_{jl}\delta_{ik},$$
$$a_{ij} = \vartheta_1\delta_{ij}, \quad b_{ij} = \vartheta_2\delta_{ij}, \quad c_{ij} = \Re_1\delta_{ij},$$
$$d_{ij} = \Re_2\delta_{ij}, \quad f_{ij} = \Re_3\delta_{ij},$$
$$A_{ij} = A_1\delta_{ij}, \quad B_{ij} = A_2\delta_{ij}, \quad C_{ij} = A_3\delta_{ij},$$
$$D_{ij} = A_4\delta_{ij}, \quad E_{ij} = A_5\delta_{ij}, \quad F_{ij} = A_6\delta_{ij},$$

where δ_{ij} is Kronecker's delta and $\lambda, \mu, K^*,\ \alpha, \beta, \gamma, K, D, J, \vartheta_1, \vartheta_2, A_1, \ldots, A_6,\ \Re_1, \Re_2, \Re_3$ are material constants, in the equations (24)-(26), (30) and (32), the governing equations for homogeneous isotropic generalized micropolar thermoelastic diffusion with triple porosity in absence of body forces and couples are obtained as

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$$(\mu + K^*)\Delta\mathbf{u} + (\lambda + \mu)\nabla \operatorname{div} \mathbf{u} + K^* \operatorname{curl} \boldsymbol{\varphi} + \Re_i \nabla\phi_i - \vartheta_1\nabla\theta - \vartheta_2\nabla C = \rho\ddot{\mathbf{u}},$$

$$K^* \operatorname{curl} \mathbf{u} + (\gamma\Delta - 2K^*)\boldsymbol{\varphi} + (\alpha + \beta)\,\nabla \operatorname{div} \boldsymbol{\varphi} = \rho J\ddot{\boldsymbol{\varphi}},$$

$$\begin{aligned}
-\Re_1 \operatorname{div} \mathbf{u} + (A_1\,\Delta - \alpha_1)\phi_1 + (A_4\,\Delta - \alpha_4)\phi_2 + (A_6\,\Delta - \alpha_6)\phi_3 + \ell_1\theta + \varepsilon_1 C &= \rho\kappa_1\ddot{\phi}_1,\\
-\Re_2 \operatorname{div} \mathbf{u} + (A_4\,\Delta - \alpha_4)\phi_1 + (A_2\,\Delta - \alpha_2)\phi_2 + (A_5\,\Delta - \alpha_5)\phi_3 + \ell_2\theta + \varepsilon_2 C &= \rho\kappa_2\ddot{\phi}_2,\\
-\Re_3 \operatorname{div} \mathbf{u} + (A_6\,\Delta - \alpha_6)\phi_1 + (A_5\,\Delta - \alpha_5)\phi_2 + (A_3\,\Delta - \alpha_3)\phi_3 + \ell_3\theta + \varepsilon_3 C &= \rho\kappa_3\ddot{\phi}_3,
\end{aligned}$$

$$\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)\left[T_0(\vartheta_1 \operatorname{div} \mathbf{u} + \ell_i\phi_i + aC) + \rho C_e\theta\right] = K\Delta\theta,$$

$$D\Delta[\vartheta_2 \operatorname{div} \mathbf{u} + \varepsilon_i\phi_i + a\theta - bC] + \left(\frac{\partial}{\partial t} + \tau^0\frac{\partial^2}{\partial t^2}\right)C = 0, \tag{33}$$

where Δ, ∇ are respectively, Laplacian and Del operators.
In the upcoming sections, the chemical potential has been used as a state variable rather than concentration. In isotropic medium, equation (23) becomes

$$P = -\vartheta_2 \operatorname{div} \mathbf{u} - \varepsilon_k\phi_k - a\theta + bC. \tag{34}$$

The system of equations (33) with the aid of equation (34) can be rewritten as

$$(\mu + K^*)\Delta\mathbf{u} + (\lambda' + \mu)\nabla \operatorname{div} \mathbf{u} + K^* \operatorname{curl} \boldsymbol{\varphi} + \sigma_i \nabla\phi_i - \zeta_1\nabla\theta - \zeta_2\nabla P = \rho\ddot{\mathbf{u}},$$

$$K^* \operatorname{curl} \mathbf{u} + (\gamma\Delta - 2K^*)\boldsymbol{\varphi} + (\alpha + \beta)\,\nabla \operatorname{div} \boldsymbol{\varphi} = \rho J\ddot{\boldsymbol{\varphi}},$$

$$\begin{aligned}
-\sigma_1 \operatorname{div} \mathbf{u} + (A_1\,\Delta - \beta_1)\phi_1 + (A_4\,\Delta - \beta_4)\phi_2 + (A_6\,\Delta - \beta_6)\phi_3 + \xi_1\theta + v_1 P &= \rho\kappa_1\ddot{\phi}_1,\\
-\sigma_2 \operatorname{div} \mathbf{u} + (A_4\,\Delta - \beta_4)\phi_1 + (A_2\,\Delta - \beta_2)\phi_2 + (A_5\,\Delta - \beta_5)\phi_3 + \xi_2\theta + v_2 P &= \rho\kappa_2\ddot{\phi}_2,\\
-\sigma_3 \operatorname{div} \mathbf{u} + (A_6\,\Delta - \beta_6)\phi_1 + (A_5\,\Delta - \beta_5)\phi_2 + (A_3\,\Delta - \beta_3)\phi_3 + \xi_3\theta + v_3 P &= \rho\kappa_3\ddot{\phi}_3,
\end{aligned}$$

$$\begin{aligned}
-\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T_0\left[\zeta_1 \operatorname{div} \mathbf{u} + \xi_i\phi_i + \eta\theta + \varsigma P\right] + K\Delta\theta &= 0,\\
-\left(\frac{\partial}{\partial t} + \tau^0\frac{\partial^2}{\partial t^2}\right)\left[\zeta_2 \operatorname{div} \mathbf{u} + v_i\phi_i + \varsigma\theta + \varpi P\right] + D\Delta P &= 0,
\end{aligned} \tag{35}$$

where

$$\varpi = b^{-1}, \quad \zeta_2 = \varpi\vartheta_2, \quad \zeta_1 = \vartheta_1 + a\,\zeta_2, \quad \sigma_i = \Re_i - \varepsilon_i\zeta_2, \quad \lambda' = \lambda - \zeta_2\vartheta_2,$$
$$\varsigma = a\varpi, \quad v_i = \varepsilon_i\varpi, \quad \beta_i = \alpha_i - \varepsilon_i v_i, \quad \beta_4 = \alpha_4 - \varepsilon_1 v_2, \quad \beta_5 = \alpha_5 - \varepsilon_2 v_3,$$
$$\beta_6 = \alpha_6 - \varepsilon_3 v_1, \quad \xi_i = \ell_i + \varsigma\varepsilon_i, \quad \eta = \frac{\rho C_e}{T_0} + a\varsigma \quad i = 1, 2, 3.$$

III. Steady Oscillations

Let $\mathbf{x} = (x_1, x_2, x_3)$ be the point of the Euclidean three-dimensional space E^3,

$$|\mathbf{x}| = (x_1^2 + x_2^2 + x_3^2)^{\frac{1}{2}}, \quad \mathbf{D_x} = (\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}).$$

The displacement vector, microrotation vector, volume fraction fields, temperature change and chemical potential functions are assumed as:

$$\Big[\mathbf{u}(\mathbf{x},t), \boldsymbol{\varphi}(\mathbf{x},t), \boldsymbol{\phi}(\mathbf{x},t), \theta(\mathbf{x},t), P(\mathbf{x},t)\Big] = \mathrm{Re}\Big[(\mathbf{u}^*, \boldsymbol{\varphi}^*, \boldsymbol{\phi}^*, \theta^*, P^*)e^{-\iota\omega t}\Big], \tag{36}$$

where ω is oscillation frequency.
Using equation (36) in the system of equations (35) and omitting asterisk (*) for simplicity, the system of equations of steady oscillations are obtained as

$$(\mu + K^*)\Delta\mathbf{u} + [(\lambda' + \mu)\nabla\,\mathrm{div} + \rho\omega^2]\mathbf{u} + K^*\,\mathrm{curl}\,\boldsymbol{\varphi} + \sigma_i\nabla\phi_i - \zeta_1\nabla\theta - \zeta_2\nabla P = \mathbf{0},$$

$$K^*\,\mathrm{curl}\,\mathbf{u} + \gamma\Delta\boldsymbol{\varphi} + [(\alpha + \beta)\,\nabla\,\mathrm{div} - 2K^* + \rho J\omega^2]\boldsymbol{\varphi} = \mathbf{0},$$

$$\begin{aligned}
-\sigma_1\,\mathrm{div}\,\mathbf{u} + (A_1\,\Delta - \gamma_1)\phi_1 + (A_4\,\Delta - \beta_4)\phi_2 + (A_6\,\Delta - \beta_6)\phi_3 + \xi_1\theta + v_1 P &= 0,\\
-\sigma_2\,\mathrm{div}\,\mathbf{u} + (A_4\,\Delta - \beta_4)\phi_1 + (A_2\,\Delta - \gamma_2)\phi_2 + (A_5\,\Delta - \beta_5)\phi_3 + \xi_2\theta + v_2 P &= 0,\\
-\sigma_3\,\mathrm{div}\,\mathbf{u} + (A_6\,\Delta - \beta_6)\phi_1 + (A_5\,\Delta - \beta_5)\phi_2 + (A_3\,\Delta - \gamma_3)\phi_3 + \xi_3\theta + v_3 P &= 0,
\end{aligned}$$

$$\tau_1 T_0[\zeta_1\,\mathrm{div}\,\mathbf{u} + \xi_i\phi_i] + [K\Delta + \tau_1\eta T_0]\theta + \tau_1\varsigma T_0 P = 0,$$

$$\tau^1[\zeta_2\,\mathrm{div}\,\mathbf{u} + v_i\phi_i + \varsigma\theta] + [D\Delta + \tau^1\varpi]P = 0, \tag{37}$$

where $\gamma_i = \beta_i - \rho\kappa_i\omega^2,\ \tau_1 = \iota\omega(1 - \iota\omega\tau_0),\ \tau^1 = \iota\omega(1 - \iota\omega\tau^0)$.

If we replace ω by $-\iota\tau$, where τ is a complex number and $Re(\tau) > 0$ in the equations (37), we obtain the system of equations of pseudo-oscillations as:

$$(\mu + K^*)\Delta\mathbf{u} + [(\lambda' + \mu)\nabla\,\mathrm{div} - \rho\tau^2]\mathbf{u} + K^*\,\mathrm{curl}\,\boldsymbol{\varphi} + \sigma_i\nabla\phi_i - \zeta_1\nabla\theta - \zeta_2\nabla P = \mathbf{0},$$

$$K^*\,\mathrm{curl}\,\mathbf{u} + \gamma\Delta\boldsymbol{\varphi} + [(\alpha + \beta)\,\nabla\,\mathrm{div} - 2K^* - \rho J\tau^2]\boldsymbol{\varphi} = \mathbf{0},$$

$$\begin{aligned}
-\sigma_1\,\mathrm{div}\,\mathbf{u} + (A_1\,\Delta - \tilde{\gamma}_1)\phi_1 + (A_4\,\Delta - \beta_4)\phi_2 + (A_6\,\Delta - \beta_6)\phi_3 + \xi_1\theta + v_1 P &= 0,\\
-\sigma_2\,\mathrm{div}\,\mathbf{u} + (A_4\,\Delta - \beta_4)\phi_1 + (A_2\,\Delta - \tilde{\gamma}_2)\phi_2 + (A_5\,\Delta - \beta_5)\phi_3 + \xi_2\theta + v_2 P &= 0,\\
-\sigma_3\,\mathrm{div}\,\mathbf{u} + (A_6\,\Delta - \beta_6)\phi_1 + (A_5\,\Delta - \beta_5)\phi_2 + (A_3\,\Delta - \tilde{\gamma}_3)\phi_3 + \xi_3\theta + v_3 P &= 0,
\end{aligned}$$

$$\tilde{\tau}_1 T_0[\zeta_1\,\mathrm{div}\,\mathbf{u} + \xi_i\phi_i] + [K\Delta + \tilde{\tau}_1\eta T_0]\theta + \tilde{\tau}_1\varsigma T_0 P = 0,$$

$$\tilde{\tau}^1[\zeta_2\,\mathrm{div}\,\mathbf{u} + v_i\phi_i + \varsigma\theta] + [D\Delta + \tilde{\tau}^1\varpi]P = 0, \tag{38}$$

where $\tilde{\gamma}_i = \beta_i + \rho\kappa_i\tau^2,\ \tilde{\tau}_1 = \tau(1 - \tau\tau_0),\ \tilde{\tau}^1 = \tau(1 - \tau\tau^0)$.

If we put $\rho = 0$ i.e. neglecting inertial effect in the equations (37), we obtain the system of equations of quasi-static oscillations as:

$$[(\mu + K^*)\Delta + (\lambda' + \mu)\nabla\,\mathrm{div}]\mathbf{u} + K^*\,\mathrm{curl}\,\boldsymbol{\varphi} + \sigma_i\nabla\phi_i - \zeta_1\nabla\theta - \zeta_2\nabla P = \mathbf{0},$$

$$K^*\,\mathrm{curl}\,\mathbf{u} + \gamma\Delta\boldsymbol{\varphi} + [(\alpha+\beta)\,\nabla\,\mathrm{div} - 2K^*]\boldsymbol{\varphi} = \mathbf{0},$$

$$\begin{aligned}
-\sigma_1\,\mathrm{div}\,\mathbf{u} + (A_1\,\Delta - \beta_1)\phi_1 + (A_4\,\Delta - \beta_4)\phi_2 + (A_6\,\Delta - \beta_6)\phi_3 + \xi_1\theta + v_1 P &= 0,\\
-\sigma_2\,\mathrm{div}\,\mathbf{u} + (A_4\,\Delta - \beta_4)\phi_1 + (A_2\,\Delta - \beta_2)\phi_2 + (A_5\,\Delta - \beta_5)\phi_3 + \xi_2\theta + v_2 P &= 0,\\
-\sigma_3\,\mathrm{div}\,\mathbf{u} + (A_6\,\Delta - \beta_6)\phi_1 + (A_5\,\Delta - \beta_5)\phi_2 + (A_3\,\Delta - \beta_3)\phi_3 + \xi_3\theta + v_3 P &= 0,
\end{aligned}$$

$$\tau_1 T_0[\zeta_1\,\mathrm{div}\,\mathbf{u} + \xi_i\phi_i] + [K\Delta + \tau_1 a\varsigma T_0]\theta + \tau_1\varsigma T_0 P = 0,$$

$$\tau^1[\zeta_2\,\mathrm{div}\,\mathbf{u} + v_i\phi_i + \varsigma\theta] + [D\Delta + \tau^1\varpi]P = 0. \tag{39}$$

If we put $\omega = 0$ in the equations (37), we obtain the system of equations in equilibrium theory of micropolar thermoelastic diffusion with triple porosity as:

$$[(\mu + K^*)\Delta + (\lambda' + \mu)\nabla\,\mathrm{div}]\mathbf{u} + K^*\,\mathrm{curl}\,\boldsymbol{\varphi} + \sigma_i\nabla\phi_i - \zeta_1\nabla\theta - \zeta_2\nabla P = \mathbf{0},$$

$$K^*\,\mathrm{curl}\,\mathbf{u} + \gamma\Delta\boldsymbol{\varphi} + [(\alpha+\beta)\,\nabla\,\mathrm{div} - 2K^*]\boldsymbol{\varphi} = \mathbf{0},$$

$$\begin{aligned}
-\sigma_1\,\mathrm{div}\,\mathbf{u} + (A_1\,\Delta - \beta_1)\phi_1 + (A_4\,\Delta - \beta_4)\phi_2 + (A_6\,\Delta - \beta_6)\phi_3 + \xi_1\theta + v_1 P &= 0,\\
-\sigma_2\,\mathrm{div}\,\mathbf{u} + (A_4\,\Delta - \beta_4)\phi_1 + (A_2\,\Delta - \beta_2)\phi_2 + (A_5\,\Delta - \beta_5)\phi_3 + \xi_2\theta + v_2 P &= 0,\\
-\sigma_3\,\mathrm{div}\,\mathbf{u} + (A_6\,\Delta - \beta_6)\phi_1 + (A_5\,\Delta - \beta_5)\phi_2 + (A_3\,\Delta - \beta_3)\phi_3 + \xi_3\theta + v_3 P &= 0,
\end{aligned}$$

$$\begin{aligned}
K\Delta\theta &= 0,\\
D\Delta P &= 0.
\end{aligned} \tag{40}$$

We introduce the second order matrix differential operators with constant coefficients

$$\mathbf{F}^{(i)}(\mathbf{D_x}) = \left(F^{(i)}_{gh}(\mathbf{D_x})\right)_{11\times 11},$$

where $F^{(i)}_{gh}(\mathbf{D_x}),\ g,h = 1,...,11$ are given in Appendix A.

Here $i = 1,2,3,4$ corresponds to static, pseudo-, quasi-static oscillations and equilibrium theory of micropolar thermoelastic diffusion with triple porosity respectively. The matrices $\mathbf{F}^{(i)}(\mathbf{D_x}),\ i = 2,3,4$ can be obtained from matrix $\mathbf{F}^{(1)}(\mathbf{D_x})$ by taking $\omega = -\iota\tau,\ \rho = 0$ and $\omega = 0$ respectively.

and

$$\tilde{\mathbf{F}}(\mathbf{D_x}) = \left(\tilde{F}_{gh}(\mathbf{D_x})\right)_{11\times 11},$$

where $\tilde{F}_{gh}(\mathbf{D_x}),\ g,h = 1,...,11$ are given in Appendix A.

The system of equations (37)-(40) can be represented as

$$\mathbf{F}^{(i)}(\mathbf{D_x})\mathbf{U}(\mathbf{x}) = \mathbf{0}, \quad i = 1, 2, 3, 4,$$

where $\mathbf{U} = (\mathbf{u}, \boldsymbol{\varphi}, \boldsymbol{\phi}, \theta, P)$ is a eleven-component vector function on E^3. The matrix $\tilde{\mathbf{F}}(\mathbf{D_x})$ is called the principal part of operator $\mathbf{F}^{(i)}(\mathbf{D_x})$.

Definition 1: The operator $\mathbf{F}^{(i)}(\mathbf{D_x}),\ i = 1, 2, 3, 4$ is said to be elliptic if $|\tilde{\mathbf{F}}(\mathbf{m})| \neq 0$, where $\mathbf{m} = (m_1, m_2, m_3)$.

Since $|\tilde{\mathbf{F}}(\mathbf{m})| = \tilde{\mu}^2\gamma^2\tilde{\lambda}\tilde{\alpha}KD\varrho|\mathbf{m}|^{22}, \quad \tilde{\lambda} = \lambda' + 2\mu + K^*, \quad \tilde{\alpha} = \alpha + \beta + \gamma, \quad \varrho = \begin{vmatrix} A_1 & A_4 & A_6 \\ A_4 & A_2 & A_5 \\ A_6 & A_5 & A_3 \end{vmatrix}$, therefore operator $\mathbf{F}^{(i)}(\mathbf{D_x})$ is an elliptic differential operator if

$$\tilde{\mu}\gamma\tilde{\lambda}\tilde{\alpha}KD\varrho \neq 0. \tag{41}$$

Definition 2: The fundamental solutions of the system of equations (37)-(40) (fundamental matrices of operators $\mathbf{F}^{(i)}$) are the matrices $\mathbf{G}^{(i)}(\mathbf{x}) = \left(G^{(i)}_{gh}(\mathbf{x})\right)_{11\times 11}$ satisfying conditions

$$\mathbf{F}^{(i)}(\mathbf{D_x})\mathbf{G}^{(i)}(\mathbf{x}) = \delta(\mathbf{x})\,\mathbf{I}(\mathbf{x}), \quad i = 1, 2, 3, 4, \tag{42}$$

where $\delta(\mathbf{x})$ is the Dirac delta, $\mathbf{I} = (\delta_{gh})_{11\times 11}$ is the unit matrix and $\mathbf{x} \in \mathrm{E}^3$.

IV. Construction of $\mathbf{G}^{(1)}(\mathbf{x})$ in terms of Elementary Functions

Let us consider the system of non-homogeneous equations

$$\tilde{\mu}\Delta\mathbf{u} + [(\lambda' + \mu)\nabla\,\mathrm{div} + \rho\omega^2]\mathbf{u} + K^*\,\mathrm{curl}\,\boldsymbol{\varphi} - \sigma_i\nabla\phi_i + \tau_1\zeta_1T_0\nabla\theta + \tau^1\zeta_2\nabla P = \mathbf{H}, \tag{43}$$

$$K^*\,\mathrm{curl}\,\mathbf{u} + [\gamma\Delta + \tilde{K} + (\alpha + \beta)\,\nabla\,\mathrm{div}]\boldsymbol{\varphi} = \mathbf{V}, \tag{44}$$

$$\sigma_1\,\mathrm{div}\,\mathbf{u} + (A_1\,\Delta - \gamma_1)\phi_1 + (A_4\,\Delta - \beta_4)\phi_2 + (A_6\,\Delta - \beta_6)\phi_3 + \tau_1T_0\xi_1\theta + \tau^1v_1P = X_1, \tag{45}$$

$$\sigma_2\,\mathrm{div}\,\mathbf{u} + (A_4\,\Delta - \beta_4)\phi_1 + (A_2\,\Delta - \gamma_2)\phi_2 + (A_5\,\Delta - \beta_5)\phi_3 + \tau_1T_0\xi_2\theta + \tau^1v_2P = X_2, \tag{46}$$

$$\sigma_3\,\mathrm{div}\,\mathbf{u} + (A_6\,\Delta - \beta_6)\phi_1 + (A_5\,\Delta - \beta_5)\phi_2 + (A_3\,\Delta - \gamma_3)\phi_3 + \tau_1T_0\xi_3\theta + \tau^1v_3P = X_3, \tag{47}$$

$$-\zeta_1\,\mathrm{div}\,\mathbf{u} + \xi_i\phi_i + [K\Delta + \tau_1\eta T_0]\theta + \tau^1\varsigma P = Y, \tag{48}$$

$$-\zeta_2\,\mathrm{div}\,\mathbf{u} + v_i\phi_i + \tau_1\varsigma T_0\theta + [D\Delta + \tau^1\varpi]P = Z, \tag{49}$$

where $\mathbf{H}, \mathbf{V}$ are three-component vector function on E^3; X_i, Y and Z are scalar functions on E^3.

The system of equations (43)-(49) may also be written in the form

$$\mathbf{F}^{(1)^{tr}}(\mathbf{D_x})\mathbf{U}(\mathbf{x}) = \mathbf{Q}(\mathbf{x}), \tag{50}$$

where $\mathbf{F}^{(1)^{tr}}$ is the transpose of matrix $\mathbf{F}^{(1)}$, $\mathbf{Q} = (\mathbf{H}, \mathbf{V}, X_i, Y, Z)$ and $\mathbf{x} \in \mathrm{E}^3$.

Applying operator div to the equations (43) and (44), we obtain

$$[\tilde{\lambda}\Delta + \rho\omega^2]\,\mathrm{div}\,\mathbf{u} - \sigma_i\Delta\phi_i + \tau_1\zeta_1T_0\Delta\theta + \tau^1\zeta_2\Delta P = \mathrm{div}\,\mathbf{H}, \tag{51}$$

$$[\tilde{\alpha}\Delta + \tilde{K}]\,\mathrm{div}\,\boldsymbol{\varphi} = \mathrm{div}\,\mathbf{V}. \tag{52}$$

The equations (45)-(49) and (51) may be expressed in the form

$$\mathbf{N}^{(1)}(\Delta)\mathbf{S} = \tilde{\mathbf{Q}}, \tag{53}$$

where $\mathbf{N}^{(1)}$, $\mathbf{S}$ and $\tilde{\mathbf{Q}}$ are given in Appendix B.

The equations (53) may also be written in determinant form as

$$\Gamma^{(1)}(\Delta)\mathbf{S} = \boldsymbol{\Psi}, \tag{54}$$

where $\Gamma^{(1)}$ and $\boldsymbol{\Psi}$ are defined in Appendix B.

On expanding $\Gamma^{(1)}(\Delta)$, we see that

$$\Gamma^{(1)}(\Delta) = \prod_{i=1}^{6}(\Delta + \lambda_i^2),$$

where $\lambda_i^2, \quad i = 1, \ldots, 6$ are the roots of the equation $\Gamma^{(1)}(-m) = 0$(with respect to m).

Applying operators $\gamma\Delta + \tilde{K}$ and K^* curl to the equations (43) and (44), respectively, we obtain

$$\begin{aligned}(\gamma\Delta + \tilde{K})\Big[\tilde{\mu}\Delta + (\lambda' + \mu)\,\nabla\,\mathrm{div} + \rho\omega^2\Big]\mathbf{u} + (\gamma\Delta + \tilde{K})K^*\,\mathrm{curl}\,\boldsymbol{\varphi} \\ = (\gamma\Delta + \tilde{K})\Big[\mathbf{H} + \sigma_i\,\nabla\,\phi_i - \tau_1\zeta_1 T_0\,\nabla\,\theta - \tau^1\zeta_2\,\nabla\,P\Big],\end{aligned} \tag{55}$$

and

$$(\gamma\Delta + \tilde{K})K^*\,\mathrm{curl}\,\boldsymbol{\varphi} = -K^{*^2}\,\mathrm{curl}\,\mathrm{curl}\,\mathbf{u} + K^*\,\mathrm{curl}\,\mathbf{V}. \tag{56}$$

Now

$$\mathrm{curl}\,\mathrm{curl}\,\mathbf{u} = \nabla\,\mathrm{div}\,\mathbf{u} - \Delta\mathbf{u}. \tag{57}$$

Using equations (56) and (57) in equation (55) and applying operator $\Gamma^{(1)}(\Delta)$ to the resulting equation with the help of equation (54), we get

$$\Gamma^{(1)}(\Delta)\Lambda^{(1)}(\Delta)\mathbf{u} = \boldsymbol{\Psi}', \tag{58}$$

where $\Lambda^{(1)}(\Delta)$ and $\boldsymbol{\Psi}'$ are given in Appendix B.

It can be seen that

$$\Lambda^{(1)}(\Delta) = (\Delta + \lambda_7^2)(\Delta + \lambda_8^2),$$

where λ_7^2, λ_8^2 are the roots of the equation $\Lambda^{(1)}(-m) = 0$(with respect to m).
From equation (52), it follows that

$$(\Delta + \lambda_9^2)\,\mathrm{div}\,\boldsymbol{\varphi} = \frac{1}{\tilde{\alpha}}\,\mathrm{div}\,\mathbf{V} = \Psi_7, \lambda_9^2 = \frac{\tilde{K}}{\tilde{\alpha}}. \tag{59}$$

Applying operators K^* curl and $\tilde{\mu}\Delta + \rho\omega^2$ to the equations (43) and (44), respectively, we acquire

$$[\tilde{\mu}\Delta + \rho\omega^2]K^*\,\mathrm{curl}\,\mathbf{u} = K^*\,\mathrm{curl}\,\mathbf{H} - K^{*^2}\,\mathrm{curl}\,\mathrm{curl}\,\boldsymbol{\varphi}, \tag{60}$$

and

$$(\tilde{\mu}\Delta + \rho\omega^2)(\gamma\Delta + \tilde{K})\boldsymbol{\varphi} + (\alpha + \beta)(\tilde{\mu}\Delta + \rho\omega^2)\,\nabla\,\mathrm{div}\,\boldsymbol{\varphi} + K^*(\tilde{\mu}\Delta + \rho\omega^2)\,\mathrm{curl}\,\mathbf{u} = (\tilde{\mu}\Delta + \rho\omega^2)\mathbf{V}. \tag{61}$$

Now

$$\mathrm{curl}\,\mathrm{curl}\,\boldsymbol{\varphi} = \nabla\,\mathrm{div}\,\boldsymbol{\varphi} - \Delta\boldsymbol{\varphi}. \tag{62}$$

Using equations (60) and (62) in equation (61) and applying the operator $\Delta + \lambda_9^2$ to the resulting equation with the help of equation (59), we get

$$\Lambda^{(1)}(\Delta)(\Delta + \lambda_9^2)\boldsymbol{\varphi} = \boldsymbol{\Psi}'', \tag{63}$$

where $\boldsymbol{\Psi}''$ is defined in Appendix B.

From equations (54), (58) and (63), we obtain

$$\boldsymbol{\Theta}^{(1)}(\Delta)\mathbf{U}(\mathbf{x}) = \hat{\boldsymbol{\Psi}}(\mathbf{x}), \tag{64}$$

where $\hat{\boldsymbol{\Psi}}$ and $\boldsymbol{\Theta}^{(1)}(\Delta)$ are given in Appendix C.

The expressions for $\boldsymbol{\Psi}'$, $\boldsymbol{\Psi}''$ and $\Psi_p, p = 2, \ldots, 6$ can be rewritten in the form

$$\begin{aligned}
\boldsymbol{\Psi}' &= \left[\frac{1}{\tilde{N}}(\gamma\Delta + \tilde{K})\Gamma^{(1)}(\Delta)\mathbf{J} + w_{11}^{(1)}(\Delta)\,\nabla\,\mathrm{div}\right]\mathbf{H} + w_{21}^{(1)}(\Delta)\,\mathrm{curl}\,\mathbf{V} + \sum_{i=2}^{6} w_{i+1;1}^{(1)}(\Delta)\,\nabla\,\chi_i, \\
\boldsymbol{\Psi}'' &= w_{12}^{(1)}(\Delta)\,\mathrm{curl}\,\mathbf{H} + \left[\frac{1}{\tilde{N}}(\Delta + \lambda_9^2)(\tilde{\mu}\Delta + \rho\omega^2)\mathbf{J} + w_{22}^{(1)}(\Delta)\,\nabla\,\mathrm{div}\right]\mathbf{V}, \\
\Psi_p &= w_{1;p+1}^{(1)}(\Delta)\,\mathrm{div}\,\mathbf{H} + \sum_{i=2}^{6} w_{i+1;p+1}^{(1)}(\Delta)\chi_i, \quad p = 2, ..., 6,
\end{aligned} \tag{65}$$

where $\boldsymbol{J} = (\delta_{ij})_{3\times 3}$ and $w_{ij}^{(1)}(\Delta),\ i, j = 1, \ldots, 7$ are defined in Appendix C.

From equations (65), we have

$$\hat{\boldsymbol{\Psi}}(\mathbf{x}) = \mathbf{R}^{tr}(\mathbf{D_x})\mathbf{Q}(\mathbf{x}), \tag{66}$$

where $\mathbf{R}(\mathbf{D_x})$ is given in Appendix C.

From equations (50), (64) and (66), we obtain

$$\mathbf{F}^{(1)}(\mathbf{D_x})\mathbf{R}(\mathbf{D_x}) = \boldsymbol{\Theta}^{(1)}(\Delta). \tag{67}$$

We assume that

$$\lambda_p^2 \neq \lambda_l^2 \neq 0 \quad p, l = 1, \ldots, 9 \quad p \neq l.$$

Let

$$\begin{aligned}
\mathbf{Y}^{(1)}(\mathbf{x}) &= \left(Y_{ij}^{(1)}(\mathbf{x})\right)_{11\times 11}, \\
Y_{pp}^{(1)}(\mathbf{x}) &= \sum_{g=1}^{8} r_{1g}^{(1)}\varsigma_g(\mathbf{x}), \\
Y_{p+3;p+3}^{(1)}(\mathbf{x}) &= \sum_{g=7}^{9} r_{2g}^{(1)}\varsigma_g(\mathbf{x}),
\end{aligned}$$

$$
\begin{aligned}
Y^{(1)}_{ll}(\mathbf{x}) &= \sum_{g=1}^{6} r^{(1)}_{3g}\varsigma_g(\mathbf{x}), Y^{(1)}_{ij}(\mathbf{x}) = 0,\\
p &= 1,2,3 \quad l=7,\dots,11 \quad i,j=1,\dots,11 \quad i\neq j
\end{aligned}
$$

where

$$
\varsigma_g(\mathbf{x}) = -\frac{e^{\iota\lambda_g|\mathbf{x}|}}{4\pi|\mathbf{x}|}, \quad r^{(1)}_{1l} = \prod_{i=1,i\neq l}^{8}(\lambda_i^2-\lambda_l^2)^{-1}, \quad r^{(1)}_{2h} = \prod_{i=7,i\neq h}^{9}(\lambda_i^2-\lambda_h^2)^{-1},
$$

$$
r^{(1)}_{3e} = \prod_{i=1,i\neq e}^{6}(\lambda_i^2-\lambda_e^2)^{-1}, \quad g=1,\dots,9 \quad l=1,\dots,8 \quad h=7,8,9 \quad e=1,\dots,6. \tag{68}
$$

Lemma 1: The matrix $\mathbf{Y}^{(1)}$ defined above is the fundamental matrix of operator $\mathbf{\Theta}^{(1)}(\Delta)$, i.e.

$$
\mathbf{\Theta}^{(1)}(\Delta)\mathbf{Y}^{(1)}(\mathbf{x}) = \delta(\mathbf{x})\,\mathbf{I}(\mathbf{x}). \tag{69}
$$

Proof: To prove the lemma, it is sufficient to prove that

$$
\Gamma^{(1)}(\Delta)\Lambda^{(1)}(\Delta)Y^{(1)}_{11}(\mathbf{x}) = \delta(\mathbf{x}), \tag{70}
$$

$$
\Lambda^{(1)}(\Delta)(\Delta+\lambda_9^2)Y^{(1)}_{44}(\mathbf{x}) = \delta(\mathbf{x}), \tag{71}
$$

$$
\Gamma^{(1)}(\Delta)Y^{(1)}_{77}(\mathbf{x}) = \delta(\mathbf{x}). \tag{72}
$$

Consider

$$
\sum_{i=1}^{8} r^{(1)}_{1i} = \frac{\sum_{j=1}^{8}(-1)^j z_j}{z_9},
$$

where $z_i,\ i=1,\dots,9$ are given in Appendix D.

On simplifying the right hand side of above relation, we obtain

$$
\sum_{i=1}^{8} r^{(1)}_{1i} = 0. \tag{73}
$$

Similarly, we find that

$$
\begin{aligned}
&\sum_{i=2}^{8} r^{(1)}_{1i}(\lambda_1^2-\lambda_i^2) = 0, \quad \sum_{i=3}^{8} r^{(1)}_{1i}\left[\prod_{j=1}^{2}(\lambda_j^2-\lambda_i^2)\right] = 0,\\
&\sum_{i=4}^{8} r^{(1)}_{1i}\left[\prod_{j=1}^{3}(\lambda_j^2-\lambda_i^2)\right] = 0, \quad \sum_{i=5}^{8} r^{(1)}_{1i}\left[\prod_{j=1}^{4}(\lambda_j^2-\lambda_i^2)\right] = 0,\\
&\sum_{i=6}^{8} r^{(1)}_{1i}\left[\prod_{j=1}^{5}(\lambda_j^2-\lambda_i^2)\right] = 0, \quad \sum_{i=7}^{8} r^{(1)}_{1i}\left[\prod_{j=1}^{6}(\lambda_j^2-\lambda_i^2)\right] = 0,\\
&\prod_{j=1}^{7} r^{(1)}_{18}(\lambda_j^2-\lambda_8^2) = 1.
\end{aligned} \tag{74}
$$

Also,

$$(\Delta + \lambda_p^2)\varsigma_g(\mathbf{x}) = \delta(\mathbf{x}) + (\lambda_p^2 - \lambda_g^2)\varsigma_g(\mathbf{x}) \quad p, g = 1, \ldots, 9. \tag{75}$$

Now consider

$$\begin{aligned}
\Gamma^{(1)}(\Delta)\Lambda^{(1)}(\Delta)Y_{11}^{(1)}(\mathbf{x}) &= \prod_{i=1}^{8}(\Delta + \lambda_i^2)\sum_{g=1}^{8} r_{1g}^{(1)}\varsigma_g(\mathbf{x}) \\
&= \prod_{i=2}^{8}(\Delta + \lambda_i^2)\sum_{g=1}^{8} r_{1g}^{(1)}\Big[\delta(\mathbf{x}) + (\lambda_1^2 - \lambda_g^2)\varsigma_g(\mathbf{x})\Big] \\
&= \prod_{i=2}^{8}(\Delta + \lambda_i^2)\Big[\delta(\mathbf{x})\sum_{g=1}^{8} r_{1g}^{(1)} + \sum_{g=2}^{8} r_{1g}^{(1)}(\lambda_1^2 - \lambda_g^2)\varsigma_g(\mathbf{x})\Big].
\end{aligned}$$

Using equations (73)-(75) in the above relation, we obtain

$$\begin{aligned}
\Gamma^{(1)}(\Delta)\Lambda^{(1)}(\Delta)Y_{11}^{(1)}(\mathbf{x}) &= \prod_{i=2}^{8}(\Delta + \lambda_i^2)\Big[\sum_{g=2}^{8} r_{1g}^{(1)}(\lambda_1^2 - \lambda_g^2)\varsigma_g(\mathbf{x})\Big] \\
&= \prod_{i=3}^{8}(\Delta + \lambda_i^2)\Big[\sum_{g=2}^{8} r_{1g}^{(1)}(\lambda_1^2 - \lambda_g^2)\Big[\delta(\mathbf{x}) + (\lambda_2^2 - \lambda_g^2)\varsigma_g(\mathbf{x})\Big]\Big] \\
&= \prod_{i=3}^{8}(\Delta + \lambda_i^2)\Big[\sum_{g=3}^{8} r_{1g}^{(1)}\Big[\prod_{j=1}^{2}(\lambda_j^2 - \lambda_g^2)\Big]\varsigma_g(\mathbf{x})\Big] \\
&= \prod_{i=4}^{8}(\Delta + \lambda_i^2)\Big[\sum_{g=3}^{8} r_{1g}^{(1)}\Big[\prod_{j=1}^{2}(\lambda_j^2 - \lambda_g^2)\Big]\Big[\delta(\mathbf{x}) + (\lambda_3^2 - \lambda_g^2)\varsigma_g(\mathbf{x})\Big]\Big] \\
&= \prod_{i=4}^{8}(\Delta + \lambda_i^2)\Big[\sum_{g=4}^{8} r_{1g}^{(1)}\Big[\prod_{j=1}^{3}(\lambda_j^2 - \lambda_g^2)\Big]\varsigma_g(\mathbf{x})\Big] \\
&= \prod_{i=5}^{8}(\Delta + \lambda_i^2)\Big[\sum_{g=4}^{8} r_{1g}^{(1)}\Big[\prod_{j=1}^{3}(\lambda_j^2 - \lambda_g^2)\Big]\Big[\delta(\mathbf{x}) + (\lambda_4^2 - \lambda_g^2)\varsigma_g(\mathbf{x})\Big]\Big] \\
&= \prod_{i=5}^{8}(\Delta + \lambda_i^2)\Big[\sum_{g=5}^{8} r_{1g}^{(1)}\Big[\prod_{j=1}^{4}(\lambda_j^2 - \lambda_g^2)\Big]\varsigma_g(\mathbf{x})\Big] \\
&= \prod_{i=6}^{8}(\Delta + \lambda_i^2)\Big[\sum_{g=5}^{8} r_{1g}^{(1)}\Big[\prod_{j=1}^{4}(\lambda_j^2 - \lambda_g^2)\Big]\Big[\delta(\mathbf{x}) + (\lambda_5^2 - \lambda_g^2)\varsigma_g(\mathbf{x})\Big]\Big] = \\
&= \prod_{i=6}^{8}(\Delta + \lambda_i^2)\Big[\sum_{g=6}^{8} r_{1g}^{(1)}\Big[\prod_{j=1}^{5}(\lambda_j^2 - \lambda_g^2)\Big]\varsigma_g(\mathbf{x})\Big] \\
&= \prod_{i=7}^{8}(\Delta + \lambda_i^2)\Big[\sum_{g=6}^{8} r_{1g}^{(1)}\Big[\prod_{j=1}^{5}(\lambda_j^2 - \lambda_g^2)\Big]\Big[\delta(\mathbf{x}) + (\lambda_6^2 - \lambda_g^2)\varsigma_g(\mathbf{x})\Big]\Big] \\
&= \prod_{i=7}^{8}(\Delta + \lambda_i^2)\Big[\sum_{g=7}^{8} r_{1g}^{(1)}\Big[\prod_{j=1}^{6}(\lambda_j^2 - \lambda_g^2)\Big]\varsigma_g(\mathbf{x})\Big] \\
&= (\Delta + \lambda_8^2)\Big[\sum_{g=7}^{8} r_{1g}^{(1)}\Big[\prod_{j=1}^{6}(\lambda_j^2 - \lambda_g^2)\Big]\Big[\delta(\mathbf{x}) + (\lambda_7^2 - \lambda_g^2)\varsigma_g(\mathbf{x})\Big]\Big] \\
&= (\Delta + \lambda_8^2)\varsigma_8(\mathbf{x}) = \delta(\mathbf{x}).
\end{aligned}$$

Equations (71) and (72) can be proved in the similar way.

We introduce the matrix

$$\mathbf{G}^{(1)}(\mathbf{x}) = \mathbf{R}(\mathbf{D_x})\mathbf{Y}^{(1)}(\mathbf{x}). \tag{76}$$

From equations (67), (69) and (76), we obtain

$$\mathbf{F}^{(1)}(\mathbf{D_x})\mathbf{G}^{(1)}(\mathbf{x}) = \mathbf{F}^{(1)}(\mathbf{D_x})\mathbf{R}(\mathbf{D_x})\mathbf{Y}^{(1)}(\mathbf{x}) = \mathbf{\Theta}^{(1)}(\Delta)\mathbf{Y}^{(1)}(\mathbf{x}) = \delta(\mathbf{x})\,\mathbf{I}(\mathbf{x}).$$

Hence, $\mathbf{G}^{(1)}(\mathbf{x})$ is a solution to equation (42) for $i = 1$.

Theorem 1: If the condition (41) is satisfied, then the matrix $\mathbf{G}^{(1)}(\mathbf{x})$ defined by the equation (76) is the fundamental solution of the system of equations (37) and the matrix $\mathbf{G}^{(1)}(\mathbf{x})$ is represented in the following form:

$$\mathbf{G}^{(1)}(\mathbf{x}) = \Big(H_{pq}(\mathbf{x})\Big)_{11\times 11} = \begin{pmatrix} \mathbf{H}^{(1)} & \mathbf{H}^{(2)} & \mathbf{H}^{(5)} \\ \mathbf{H}^{(3)} & \mathbf{H}^{(4)} & \mathbf{H}^{(6)} \\ \mathbf{H}^{(7)} & \mathbf{H}^{(8)} & \mathbf{H}^{(9)} \end{pmatrix}_{11\times 11},$$

where $\mathbf{H}^{(p)}(\mathbf{x})$, $p = 1, \ldots, 9$ are given in Appendix D.

V. Construction of Matrices $\mathbf{G}^{(i)}(\mathbf{x})$ $\quad i = 2, 3, 4$

V. 1. Pseudo-Oscillations

We introduce the matrix

$$\mathbf{G}^{(2)}(\mathbf{x}) = \mathbf{T}(\mathbf{D_x})\mathbf{Y}^{(2)}(\mathbf{x}), \tag{77}$$

where, the matrices $\mathbf{T}(\mathbf{D_x})$ and $\mathbf{Y}^{(2)}(\mathbf{x})$ can be obtained from matrices $\mathbf{R}(\mathbf{D_x})$ and $\mathbf{Y}^{(1)}(\mathbf{x})$ respectively by taking $\omega = -\iota\tau$ and repeating the above procedure after equation (42).

Theorem 2: If the condition (41) is satisfied, then the matrix $\mathbf{G}^{(2)}(\mathbf{x})$ defined by the equation (77) is the fundamental solution of the system of equations (38).

V. 2. Quasi-Static Oscillations

In this case, the matrix $\mathbf{N}^{(3)}(\Delta)$, operators $\Gamma^{(3)}(\Delta), \Lambda^{(3)}(\Delta)$ and matrix operators $\mathbf{\Theta}^{(3)}(\Delta)$, $\mathbf{M}(\mathbf{D_x})$, $\mathbf{Y}^{(3)}(\mathbf{x})$ and $\mathbf{G}^{(3)}(\mathbf{x})$ are obtained as

(i)
$$\hat{\mathbf{N}}^{(3)}(\Delta) = \Big(\hat{N}^{(3)}_{gh}(\Delta)\Big)_{6\times 6} =$$

$$\begin{pmatrix} \tilde{\lambda} & -\sigma_1 & -\sigma_2 & -\sigma_3 & \tau_1\zeta_1 T_0 & \tau^1\zeta_2 \\ \sigma_1 & A_1\Delta - \beta_1 & A_4\Delta - \beta_4 & A_6\Delta - \beta_6 & \tau_1\xi_1 T_0 & \tau^1 v_1 \\ \sigma_2 & A_4\Delta - \beta_4 & A_2\Delta - \beta_2 & A_5\Delta - \beta_5 & \tau_1\xi_2 T_0 & \tau^1 v_2 \\ \sigma_3 & A_6\Delta - \beta_6 & A_5\Delta - \beta_5 & A_3\Delta - \beta_3 & \tau_1\xi_3 T_0 & \tau^1 v_3 \\ -\zeta_1 & \xi_1 & \xi_2 & \xi_3 & K\Delta + \tau_1 a\varsigma T_0 & \tau^1\varsigma \\ -\zeta_2 & v_1 & v_2 & v_3 & \tau_1\varsigma T_0 & D\Delta + \tau^1\varpi \end{pmatrix}_{6\times 6}$$

$$\mathbf{N}^{(3)}(\Delta) = \Big(N^{(3)}_{gh}(\Delta)\Big)_{6\times 6} = \Delta\Big(\hat{N}^{(3)}_{gh}(\Delta)\Big)_{6\times 6},$$

(ii)
$$\Gamma^{(3)}(\Delta) = \Delta\prod_{i=1}^{5}(\Delta + \tilde{\mu}_i^2), \Lambda^{(3)}(\Delta) = \Delta(\Delta + \tilde{\mu}_6^2) = \frac{1}{\tilde{N}}\begin{vmatrix} \tilde{\mu}\Delta & K^*\Delta \\ -K^* & \gamma\Delta - 2K^* \end{vmatrix}, \quad \tilde{\mu}_7^2 = -\frac{2K^*}{\tilde{\alpha}},$$

where $\tilde{\mu}_i^2, \quad i = 1, \dots, 5$ are the roots of the equation $|\hat{\mathbf{N}}^{(3)}(-m)| = 0$(with respect to m).

$$
\begin{aligned}
(iii) \qquad \boldsymbol{\Theta}^{(3)}(\Delta) &= \Big(\Theta_{gh}^{(3)}(\Delta)\Big)_{11\times 11}, \\
\Theta_{pp}^{(3)}(\Delta) &= \Gamma^{(3)}(\Delta)\Lambda^{(3)}(\Delta) = \Delta^2 \prod_{i=1}^{6}(\Delta + \tilde{\mu}_i^2), \\
\Theta_{p+3;p+3}^{(3)}(\Delta) &= \Lambda^{(3)}(\Delta)(\Delta + \tilde{\mu}_7^2) = \Delta \prod_{i=6}^{7}(\Delta + \tilde{\mu}_i^2), \\
\Theta_{ll}^{(3)}(\Delta) &= \Gamma^{(3)}(\Delta) = \Delta \prod_{i=1}^{5}(\Delta + \tilde{\mu}_i^2), \\
\Theta_{gh}^{(3)}(\Delta) &= 0, \\
p &= 1,2,3 \quad g,h = 1,\dots,11 \quad l = 7,\dots,11 \quad g \neq h
\end{aligned}
$$

$$
\begin{aligned}
(iv) \qquad w_{11}^{(3)}(\Delta) &= -\tfrac{1}{\tilde{A}\tilde{N}}\Bigg\{\Big[(\lambda' + \mu)(\gamma\Delta - 2K^*) - {K^*}^2\Big]\tilde{N}_{11}^{(3)}(\Delta) + (\gamma\Delta - 2K^*) \times \\
&\qquad \Big[-\sigma_k \tilde{N}_{1;k+1}^{(3)}(\Delta) + \tau_1\zeta_1 T_0 \tilde{N}_{15}^{(3)}(\Delta) + \tau^1\zeta_2\tilde{N}_{16}^{(3)}(\Delta)\Big]\Bigg\}, \\
w_{p+1;1}^{(3)}(\Delta) &= -\tfrac{1}{\tilde{A}\tilde{N}}\Bigg\{\Big[(\lambda' + \mu)(\gamma\Delta - 2K^*) - {K^*}^2\Big]\tilde{N}_{p1}^{(3)}(\Delta) + (\gamma\Delta - 2K^*) \times \\
&\qquad \Big[-\sigma_k \tilde{N}_{p;k+1}^{(3)}(\Delta) + \tau_1\zeta_1 T_0 \tilde{N}_{p5}^{(3)}(\Delta) + \tau^1\zeta_2\tilde{N}_{p6}^{(3)}(\Delta)\Big]\Bigg\}, \\
w_{21}^{(3)}(\Delta) &= -\tfrac{K^*\Gamma^{(3)}(\Delta)}{\tilde{N}}, \quad w_{12}^{(3)}(\Delta) = -\tfrac{K^*(\Delta + \tilde{\mu}_7^2)}{\tilde{N}}, \quad w_{22}^{(3)}(\Delta) = -\tfrac{(\alpha+\beta)\tilde{\mu}\Delta - {K^*}^2}{\tilde{N}\tilde{\alpha}}, \\
w_{1;p+1}^{(3)}(\Delta) &= \tfrac{\tilde{N}_{1p}^{(3)}(\Delta)}{\tilde{A}}, \quad w_{i+1;p+1}^{(3)}(\Delta) = \tfrac{\tilde{N}_{ip}^{(3)}(\Delta)}{\tilde{A}}, \\
w_{p+1;2}^{(3)}(\Delta) &= w_{2;p+1}^{(3)}(\Delta) = 0 \quad i,p = 2,\dots,6
\end{aligned}
$$

where $\tilde{N}_{ij}^{(3)}$ is the cofactor of the element $N_{ij}^{(3)}$ of the matrix $\mathbf{N}^{(3)}$.

$$
\begin{aligned}
(v) \qquad \mathbf{M}(\mathbf{D_x}) &= \Big(M_{gh}(\mathbf{D_x})\Big)_{11\times 11} = \begin{pmatrix} \mathbf{M}^{(1)} & \mathbf{M}^{(2)} & \mathbf{M}^{(5)} \\ \mathbf{M}^{(3)} & \mathbf{M}^{(4)} & \mathbf{M}^{(6)} \\ \mathbf{M}^{(7)} & \mathbf{M}^{(8)} & \mathbf{M}^{(9)} \end{pmatrix}_{11\times 11}, \\
\mathbf{M}^{(i)}(\mathbf{D_x}) &= \Big(M_{gh}^{(i)}(\mathbf{D_x})\Big)_{3\times 3}, \qquad \mathbf{M}^{(j)}(\mathbf{D_x}) = \Big(M_{gh}^{(j)}(\mathbf{D_x})\Big)_{3\times 5}, \\
\mathbf{M}^{(l)}(\mathbf{D_x}) &= \Big(M_{gh}^{(l)}(\mathbf{D_x})\Big)_{5\times 3}, \qquad \mathbf{M}^{(9)}(\mathbf{D_x}) = \Big(M_{gh}^{(9)}(\mathbf{D_x})\Big)_{5\times 5}, \\
M_{gh}^{(1)}(\mathbf{D_x}) &= \frac{1}{\tilde{N}}(\gamma\Delta - 2K^*)\Gamma^{(3)}(\Delta)\delta_{gh} + w_{11}^{(3)}(\Delta)\frac{\partial^2}{\partial x_g \partial x_h}, \\
M_{gh}^{(2)}(\mathbf{D_x}) &= w_{12}^{(3)}(\Delta)\sum_{p=1}^{3}\varepsilon_{gph}\frac{\partial}{\partial x_p}, M_{gh}^{(3)}(\mathbf{D_x}) = w_{21}^{(3)}(\Delta)\sum_{p=1}^{3}\varepsilon_{gph}\frac{\partial}{\partial x_p}, \\
M_{gh}^{(4)}(\mathbf{D_x}) &= \frac{1}{\tilde{N}}(\Delta + \tilde{\mu}_7^2)\tilde{\mu}\Delta\,\delta_{gh} + w_{22}^{(3)}(\Delta)\frac{\partial^2}{\partial x_g \partial x_h}, \\
M_{gh}^{(5)}(\mathbf{D_x}) &= w_{1;h+2}^{(3)}(\Delta)\frac{\partial}{\partial x_g}, M_{gh}^{(6)}(\mathbf{D_x}) = M_{gh}^{(8)}(\mathbf{D_x}) = 0, \\
M_{gh}^{(7)}(\mathbf{D_x}) &= w_{g+2;1}^{(3)}(\Delta)\frac{\partial}{\partial x_h}, M_{gh}^{(9)}(\mathbf{D_x}) = w_{g+2;h+2}^{(3)}(\Delta) \quad i = 1,...,4 \quad j = 5,6 \quad l = 7,8.
\end{aligned}
$$

$$
\begin{aligned}
(vi) \qquad \mathbf{Y}^{(3)}(\mathbf{x}) &= \Big(Y^{(3)}_{ij}(\mathbf{x})\Big)_{11\times 11}, \quad Y^{(3)}_{pp}(\mathbf{x}) = r^{(3)}_{11}\varsigma^*_1(\mathbf{x}) + r^{(3)}_{12}\varsigma^*_2(\mathbf{x}) + \sum_{g=1}^{6} r^{(3)}_{1;g+2}\tilde{\varsigma}_g(\mathbf{x}),\\
Y^{(3)}_{p+3;p+3}(\mathbf{x}) &= r^{(3)}_{21}\varsigma^*_1(\mathbf{x}) + \sum_{g=6}^{7} r^{(3)}_{2;g-4}\tilde{\varsigma}_g(\mathbf{x}), \quad Y^{(3)}_{ll}(\mathbf{x}) = r^{(3)}_{31}\varsigma^*_1(\mathbf{x}) + \sum_{g=1}^{5} r^{(3)}_{3;g+1}\tilde{\varsigma}_g(\mathbf{x}),\\
Y^{(3)}_{ij}(\mathbf{x}) &= 0 \quad p=1,2,3 \quad l=7,\dots,11 \quad i,j=1,\dots,11 \quad i\neq j,
\end{aligned}
$$

where

$$
\begin{aligned}
\varsigma^*_1(\mathbf{x}) &= -\frac{1}{4\pi|\mathbf{x}|}, \qquad \varsigma^*_2(\mathbf{x}) = -\frac{|\mathbf{x}|}{8\pi}, \qquad \tilde{\varsigma}_g(\mathbf{x}) = -\frac{e^{\iota\tilde{\mu}_g|\mathbf{x}|}}{4\pi|\mathbf{x}|} \quad g=1,\dots,7,\\
r^{(3)}_{11} &= -\sum_{p=1}^{6}\Big(\prod_{j=1,j\neq p}^{6}\tilde{\mu}_j^2\Big)\prod_{i=1}^{6}\tilde{\mu}_i^{-4}, \qquad r^{(3)}_{12} = \prod_{i=1}^{6}\tilde{\mu}_i^{-2}, \qquad r^{(3)}_{1;k+2} = \tilde{\mu}_k^{-4}\prod_{i=1,i\neq k}^{6}(\tilde{\mu}_i^2-\tilde{\mu}_k^2)^{-1},\\
r^{(3)}_{21} &= \prod_{i=6}^{7}\tilde{\mu}_i^{-2}, \qquad r^{(3)}_{2;q-4} = -\tilde{\mu}_q^{-2}\prod_{i=6,i\neq q}^{7}(\tilde{\mu}_i^2-\tilde{\mu}_q^2)^{-1}, \qquad r^{(3)}_{31} = \prod_{i=1}^{5}\tilde{\mu}_i^{-2},\\
r^{(3)}_{3;l+1} &= -\tilde{\mu}_l^{-2}\prod_{i=1,i\neq l}^{5}(\tilde{\mu}_i^2-\tilde{\mu}_l^2)^{-1}, \qquad k=1,\dots,6 \quad q=6,7 \quad l=1,\dots,5.
\end{aligned}
$$

On introducing the matrix

$$
\mathbf{G}^{(3)}(\mathbf{x}) = \mathbf{M}(\mathbf{D_x})\mathbf{Y}^{(3)}(\mathbf{x}), \tag{78}
$$

we obtain

$$
\mathbf{F}^{(3)}(\mathbf{D_x})\mathbf{G}^{(3)}(\mathbf{x}) = \mathbf{F}^{(3)}(\mathbf{D_x})\mathbf{M}(\mathbf{D_x})\mathbf{Y}^{(3)}(\mathbf{x}) = \mathbf{\Theta}^{(3)}(\Delta)\mathbf{Y}^{(3)}(\mathbf{x}) = \delta(\mathbf{x})\,\mathbf{I}(\mathbf{x}).
$$

Hence, $\mathbf{G}^{(3)}(\mathbf{x})$ is a fundamental solution to equation (42) for $i=3$.

Theorem 3: If the condition (41) is satisfied, then the matrix $\mathbf{G}^{(3)}(\mathbf{x})$ defined by the equation (78) is the fundamental solution of the system of equations (39).

Equilibrium Theory

In this case, the matrix $\mathbf{N}^{(4)}(\Delta)$, operators $\Gamma^{(4)}(\Delta), \Lambda^{(4)}(\Delta)$ and matrix operators $\mathbf{\Theta}^{(4)}(\Delta)$, $\mathbf{Z}(\mathbf{D_x})$, $\mathbf{Y}^{(4)}(\mathbf{x})$ and $\mathbf{G}^{(4)}(\mathbf{x})$ are obtained as

$$
\begin{aligned}
(i) \quad \hat{\mathbf{N}}^{(4)}(\Delta) &= \Big(\hat{N}^{(4)}_{gh}(\Delta)\Big)_{4\times 4} = \begin{pmatrix} \tilde{\lambda} & -\sigma_1 & -\sigma_2 & -\sigma_3\\ \sigma_1 & A_1\Delta-\beta_1 & A_4\Delta-\beta_4 & A_6\Delta-\beta_6\\ \sigma_2 & A_4\Delta-\beta_4 & A_2\Delta-\beta_2 & A_5\Delta-\beta_5\\ \sigma_3 & A_6\Delta-\beta_6 & A_5\Delta-\beta_5 & A_3\Delta-\beta_3 \end{pmatrix}_{4\times 4}\\
\mathbf{N}^{(4)}(\Delta) &= \Big(N^{(4)}_{gh}(\Delta)\Big)_{4\times 4} = \Delta\Big(\hat{N}^{(4)}_{gh}(\Delta)\Big)_{4\times 4}.\\
(ii) \quad \Gamma^{(4)}(\Delta) &= \Delta\prod_{i=1}^{3}(\Delta+\omega_i^2), \quad \Lambda^{(4)}(\Delta) = \Delta(\Delta+\omega_4^2), \quad \omega_4^2 = \tilde{\mu}_6^2, \quad \omega_5^2 = \tilde{\mu}_7^2,
\end{aligned}
$$

where $\omega_i^2, \quad i=1,2,3$ are the roots of the equation $|\hat{\mathbf{N}}^{(4)}(-m)| = 0$(with respect to m).

$$
\begin{aligned}
(iii) \quad \mathbf{\Theta}^{(4)}(\Delta) &= \Big(\Theta^{(4)}_{gh}(\Delta)\Big)_{11\times 11},\\
\Theta^{(4)}_{pp}(\Delta) &= \Gamma^{(4)}(\Delta)\Lambda^{(4)}(\Delta) = \Delta^2\prod_{i=1}^{4}(\Delta+\omega_i^2),
\end{aligned}
$$

$$\begin{aligned}
\Theta^{(4)}_{p+3;p+3}(\Delta) &= \Lambda^{(4)}(\Delta)(\Delta+\omega_5^2) = \Delta\prod_{i=4}^{5}(\Delta+\omega_i^2),\\
\Theta^{(4)}_{p+6;p+6}(\Delta) &= \Gamma^{(4)}(\Delta) = \Delta\prod_{i=1}^{3}(\Delta+\omega_i^2),\\
\Theta^{(4)}_{jj}(\Delta) &= \Gamma^{(4)}(\Delta)\Delta = \Delta^2\prod_{i=1}^{3}(\Delta+\omega_i^2), \quad \Theta^{(4)}_{gh}(\Delta) = 0,\\
p &= 1,2,3 \quad g,h = 1,\ldots,11 \quad j = 10,11 \quad g \neq h.
\end{aligned}$$

$$\begin{aligned}
(iv) \quad w^{(4)}_{11}(\Delta) &= -\frac{1}{\tilde{\lambda}\tilde{N}\varrho}\Big\{\Big[(\lambda'+\mu)(\gamma\Delta-2K^*)-K^{*^2}\Big]\tilde{N}^{(4)}_{11}(\Delta)-(\gamma\Delta-2K^*)\sigma_k\tilde{N}^{(4)}_{1;k+1}(\Delta)\Big\},\\
w^{(4)}_{l+1;1}(\Delta) &= -\frac{1}{\tilde{\lambda}\tilde{N}\varrho}\Big\{\Big[(\lambda'+\mu)(\gamma\Delta-2K^*)-K^{*^2}\Big]\tilde{N}^{(4)}_{l1}(\Delta)-(\gamma\Delta-2K^*)\sigma_k\tilde{N}^{(4)}_{l;k+1}(\Delta)\Big\},\\
w^{(4)}_{21}(\Delta) &= -\frac{K^*\Gamma^{(4)}(\Delta)}{\tilde{N}}, \quad w^{(4)}_{12}(\Delta) = -\frac{K^*(\Delta+\omega_5^2)}{\tilde{N}}, \quad w^{(4)}_{22}(\Delta) = w^{(3)}_{22}(\Delta),\\
w^{(4)}_{1;l+1}(\Delta) &= \frac{\tilde{N}^{(4)}_{1l}(\Delta)}{\varrho\tilde{\lambda}}, \quad w^{(4)}_{i+1;l+1}(\Delta) = \frac{\tilde{N}^{(4)}_{il}(\Delta)}{\varrho\tilde{\lambda}},\\
w^{(4)}_{16}(\Delta) &= \frac{1}{\varrho\tilde{\lambda}K}\Big[\zeta_1\tilde{N}^{(4)}_{11}(\Delta)-\sigma_k\tilde{N}^{(4)}_{1;k+1}(\Delta)\Big],\\
w^{(4)}_{l+1;6}(\Delta) &= \frac{1}{\varrho\tilde{\lambda}K}\Big[\zeta_1\tilde{N}^{(4)}_{l1}(\Delta)-\sigma_k\tilde{N}^{(4)}_{l;k+1}(\Delta)\Big],\\
w^{(4)}_{17}(\Delta) &= \frac{1}{\varrho\tilde{\lambda}D}\Big[\zeta_2\tilde{N}^{(4)}_{11}(\Delta)-v_k\tilde{N}^{(4)}_{1;k+1}(\Delta)\Big],\\
w^{(4)}_{l+1;7}(\Delta) &= \frac{1}{\varrho\tilde{\lambda}D}\Big[\zeta_2\tilde{N}^{(4)}_{l1}(\Delta)-v_k\tilde{N}^{(4)}_{l;k+1}(\Delta)\Big],
\end{aligned}$$

$$\begin{aligned}
w^{(4)}_{66}(\Delta) &= \Delta\prod_{i=1}^{3}(\Delta+\omega_i^2)K^{-1}, \quad w^{(4)}_{77}(\Delta) = \Delta\prod_{i=1}^{3}(\Delta+\omega_i^2)D^{-1},\\
w^{(4)}_{q+1;2}(\Delta) &= w^{(4)}_{2;q+1}(\Delta) = w^{(4)}_{6j}(\Delta) = w^{(4)}_{7p}(\Delta) = 0,\\
i,l &= 2,3,4 \quad j = 1,3,4,5,7 \quad p = 1,3,4,5,6 \quad q = 2,\ldots,6
\end{aligned}$$

where $\tilde{N}^{(4)}_{ij}$ is the cofactor of the element $N^{(4)}_{ij}$ of the matrix $\mathbf{N}^{(4)}$.

$$\begin{aligned}
(v) \qquad \mathbf{Z}(\mathbf{D_x}) &= \Big(Z_{gh}(\mathbf{D_x})\Big)_{11\times 11} = \begin{pmatrix} \mathbf{Z}^{(1)} & \mathbf{Z}^{(2)} & \mathbf{Z}^{(5)}\\ \mathbf{Z}^{(3)} & \mathbf{Z}^{(4)} & \mathbf{Z}^{(6)}\\ \mathbf{Z}^{(7)} & \mathbf{Z}^{(8)} & \mathbf{Z}^{(9)} \end{pmatrix}_{11\times 11},\\
\mathbf{Z}^{(i)}(\mathbf{D_x}) &= \Big(Z^{(i)}_{gh}(\mathbf{D_x})\Big)_{3\times 3}, \quad \mathbf{Z}^{(j)}(\mathbf{D_x}) = \Big(Z^{(j)}_{gh}(\mathbf{D_x})\Big)_{3\times 5},\\
\mathbf{Z}^{(l)}(\mathbf{D_x}) &= \Big(Z^{(l)}_{gh}(\mathbf{D_x})\Big)_{5\times 3}, \quad \mathbf{Z}^{(9)}(\mathbf{D_x}) = \Big(Z^{(9)}_{gh}(\mathbf{D_x})\Big)_{5\times 5},\\
Z^{(1)}_{gh}(\mathbf{D_x}) &= \frac{1}{\tilde{N}}(\gamma\Delta-2K^*)\Gamma^{(4)}(\Delta)\delta_{gh} + w^{(4)}_{11}(\Delta)\frac{\partial^2}{\partial x_g\partial x_h},\\
Z^{(2)}_{gh}(\mathbf{D_x}) &= w^{(4)}_{12}(\Delta)\sum_{p=1}^{3}\varepsilon_{gph}\frac{\partial}{\partial x_p}, \quad Z^{(3)}_{gh}(\mathbf{D_x}) = w^{(4)}_{21}(\Delta)\sum_{p=1}^{3}\varepsilon_{gph}\frac{\partial}{\partial x_p},\\
Z^{(4)}_{gh}(\mathbf{D_x}) &= \frac{1}{\tilde{N}}(\Delta+\omega_5^2)\tilde{\mu}\Delta\,\delta_{gh} + w^{(4)}_{22}(\Delta)\frac{\partial^2}{\partial x_g\partial x_h},
\end{aligned}$$

$$\begin{aligned} Z_{gh}^{(5)}(\mathbf{D_x}) &= w_{1;h+2}^{(4)}(\Delta)\frac{\partial}{\partial x_g}, \quad Z_{gh}^{(6)}(\mathbf{D_x}) = Z_{gh}^{(8)}(\mathbf{D_x}) = 0, \\ Z_{gh}^{(7)}(\mathbf{D_x}) &= w_{g+2;1}^{(4)}(\Delta)\frac{\partial}{\partial x_h}, \quad Z_{gh}^{(9)}(\mathbf{D_x}) = w_{g+2;h+2}^{(4)}(\Delta) \quad i=1,...,4 \quad j=5,6 \quad l=7,8. \end{aligned}$$

$$\begin{aligned} (vi) \qquad \mathbf{Y}^{(4)}(\mathbf{x}) &= \left(Y_{ij}^{(4)}(\mathbf{x})\right)_{11\times 11}, \quad Y_{pp}^{(4)}(\mathbf{x}) = r_{11}^{(4)}\varsigma_1^*(\mathbf{x}) + r_{12}^{(4)}\varsigma_2^*(\mathbf{x}) + \sum_{g=1}^{4} r_{1;g+2}^{(4)}\hat{\varsigma}_g(\mathbf{x}), \\ Y_{p+3;p+3}^{(4)}(\mathbf{x}) &= r_{21}^{(4)}\varsigma_1^*(\mathbf{x}) + \sum_{g=4}^{5} r_{2;g-2}^{(4)}\hat{\varsigma}_g(\mathbf{x}), \quad Y_{p+6;p+6}^{(4)}(\mathbf{x}) = r_{31}^{(4)}\varsigma_1^*(\mathbf{x}) + \sum_{g=1}^{3} r_{3;g+1}^{(4)}\hat{\varsigma}_g(\mathbf{x}) \\ Y_{ee}^{(4)}(\mathbf{x}) &= r_{41}^{(4)}\varsigma_1^*(\mathbf{x}) + r_{42}^{(4)}\varsigma_2^*(\mathbf{x}) + \sum_{g=1}^{3} r_{4;g+2}^{(4)}\hat{\varsigma}_g(\mathbf{x}), \\ Y_{ij}^{(4)}(\mathbf{x}) &= 0 \quad p=1,2,3 \quad e=10,11 \quad i,j=1,\ldots,11 \quad i\neq j, \end{aligned}$$

where

$$\begin{aligned} \hat{\varsigma}_g(\mathbf{x}) &= -\frac{e^{\iota\omega_g|\mathbf{x}|}}{4\pi|\mathbf{x}|} \quad g=1,\ldots,5, \\ r_{11}^{(4)} &= -\sum_{p=1}^{4}\left(\prod_{j=1,j\neq p}^{4}\omega_j^2\right)\prod_{i=1}^{4}\omega_i^{-4}, \quad r_{12}^{(4)} = \prod_{i=1}^{4}\omega_i^{-2}, r_{1;l+2}^{(4)} = \omega_l^{-4}\prod_{i=1,i\neq l}^{4}(\omega_i^2-\omega_l^2)^{-1}, \\ r_{21}^{(4)} &= \prod_{i=4}^{5}\omega_i^{-2}, r_{2;q-2}^{(4)} = -\omega_q^{-2}\prod_{i=4,i\neq q}^{5}(\omega_i^2-\omega_q^2)^{-1}, \quad r_{31}^{(4)} = \prod_{i=1}^{3}\omega_i^{-2}, \end{aligned}$$

$$\begin{aligned} r_{3;e+1}^{(4)} &= -\omega_e^{-2}\prod_{i=1,i\neq e}^{3}(\omega_i^2-\omega_e^2)^{-1}, \\ r_{41}^{(4)} &= -\frac{\omega_1^2\omega_2^2+\omega_1^2\omega_3^2+\omega_2^2\omega_3^2}{\omega_1^4\omega_2^4\omega_3^4}, \quad r_{42}^{(4)} = \prod_{i=1}^{3}\omega_i^{-2}, \\ r_{4;e+2}^{(4)} &= \omega_e^{-4}\prod_{i=1,i\neq e}^{3}(\omega_i^2-\omega_e^2)^{-1}, \quad l=1,....,4 \quad q=4,5 \quad e=1,2,3. \end{aligned}$$

If we introduce the matrix

$$\mathbf{G}^{(4)}(\mathbf{x}) = \mathbf{R}^{(4)}(\mathbf{D_x})\mathbf{Y}^{(4)}(\mathbf{x}). \tag{79}$$

then, we obtain

$$\mathbf{F}^{(4)}(\mathbf{D_x})\mathbf{G}^{(4)}(\mathbf{x}) = \mathbf{F}^{(4)}(\mathbf{D_x})\mathbf{Z}(\mathbf{D_x})\mathbf{Y}^{(4)}(\mathbf{x}) = \mathbf{\Theta}^{(4)}(\Delta)\mathbf{Y}^{(4)}(\mathbf{x}) = \delta(\mathbf{x})\,\mathbf{I}(\mathbf{x}).$$

Hence, $\mathbf{G}^{(4)}(\mathbf{x})$ is a solution to equation (42) for $i=4$.
Theorem 4: If the condition (41) is satisfied, then the matrix $\mathbf{G}^{(4)}(\mathbf{x})$ defined by the equation (79) is the fundamental solution of the system of equations (40).

VI. Basic Properties of $\mathbf{G}^{(1)}(\mathbf{x})$

Theorem 5: Each column of the matrix $\mathbf{G}^{(1)}(\mathbf{x})$ is a solution of the system of equations (37) at every point $\mathbf{x} \in \mathrm{E}^3$ except the origin.

Theorem 6: If the condition (41) is satisfied, then the fundamental solution of the system of equations $\tilde{\mathbf{F}}(\mathbf{D_x})\mathbf{U}(\mathbf{x}) = \mathbf{0}$ is the matrix

$$\mathbf{W}(\mathbf{x}) = \Big(W_{gh}(\mathbf{x})\Big)_{11\times 11} = \begin{pmatrix} \mathbf{W}^{(1)} & \mathbf{W}^{(2)} & \mathbf{W}^{(5)} \\ \mathbf{W}^{(3)} & \mathbf{W}^{(4)} & \mathbf{W}^{(6)} \\ \mathbf{W}^{(7)} & \mathbf{W}^{(8)} & \mathbf{W}^{(9)} \end{pmatrix}_{11\times 11},$$

where $\mathbf{W}^{(p)}(\mathbf{x}),\ p = 1,\dots,9$ are defined in Appendix E.

Lemma 2: If condition (41) is satisfied, then

$$\Delta w_{11}^{(1)}(\Delta) = \frac{1}{\tilde{A}}\Lambda^{(1)}(\Delta)\tilde{N}_{11}^{(1)}(\Delta) - \frac{1}{\tilde{N}}\Gamma^{(1)}(\Delta)(\gamma\Delta + \tilde{K}), \tag{80}$$

$$\Delta w_{22}^{(1)}(\Delta) = \frac{1}{\tilde{\alpha}}\Big[\Lambda^{(1)}(\Delta) - \frac{1}{\tilde{N}}(\tilde{\alpha}\Delta + \tilde{K})(\tilde{\mu}\Delta + \rho\omega^2)\Big]. \tag{81}$$

Proof: Consider

$$w_{11}^{(1)}(\Delta) = -\frac{1}{\tilde{A}\tilde{N}}\bigg\{\Big[(\lambda'+\mu)(\gamma\Delta+\tilde{K}) - K^{*^2}\Big]\tilde{N}_{11}^{(1)}(\Delta) + (\gamma\Delta+\tilde{K})\times\Big[-\sigma_k\tilde{N}_{1;k+1}^{(1)}(\Delta) + \tau_1\zeta_1T_0\tilde{N}_{15}^{(1)}(\Delta) + \tau^1\zeta_2\tilde{N}_{16}^{(1)}(\Delta)\Big]\bigg\}.$$

Now

$$\Gamma^{(1)}(\Delta) = \frac{1}{\tilde{A}}\,|\mathbf{N}^{(1)}(\Delta)| = \frac{1}{\tilde{A}}\Big\{[\tilde{\lambda}\Delta + \rho\omega^2]\tilde{N}_{11}^{(1)} - \sigma_k\Delta\tilde{N}_{1;k+1}^{(1)} + \tau_1\zeta_1T_0\Delta\tilde{N}_{15}^{(1)}(\Delta) + \tau^1\zeta_2\Delta\tilde{N}_{16}^{(1)}(\Delta)\Big\}.$$

Therefore,

$$\Delta w_{11}^{(1)}(\Delta) = -\frac{1}{\tilde{A}\tilde{N}}\bigg\{\Big[(\lambda'+\mu)(\gamma\Delta+\tilde{K}) - K^{*^2}\Big]\Delta\tilde{N}_{11}^{(1)}(\Delta) + (\gamma\Delta+\tilde{K})\times \Big[-\sigma_k\Delta\tilde{N}_{1;k+1}^{(1)}(\Delta) + \tau_1\zeta_1T_0\Delta\tilde{N}_{15}^{(1)}(\Delta) + \tau^1\zeta_2\Delta\tilde{N}_{16}^{(1)}(\Delta)\Big]\bigg\}$$

$$\begin{aligned}\Delta w_{11}^{(1)}(\Delta) &= -\frac{1}{\tilde{A}\tilde{N}}\bigg\{\Big[\tilde{A}\Gamma^{(1)}(\Delta) - (\tilde{\lambda}\Delta + \rho\omega^2)\tilde{N}_{11}^{(1)}\Big](\gamma\Delta+\tilde{K}) + \Big[(\lambda'+\mu)(\gamma\Delta+\tilde{K}) - K^{*^2}\Big]\Delta\tilde{N}_{11}^{(1)}\bigg\} \\ &= \frac{1}{\tilde{A}}\Lambda^{(1)}(\Delta)\tilde{N}_{11}^{(1)}(\Delta) - \frac{1}{\tilde{N}}\Gamma^{(1)}(\Delta)(\gamma\Delta+\tilde{K}).\end{aligned}$$

Also taking the R.H.S. of equation (81),

$$\frac{1}{\tilde{N}\tilde{\alpha}}\bigg\{\Big[(\tilde{\mu}\Delta+\rho\omega^2)(\gamma\Delta+\tilde{K}) + K^{*^2}\Delta\Big] - (\tilde{\alpha}\Delta+\tilde{K})(\tilde{\mu}\Delta+\rho\omega^2)\bigg\}$$

$$= -\frac{1}{\tilde{N}\tilde{\alpha}}\Delta\Big[(\alpha+\beta)(\tilde{\mu}\Delta+\rho\omega^2) - K^{*^2}\Big] = \Delta w_{22}^{(1)}(\Delta).$$

Theorem 7: If condition (41) is satisfied and $\mathbf{x} \in \mathrm{E}^3 - \{\mathbf{0}\}$, then

$$\mathbf{H}^{(1)}(\mathbf{x}) = \nabla\,\mathrm{div}\sum_{j=1}^{6} x_{1j}\varsigma_j(\mathbf{x}) - \mathrm{curl}\,\mathrm{curl}\sum_{e=6}^{8} x_{1e}\varsigma_e(\mathbf{x}),$$

$$\mathbf{H}^{(2)}(\mathbf{x}) = \mathbf{H}^{(3)}(\mathbf{x}) = \mathrm{curl}\sum_{e=7}^{8} x_{2e}\varsigma_e(\mathbf{x}),$$

$$\mathbf{H}^{(4)}(\mathbf{x}) = \nabla\,\mathrm{div}\,[x_{49}\varsigma_9(\mathbf{x})] - \mathrm{curl}\,\mathrm{curl}\sum_{e=7}^{8} x_{4e}\varsigma_e(\mathbf{x}),$$

$$H_{iq}^{(5)}(\mathbf{x}) = \frac{\partial}{\partial x_i}\sum_{j=1}^{6} x_{5qj}\varsigma_j(\mathbf{x}), \quad \mathbf{H}^{(6)}(\mathbf{x}) = \mathbf{H}^{(8)}(\mathbf{x}) = \mathbf{0},$$

$$H_{qi}^{(7)}(\mathbf{x}) = \frac{\partial}{\partial x_i}\sum_{j=1}^{6} x_{7qj}\varsigma_j(\mathbf{x}), \quad H_{ql}^{(9)}(\mathbf{x}) = \sum_{j=1}^{6} x_{9qlj}\varsigma_j(\mathbf{x}) \quad i = 1,2,3 \quad q,l = 1,\dots,5,$$

where

$$x_{1j} = -\frac{r^{(1)}_{3j}}{\tilde{A}\lambda_j^2}\tilde{N}^{(1)}_{11}(-\lambda_j^2), \quad x_{1e} = \frac{(-1)^{e+1}(\gamma\lambda_e^2 - \tilde{K})}{\tilde{N}\lambda_e^2(\lambda_8^2 - \lambda_7^2)}, \quad x_{2e} = \frac{(-1)^e K^*}{\tilde{N}(\lambda_8^2 - \lambda_7^2)},$$
$$x_{4e} = \frac{(-1)^{e+1}(\tilde{\mu}\lambda_e^2 - \rho\omega^2)}{\tilde{N}\lambda_e^2(\lambda_8^2 - \lambda_7^2)}, \quad x_{49} = -\frac{1}{\tilde{\alpha}\lambda_9^2}, \quad x_{5qj} = \frac{r^{(1)}_{3j}}{\tilde{A}}\tilde{N}^{(1)}_{1;q+1}(-\lambda_j^2),$$
$$x_{7qj} = -\frac{r^{(1)}_{3j}}{\tilde{A}\lambda_j^2}\tilde{N}^{(1)}_{q+1;1}(-\lambda_j^2), \quad x_{9qlj} = \frac{r^{(1)}_{3j}}{\tilde{A}}\tilde{N}^{(1)}_{q+1;l+1}(-\lambda_j^2) \quad j = 1,\ldots,6 \quad q,l = 1,\ldots,5 \quad e = 7,8. \tag{82}$$

Proof: From equation (75),

$$\Delta\varsigma_j(\mathbf{x}) = -\lambda_j^2\varsigma_j(\mathbf{x}) \quad j = 1,\ldots,9.$$

Thus, we have

$$-\frac{1}{\lambda_j^2}(\nabla\,\mathrm{div} - \mathrm{curl}\,\mathrm{curl})\varsigma_j(\mathbf{x}) = \mathbf{J}\,\varsigma_j(\mathbf{x}), \quad \mathbf{x} \neq \mathbf{0}$$

Consider

$$\mathbf{H}^{(1)}(\mathbf{x}) = \mathbf{R}^{(1)}(\mathbf{D_x})Y^{(1)}_{11}(\mathbf{x}) = \left\{\frac{1}{\tilde{N}}(\gamma\Delta + \tilde{K})\mathbf{J}\,\Gamma^{(1)}(\Delta) + w^{(1)}_{11}(\Delta)\,\nabla\,\mathrm{div}\right\}\sum_{j=1}^{8} r^{(1)}_{1j}\varsigma_j(\mathbf{x})$$
$$= \sum_{j=1}^{8} r^{(1)}_{1j}\left\{\left[-\frac{1}{\tilde{N}\lambda_j^2}(-\gamma\lambda_j^2 + \tilde{K})\Gamma^{(1)}(-\lambda_j^2) + w^{(1)}_{11}(-\lambda_j^2)\right]\nabla\,\mathrm{div}\right. \tag{83}$$
$$\left. + \frac{1}{\tilde{N}\lambda_j^2}(-\gamma\lambda_j^2 + \tilde{K})\Gamma^{(1)}(-\lambda_j^2)\,\mathrm{curl}\,\mathrm{curl}\right\}\varsigma_j(\mathbf{x}).$$

From equation (80), we have

$$w^{(1)}_{11}(-\lambda_j^2) = -\frac{1}{\tilde{A}\lambda_j^2}\Lambda^{(1)}(-\lambda_j^2)\tilde{N}^{(1)}_{11}(-\lambda_j^2) + \frac{1}{\tilde{N}\lambda_j^2}\Gamma^{(1)}(-\lambda_j^2)(-\gamma\lambda_j^2 + \tilde{K}).$$

Using above equation in equation (83), we get

$$\mathbf{H}^{(1)}(\mathbf{x}) = \sum_{j=1}^{8} r^{(1)}_{1j}\left\{\left[-\frac{1}{\tilde{A}\lambda_j^2}\Lambda^{(1)}(-\lambda_j^2)\tilde{N}^{(1)}_{11}(-\lambda_j^2)\right]\nabla\,\mathrm{div} + \frac{1}{\tilde{N}\lambda_j^2}(-\gamma\lambda_j^2 + \tilde{K})\Gamma^{(1)}(-\lambda_j^2)\,\mathrm{curl}\,\mathrm{curl}\right\}\varsigma_j(\mathbf{x}). \tag{84}$$

Now, and

$$\Gamma^{(1)}(-\lambda_j^2)r^{(1)}_{1j} = 0 \quad j = 1,\ldots,6 \qquad \Lambda^{(1)}(-\lambda_j^2)r^{(1)}_{1j} = r^{(1)}_{3j} \quad j = 1,\ldots,6$$
$$\Gamma^{(1)}(-\lambda_j^2)r^{(1)}_{1j} = \frac{(-1)^{j+1}}{\lambda_8^2 - \lambda_7^2} \quad j = 7,8 \qquad \Lambda^{(1)}(-\lambda_j^2)r^{(1)}_{1j} = 0 \quad j = 7,8. \tag{85}$$

By virtue of equation (85), equation (84) becomes

$$\mathbf{H}^{(1)}(\mathbf{x}) = \nabla\,\mathrm{div}\sum_{j=1}^{6}\left[-\frac{1}{\tilde{A}\lambda_j^2}r^{(1)}_{3j}\tilde{N}^{(1)}_{11}(-\lambda_j^2)\right]\varsigma_j(\mathbf{x}+)\mathrm{curl}\,\mathrm{curl}\sum_{e=7}^{8}\frac{(-1)^{e+1}(-\gamma\lambda_e^2 + \tilde{K})}{\tilde{N}\lambda_e^2(\lambda_8^2 - \lambda_7^2)}\varsigma_e(\mathbf{x})$$
$$= \nabla\,\mathrm{div}\sum_{j=1}^{6} x_{1j}\varsigma_j(\mathbf{x}) - \mathrm{curl}\,\mathrm{curl}\sum_{e=7}^{8} x_{1e}\varsigma_e(\mathbf{x})$$

The remaining formulae of above theorem can be proved in the similar way.
Lemma 3: If the condition (41) is satisfied, then

$$\begin{aligned}\sum_{j=1}^{6} r_{3j}^{(1)} &= \sum_{j=1}^{6} r_{3j}^{(1)}\lambda_j^2 = \sum_{j=1}^{6} r_{3j}^{(1)}\lambda_j^4 = \sum_{j=1}^{6} r_{3j}^{(1)}\lambda_j^6 = \sum_{j=1}^{6} r_{3j}^{(1)}\lambda_j^8 = 0,\\ \sum_{j=1}^{6} r_{3j}^{(1)}\lambda_j^{10} &= -1, \quad \sum_{j=1}^{6}\frac{r_{3j}^{(1)}}{\lambda_j^2} = \prod_{i=1}^{6}\lambda_i^{-2} = \frac{\tilde{A}}{\rho\omega^2\tilde{N}_{11}^{(1)}(0)},\end{aligned} \tag{86}$$

and

$$\sum_{j=1}^{6} x_{1j} = -(\rho\omega^2)^{-1}, \quad \sum_{j=1}^{6} x_{1j}\lambda_j^2 = -\tilde{\lambda}^{-1}, \quad \sum_{e=7}^{8} x_{1e}\lambda_e^2 = -\tilde{\mu}^{-1}. \tag{87}$$

Proof: Consider

$$\tilde{N}_{11}^{(1)}(-\lambda_j^2) = -KD\varrho\lambda_j^{10} + B_1\lambda_j^8 + B_2\lambda_j^6 + B_3\lambda_j^4 + B_4\lambda_j^2 + \tilde{N}_{11}^{(1)}(0), \tag{88}$$

where $B_p,\ p = 1, \ldots, 4$ are coefficients, independent of λ_j and skipped due to lengthy calculations.

Using equation (68), relations (86) can be proved by direct calculations.

From equations (86) and (88), we get

$$\begin{aligned}\sum_{j=1}^{6}\frac{r_{3j}^{(1)}}{\lambda_j^2}\tilde{N}_{11}^{(1)}(-\lambda_j^2) &= \sum_{j=1}^{6} r_{3j}^{(1)}[-KD\varrho\lambda_j^8 + B_1\lambda_j^6 + B_2\lambda_j^4 + B_3\lambda_j^2 + B_4 + \tilde{N}_{11}^{(1)}(0)\lambda_j^{-2}]\\ &= \tilde{N}_{11}^{(1)}(0)\sum_{j=1}^{6}\frac{r_{3j}^{(1)}}{\lambda_j^2} = \frac{\tilde{A}}{\rho\omega^2},\end{aligned}$$

and

$$\sum_{j=1}^{6} r_{3j}^{(1)}\tilde{N}_{11}^{(1)}(-\lambda_j^2) = \sum_{j=1}^{6} r_{3j}^{(1)}\big[-KD\varrho\lambda_j^{10} + B_1\lambda_j^8 + B_2\lambda_j^6 + B_3\lambda_j^4 + B_4\lambda_j^2 + \tilde{N}_{11}^{(1)}(0)\big] = KD\varrho.$$

Therefore, from equation (82), we have

$$\begin{aligned}\sum_{j=1}^{6} x_{1j} &= -\sum_{j=1}^{6}\frac{r_{3j}^{(1)}}{\tilde{A}\lambda_j^2}\tilde{N}_{11}^{(1)}(-\lambda_j^2) = -(\rho\omega^2)^{-1},\\ \sum_{j=1}^{6} x_{1j}\lambda_j^2 &= -\sum_{j=1}^{6}\frac{r_{3j}^{(1)}}{\tilde{A}}\tilde{N}_{11}^{(1)}(-\lambda_j^2) = -\frac{KD\varrho}{\tilde{A}} = -\tilde{\lambda}^{-1}.\end{aligned}$$

Also, we obtain

$$\sum_{e=7}^{8} x_{1e}\lambda_e^2 = -\frac{\gamma}{\tilde{N}} = -\tilde{\mu}^{-1}.$$

Theorem 8: The relations

$$G_{pq}^{(1)}(\mathbf{x}) - W_{pq}(\mathbf{x}) = \text{constant} + O(|\mathbf{x}|) \quad p, q = 1, \ldots, 11. \tag{89}$$

hold in the neighborhood of the origin.
Proof: Consider

$$\mathbf{H}^{(1)}(\mathbf{x}) - \mathbf{W}^{(1)}(\mathbf{x}) = \nabla \operatorname{div} \tilde{Y}_{11}(\mathbf{x}) - \operatorname{curl}\operatorname{curl} \tilde{Y}_{22}(\mathbf{x}), \tag{90}$$

where

$$\begin{aligned} \tilde{Y}_{11}(\mathbf{x}) &= \sum_{j=1}^{6} x_{1j}\varsigma_j(\mathbf{x}) - \frac{\varsigma_2^*(\mathbf{x})}{\tilde{\lambda}}, \\ \tilde{Y}_{22}(\mathbf{x}) &= \sum_{e=7}^{8} x_{1e}\varsigma_e(\mathbf{x}) - \frac{\varsigma_2^*(\mathbf{x})}{\tilde{\mu}}. \end{aligned} \tag{91}$$

From equation (91), we have

$$\begin{aligned} \tilde{Y}_{11}(\mathbf{x}) &= \sum_{j=1}^{6} \frac{-x_{1j}}{4\pi} \sum_{g=0}^{\infty} \frac{\iota^g \lambda_j^g}{g!} |\mathbf{x}|^{g-1} + \frac{|\mathbf{x}|}{8\pi\tilde{\lambda}} = -\frac{1}{8\pi}\left[2\sum_{j=1}^{6} x_{1j} \sum_{g=0}^{\infty} \frac{\iota^g \lambda_j^g}{g!}|\mathbf{x}|^{g-1} - \frac{|\mathbf{x}|}{\tilde{\lambda}}\right] \\ &= -\frac{1}{8\pi}\left[\frac{2}{|\mathbf{x}|}\sum_{j=1}^{6} x_{1j} - |\mathbf{x}|\left(\sum_{j=1}^{6} x_{1j}\lambda_j^2 + \frac{1}{\tilde{\lambda}}\right)\right] - \frac{\iota}{4\pi}\sum_{j=1}^{6} x_{1j}\lambda_j + \tilde{Y}_{33}(\mathbf{x}). \end{aligned} \tag{92}$$

Similarly,

$$\tilde{Y}_{22}(\mathbf{x}) = -\frac{1}{8\pi}\left[\frac{2}{|\mathbf{x}|}\sum_{e=7}^{8} x_{1e} - |\mathbf{x}|\left(\sum_{e=7}^{8} x_{1e}\lambda_e^2 + \frac{1}{\tilde{\mu}}\right)\right] - \frac{\iota}{4\pi}\sum_{e=7}^{8} x_{1e}\lambda_e + \tilde{Y}_{44}(\mathbf{x}), \tag{93}$$

where

$$\begin{aligned} \tilde{Y}_{33}(\mathbf{x}) &= -\frac{1}{4\pi}\sum_{j=1}^{6} x_{1j} \sum_{g=3}^{\infty} \frac{\iota^g \lambda_j^g}{g!}|\mathbf{x}|^{g-1}, \\ \tilde{Y}_{44}(\mathbf{x}) &= -\frac{1}{4\pi}\sum_{e=7}^{8} x_{1e} \sum_{g=3}^{\infty} \frac{\iota^g \lambda_e^g}{g!}|\mathbf{x}|^{g-1}. \end{aligned} \tag{94}$$

Clearly

$$\begin{aligned} \tilde{Y}_{qq}(\mathbf{x}) &= O(|\mathbf{x}|^2), \quad \frac{\partial}{\partial x_e}\tilde{Y}_{qq}(\mathbf{x}) = O(|\mathbf{x}|), \\ \frac{\partial^2}{\partial x_e \partial x_i}\tilde{Y}_{qq}(\mathbf{x}) &= \text{constant} + O(|\mathbf{x}|) \quad e, i = 1,2,3 \quad q = 3,4. \end{aligned} \tag{95}$$

Consider

$$\frac{\partial}{\partial x_i}\left(\frac{1}{|\mathbf{x}|}\right) = -\frac{x_i}{|\mathbf{x}|^3}, \quad \frac{\partial^2}{\partial x_i^2}\left(\frac{1}{|\mathbf{x}|}\right) = \left[\frac{3x_i^2}{|\mathbf{x}|^5} - \frac{1}{|\mathbf{x}|^3}\right]$$

Hence,

$$\Delta\frac{1}{|\mathbf{x}|} = \sum_{i=1}^{3}\frac{\partial^2}{\partial x_i^2}\left(\frac{1}{|\mathbf{x}|}\right) = 0.$$

Therefore,

$$(\nabla \operatorname{div} - \operatorname{curl}\operatorname{curl})\frac{1}{|\mathbf{x}|} = \mathbf{J}\Delta\frac{1}{|\mathbf{x}|} = \mathbf{0}. \tag{96}$$

Equation (90) with the aid of equations (87) and (92)-(96) becomes

$$\mathbf{H}^{(1)}(\mathbf{x}) - \mathbf{W}^{(1)}(\mathbf{x}) = \nabla \operatorname{div} \tilde{Y}_{33}(\mathbf{x}) - \operatorname{curl}\operatorname{curl} \tilde{Y}_{44}(\mathbf{x}) = \text{constant} + O(|\mathbf{x}|).$$

Similarly other formulae of equation (89) can be proved.

Therefore, matrix $\mathbf{W}(\mathbf{x})$ is the singular part of the fundamental matrix $\mathbf{G}^{(1)}(\mathbf{x})$ in the neighborhood of the origin.

VII. Summary and Conclusions

This paper focuses on deriving the linear theory of micropolar thermoelastic diffusion with triple porosity without utilizing Darcy's law. The Firstly, the governing equations are reduced in an isotropic medium. Subsequently, the fundamental solution $\mathbf{G}^{(1)}(\mathbf{x})$ for the system of equations (37) in the case of steady oscillations is obtained . Additionally, the fundamental solutions $\mathbf{G}^{(i)}(\mathbf{x}), \quad i = 2, 3, 4$ for the system of equations (38)-(40) in the cases of pseudo-static, quasi-static oscillations, and equilibrium are also obtained. The fundamental solution $\mathbf{G}^{(1)}(\mathbf{x})$ derived for the system of equations (37) allows for the investigation of three-dimensional boundary value problems in the theory of triple porosity micropolar thermoelastic diffusion elastic solids using the potential method. Furthermore, this method enables the construction of fundamental solutions for the system of equations in the linear theory of isotropic micropolar thermoelastic materials with triple porosity. By obtaining these fundamental solutions, a basis for analyzing and solving boundary value problems within the linear theory of micropolar thermoelastic diffusion with triple porosity is provided. The derived fundamental solutions contribute to understanding the behavior and properties of these materials under various conditions.

VIII. Appendix A

$$\begin{aligned}
F^{(1)}_{pq}(\mathbf{D}_{\mathbf{x}}) &= [\tilde{\mu}\Delta + \rho\omega^2]\delta_{pq} + (\lambda' + \mu)\frac{\partial^2}{\partial x_p \partial x_q},\\
F^{(1)}_{p;q+3}(\mathbf{D}_{\mathbf{x}}) &= F^{(1)}_{p+3;q}(\mathbf{D}_{\mathbf{x}}) = K^* \sum_{j=1}^{3} \varepsilon_{pjq}\frac{\partial}{\partial x_j}, \quad F^{(1)}_{p;q+6}(\mathbf{D}_{\mathbf{x}}) = -F^{(1)}_{q+6;p}(\mathbf{D}_{\mathbf{x}}) = \sigma_q \frac{\partial}{\partial x_p},\\
F^{(1)}_{p;10}(\mathbf{D}_{\mathbf{x}}) &= -\zeta_1 \frac{\partial}{\partial x_p}, \quad F^{(1)}_{p;11}(\mathbf{D}_{\mathbf{x}}) = -\zeta_2 \frac{\partial}{\partial x_p},\\
F^{(1)}_{p+3;q+3}(\mathbf{D}_{\mathbf{x}}) &= (\gamma\Delta + \tilde{K})\delta_{pq} + (\alpha + \beta)\frac{\partial^2}{\partial x_p \partial x_q}, \quad F^{(1)}_{p+3;k}(\mathbf{D}_{\mathbf{x}}) = F^{(1)}_{k;p+3}(\mathbf{D}_{\mathbf{x}}) = 0,\\
F^{(1)}_{p+6;p+6}(\mathbf{D}_{\mathbf{x}}) &= A_p\Delta - \gamma_p, \quad F^{(1)}_{78}(\mathbf{D}_{\mathbf{x}}) = F^{(1)}_{87}(\mathbf{D}_{\mathbf{x}}) = A_4\Delta - \beta_4,\\
F^{(1)}_{79}(\mathbf{D}_{\mathbf{x}}) &= F^{(1)}_{97}(\mathbf{D}_{\mathbf{x}}) = A_6\Delta - \beta_6, \quad F^{(1)}_{89}(\mathbf{D}_{\mathbf{x}}) = F^{(1)}_{98}(\mathbf{D}_{\mathbf{x}}) = A_5\Delta - \beta_5,\\
F^{(1)}_{p+6;10}(\mathbf{D}_{\mathbf{x}}) &= \xi_p, \quad F^{(1)}_{p+6;11}(\mathbf{D}_{\mathbf{x}}) = \upsilon_p, \quad F^{(1)}_{10;p}(\mathbf{D}_{\mathbf{x}}) = \tau_1\zeta_1 T_0 \frac{\partial}{\partial x_p},\\
F^{(1)}_{10;p+6}(\mathbf{D}_{\mathbf{x}}) &= \tau_1\xi_p T_0, \quad F^{(1)}_{10;10}(\mathbf{D}_{\mathbf{x}}) = K\Delta + \tau_1\eta T_0,\\
F^{(1)}_{10;11}(\mathbf{D}_{\mathbf{x}}) &= \tau_1\varsigma T_0, \quad F^{(1)}_{11;p}(\mathbf{D}_{\mathbf{x}}) = \tau^1\zeta_2\frac{\partial}{\partial x_p}, \quad F^{(1)}_{11;p+6}(\mathbf{D}_{\mathbf{x}}) = \tau^1\upsilon_p,\\
F^{(1)}_{11;10}(\mathbf{D}_{\mathbf{x}}) &= \tau^1\varsigma, \quad F^{(1)}_{11;11}(\mathbf{D}_{\mathbf{x}}) = D\Delta + \tau^1\varpi,\\
\tilde{\mu} &= \mu + K^*, \tilde{K} = -2K^* + \rho J\omega^2, \quad p, q = 1, 2, 3 \quad k = 7, \ldots, 11.\\
\tilde{F}_{pq}(\mathbf{D}_{\mathbf{x}}) &= \tilde{\mu}\Delta\delta_{pq} + (\lambda' + \mu)\frac{\partial^2}{\partial x_p \partial x_q},\\
\tilde{F}_{p+3;q+3}(\mathbf{D}_{\mathbf{x}}) &= \gamma\Delta\delta_{pq} + (\alpha + \beta)\frac{\partial^2}{\partial x_p \partial x_q},\\
\tilde{F}_{p+6;p+6}(\mathbf{D}_{\mathbf{x}}) &= A_p\Delta, \quad \tilde{F}_{78}(\mathbf{D}_{\mathbf{x}}) = \tilde{F}_{87}(\mathbf{D}_{\mathbf{x}}) = A_4\Delta, \quad \tilde{F}_{79}(\mathbf{D}_{\mathbf{x}}) = \tilde{F}_{97}(\mathbf{D}_{\mathbf{x}}) = A_6\Delta,
\end{aligned}$$

$$
\begin{aligned}
\tilde{F}_{89}(\mathbf{D_x}) &= \tilde{F}_{98}(\mathbf{D_x}) = A_5\Delta, \quad \tilde{F}_{10;10}(\mathbf{D_x}) = K\Delta,\\
\tilde{F}_{11;11}(\mathbf{D_x}) &= D\Delta, \quad \tilde{F}_{p;q+3}(\mathbf{D_x}) = \tilde{F}_{p+3;q}(\mathbf{D_x}) = 0\\
\tilde{F}_{le}(\mathbf{D_x}) &= \tilde{F}_{el}(\mathbf{D_x}) = \tilde{F}_{ij}(\mathbf{D_x}) = \tilde{F}_{ji}(\mathbf{D_x}) = \tilde{F}_{ky}(\mathbf{D_x}) = 0\\
p,q &= 1,2,3 \quad l=7,\dots,11 \quad i=7,8 \quad j=10,11 \quad k,y=9,10,11 \quad k\neq y \quad e=1,\dots,6.
\end{aligned}
$$

IX. Appendix B

$\mathbf{S} = (\operatorname{div}\mathbf{u}, \boldsymbol{\phi}, \theta, P)$, $\tilde{\mathbf{Q}} = (\chi_1,\dots,\chi_6) = (\operatorname{div}\mathbf{H}, X_i, Y, Z)$ and

$$
\mathbf{N}^{(1)}(\Delta) = \left(N^{(1)}_{gh}(\Delta)\right)_{6\times6} =
$$

$$
\begin{pmatrix}
\tilde{\lambda}\Delta+\rho\omega^2 & -\sigma_1\Delta & -\sigma_2\Delta & -\sigma_3\Delta & \tau_1\zeta_1T_0\Delta & \tau^1\zeta_2\Delta\\
\sigma_1 & A_1\Delta-\gamma_1 & A_4\Delta-\beta_4 & A_6\Delta-\beta_6 & \tau_1\xi_1T_0 & \tau^1 v_1\\
\sigma_2 & A_4\Delta-\beta_4 & A_2\Delta-\gamma_2 & A_5\Delta-\beta_5 & \tau_1\xi_2T_0 & \tau^1 v_2\\
\sigma_3 & A_6\Delta-\beta_6 & A_5\Delta-\beta_5 & A_3\Delta-\gamma_3 & \tau_1\xi_3T_0 & \tau^1 v_3\\
-\zeta_1 & \xi_1 & \xi_2 & \xi_3 & K\Delta+\tau_1\eta T_0 & \tau^1\varsigma\\
-\zeta_2 & v_1 & v_2 & v_3 & \tau_1\varsigma T_0 & D\Delta+\tau^1\varpi
\end{pmatrix}_{6\times6},
$$

$$
\mathbf{\Psi} = (\Psi_1,\dots,\Psi_6), \Psi_p = \frac{1}{\tilde{A}}\sum_{i=1}^{6}\tilde{N}^{(1)}_{ip}(\Delta)\varphi_i,
$$

$$
\Gamma^{(1)}(\Delta) = \frac{1}{\tilde{A}}\,|\mathbf{N}^{(1)}(\Delta)|, \quad \tilde{A} = \tilde{\lambda}KD\varrho \quad p=1,\dots,6,
$$

and $\tilde{N}^{(1)}_{ip}$ is the cofactor of the element $N^{(1)}_{ip}$ of the matrix $\mathbf{N}^{(1)}$.

$$
\Lambda^{(1)}(\Delta) = \frac{1}{\tilde{N}}\begin{vmatrix} \tilde{\mu}\Delta+\rho\omega^2 & K^*\Delta\\ -K^* & \gamma\Delta+\tilde{K}\end{vmatrix}, \quad \tilde{N} = \gamma\tilde{\mu},
$$

$$
\begin{aligned}
\mathbf{\Psi}' &= \frac{1}{\tilde{N}}\Big\{-\Big[(\gamma\Delta+\tilde{K})(\lambda'+\mu)-K^{*^2}\Big]\nabla\Psi_1 + (\gamma\Delta+\tilde{K})\times\\
&\quad \Big[\Gamma^{(1)}(\Delta)\mathbf{H}+\sigma_i\nabla\Psi_{i+1}-\tau_1\zeta_1T_0\nabla\Psi_5-\tau^1\zeta_2\nabla\Psi_6\Big]-K^*\Gamma^{(1)}(\Delta)\operatorname{curl}\mathbf{V}\Big\},\\
\mathbf{\Psi}'' &= \frac{1}{\tilde{N}}\Big\{-\Big[(\alpha+\beta)(\tilde{\mu}\Delta+\rho\omega^2)-K^{*^2}\Big]\nabla\Psi_7+\\
&\quad (\tilde{\mu}\Delta+\rho\omega^2)(\Delta+\lambda_9^2)\mathbf{V}-K^*(\Delta+\lambda_9^2)\operatorname{curl}\mathbf{H}\Big\}.
\end{aligned}
$$

X. Appendix C

$$
\begin{aligned}
\hat{\mathbf{\Psi}} &= (\mathbf{\Psi}', \mathbf{\Psi}'', \Psi_2,\dots,\Psi_6),\\
\mathbf{\Theta}^{(1)}(\Delta) &= \left(\Theta^{(1)}_{gh}(\Delta)\right)_{11\times11},\\
\Theta^{(1)}_{pp}(\Delta) &= \Gamma^{(1)}(\Delta)\Lambda^{(1)}(\Delta) = \prod_{i=1}^{8}(\Delta+\lambda_i^2),\\
\Theta^{(1)}_{p+3;p+3}(\Delta) &= \Lambda^{(1)}(\Delta)(\Delta+\lambda_9^2) = \prod_{i=7}^{9}(\Delta+\lambda_i^2),
\end{aligned}
$$

$$\begin{aligned}
\Theta_{ll}^{(1)}(\Delta) &= \Gamma^{(1)}(\Delta) = \prod_{i=1}^{6}(\Delta + \lambda_i^2), \Theta_{gh}^{(1)}(\Delta) = 0,\\
p &= 1,2,3 \quad g,h = 1,\dots,11 \quad l = 7,\dots,11 \quad g \neq h\\
w_{11}^{(1)}(\Delta) &= -\frac{1}{\tilde{A}\tilde{N}}\Bigg\{\Bigg[(\lambda' + \mu)(\gamma\Delta + \tilde{K}) - K^{*^2}\Bigg]\tilde{N}_{11}^{(1)}(\Delta) + (\gamma\Delta + \tilde{K}) \times\\
&\quad \Bigg[-\sigma_k \tilde{N}_{1;k+1}^{(1)}(\Delta) + \tau_1\zeta_1 T_0 \tilde{N}_{15}^{(1)}(\Delta) + \tau^1\zeta_2\tilde{N}_{16}^{(1)}(\Delta)\Bigg]\Bigg\},\\
w_{p+1;1}^{(1)}(\Delta) &= -\frac{1}{\tilde{A}\tilde{N}}\Bigg\{\Bigg[(\lambda' + \mu)(\gamma\Delta + \tilde{K}) - K^{*^2}\Bigg]\tilde{N}_{p1}^{(1)}(\Delta) + (\gamma\Delta + \tilde{K}) \times\\
&\quad \Bigg[-\sigma_k \tilde{N}_{p;k+1}^{(1)}(\Delta) + \tau_1\zeta_1 T_0 \tilde{N}_{p5}^{(1)}(\Delta) + \tau^1\zeta_2\tilde{N}_{p6}^{(1)}(\Delta)\Bigg]\Bigg\},\\
w_{21}^{(1)}(\Delta) &= -\frac{K^*\Gamma^{(1)}(\Delta)}{\tilde{N}}, \quad w_{12}^{(1)}(\Delta) = -\frac{K^*(\Delta + \lambda_9^2)}{\tilde{N}},\\
w_{22}^{(1)}(\Delta) &= -\frac{(\alpha+\beta)(\tilde{\mu}\Delta + \rho\omega^2) - K^{*^2}}{\tilde{N}\tilde{\alpha}},\\
w_{1;p+1}^{(1)}(\Delta) &= \frac{\tilde{N}_{1p}^{(1)}(\Delta)}{\tilde{A}}, \quad w_{i+1;p+1}^{(1)}(\Delta) = \frac{\tilde{N}_{ip}^{(1)}(\Delta)}{\tilde{A}},\\
w_{2;p+1}^{(1)}(\Delta) &= w_{p+1;2}^{(1)}(\Delta) = 0 \quad i,p = 2,....,6.\\
\mathbf{R}(\mathbf{D_x}) &= \Big(R_{gh}(\mathbf{D_x})\Big)_{11\times 11} = \begin{pmatrix} \mathbf{R}^{(1)} & \mathbf{R}^{(2)} & \mathbf{R}^{(5)}\\ \mathbf{R}^{(3)} & \mathbf{R}^{(4)} & \mathbf{R}^{(6)}\\ \mathbf{R}^{(7)} & \mathbf{R}^{(8)} & \mathbf{R}^{(9)} \end{pmatrix}_{11\times 11},\\
\mathbf{R}^{(i)}(\mathbf{D_x}) &= \Big(R_{gh}^{(i)}(\mathbf{D_x})\Big)_{3\times 3}, \quad \mathbf{R}^{(j)}(\mathbf{D_x}) = \Big(R_{gh}^{(j)}(\mathbf{D_x})\Big)_{3\times 5},\\
\mathbf{R}^{(l)}(\mathbf{D_x}) &= \Big(R_{gh}^{(l)}(\mathbf{D_x})\Big)_{5\times 3}, \quad \mathbf{R}^{(9)}(\mathbf{D_x}) = \Big(R_{gh}^{(9)}(\mathbf{D_x})\Big)_{5\times 5},
\end{aligned}$$

$$\begin{aligned}
R_{gh}^{(1)}(\mathbf{D_x}) &= \frac{1}{\tilde{N}}(\gamma\Delta + \tilde{K})\Gamma^{(1)}(\Delta)\delta_{gh} + w_{11}^{(1)}(\Delta)\frac{\partial^2}{\partial x_g \partial x_h},\\
R_{gh}^{(2)}(\mathbf{D_x}) &= w_{12}^{(1)}(\Delta)\sum_{p=1}^{3}\varepsilon_{gph}\frac{\partial}{\partial x_p}, \quad R_{gh}^{(3)}(\mathbf{D_x}) = w_{21}^{(1)}(\Delta)\sum_{p=1}^{3}\varepsilon_{gph}\frac{\partial}{\partial x_p},\\
R_{gh}^{(4)}(\mathbf{D_x}) &= \frac{1}{\tilde{N}}(\Delta + \lambda_9^2)(\tilde{\mu}\Delta + \rho\omega^2)\delta_{gh} + w_{22}^{(1)}(\Delta)\frac{\partial^2}{\partial x_g \partial x_h},\\
R_{gh}^{(5)}(\mathbf{D_x}) &= w_{1;h+2}^{(1)}(\Delta)\frac{\partial}{\partial x_g}, \quad R_{gh}^{(6)}(\mathbf{D_x}) = R_{gh}^{(8)}(\mathbf{D_x}) = 0,\\
R_{gh}^{(7)}(\mathbf{D_x}) &= w_{g+2;1}^{(1)}(\Delta)\frac{\partial}{\partial x_h}, \quad R_{gh}^{(9)}(\mathbf{D_x}) = w_{g+2;h+2}^{(1)}(\Delta) \quad i = 1,...,4 \quad j = 5,6 \quad l = 7,8.
\end{aligned}$$

XI. Appendix D

$$\begin{aligned}
z_1 &= \prod_{i=3}^{8}(\lambda_2^2 - \lambda_i^2)\prod_{j=4}^{8}(\lambda_3^2 - \lambda_j^2)\prod_{l=5}^{8}(\lambda_4^2 - \lambda_l^2)\prod_{p=6}^{8}(\lambda_5^2 - \lambda_p^2)\prod_{k=7}^{8}(\lambda_6^2 - \lambda_k^2)(\lambda_7^2 - \lambda_8^2),\\
z_2 &= \prod_{i=3}^{8}(\lambda_1^2 - \lambda_i^2)\prod_{j=4}^{8}(\lambda_3^2 - \lambda_j^2)\prod_{l=5}^{8}(\lambda_4^2 - \lambda_l^2)\prod_{p=6}^{8}(\lambda_5^2 - \lambda_p^2)\prod_{k=7}^{8}(\lambda_6^2 - \lambda_k^2)(\lambda_7^2 - \lambda_8^2),
\end{aligned} \tag{97}$$

$$
\begin{aligned}
z_3 &= \prod_{i=2,i\neq 3}^{8}(\lambda_1^2-\lambda_i^2)\prod_{j=4}^{8}(\lambda_2^2-\lambda_j^2)\prod_{l=5}^{8}(\lambda_4^2-\lambda_l^2)\prod_{p=6}^{8}(\lambda_5^2-\lambda_p^2)\prod_{k=7}^{8}(\lambda_6^2-\lambda_k^2)(\lambda_7^2-\lambda_8^2),\\
z_4 &= \prod_{i=2,i\neq 4}^{8}(\lambda_1^2-\lambda_i^2)\prod_{j=3,j\neq 4}^{8}(\lambda_2^2-\lambda_j^2)\prod_{l=5}^{8}(\lambda_3^2-\lambda_l^2)\prod_{p=6}^{8}(\lambda_5^2-\lambda_p^2)\prod_{k=7}^{8}(\lambda_6^2-\lambda_k^2)(\lambda_7^2-\lambda_8^2),\\
z_5 &= \prod_{i=2,i\neq 5}^{8}(\lambda_1^2-\lambda_i^2)\prod_{j=3,j\neq 5}^{8}(\lambda_2^2-\lambda_j^2)\prod_{l=4,l\neq 5}^{8}(\lambda_3^2-\lambda_l^2)\prod_{p=6}^{8}(\lambda_4^2-\lambda_p^2)\prod_{k=7}^{8}(\lambda_6^2-\lambda_k^2)(\lambda_7^2-\lambda_8^2),\\
z_6 &= \prod_{i=2,i\neq 6}^{8}(\lambda_1^2-\lambda_i^2)\prod_{j=3,j\neq 6}^{8}(\lambda_2^2-\lambda_j^2)\prod_{l=4,l\neq 6}^{8}(\lambda_3^2-\lambda_l^2)\prod_{p=5,p\neq 6}^{8}(\lambda_4^2-\lambda_p^2)\prod_{k=7}^{8}(\lambda_5^2-\lambda_k^2)(\lambda_7^2-\lambda_8^2),\\
z_7 &= \prod_{i=2,i\neq 7}^{8}(\lambda_1^2-\lambda_i^2)\prod_{j=3,j\neq 7}^{8}(\lambda_2^2-\lambda_j^2)\prod_{l=4,l\neq 7}^{8}(\lambda_3^2-\lambda_l^2)\prod_{p=5,p\neq 7}^{8}(\lambda_4^2-\lambda_p^2)\prod_{k=6,k\neq 7}^{8}(\lambda_5^2-\lambda_k^2)(\lambda_6^2-\lambda_8^2),\\
z_8 &= \prod_{i=2}^{7}(\lambda_1^2-\lambda_i^2)\prod_{j=3}^{7}(\lambda_2^2-\lambda_j^2)\prod_{l=4}^{7}(\lambda_3^2-\lambda_l^2)\prod_{p=5}^{7}(\lambda_4^2-\lambda_p^2)\prod_{k=6}^{7}(\lambda_5^2-\lambda_k^2)(\lambda_6^2-\lambda_7^2),\\
z_9 &= \prod_{i=2}^{8}(\lambda_1^2-\lambda_i^2)\prod_{j=3}^{8}(\lambda_2^2-\lambda_j^2)\prod_{l=4}^{8}(\lambda_3^2-\lambda_l^2)\prod_{p=5}^{8}(\lambda_4^2-\lambda_p^2)\prod_{q=6}^{8}(\lambda_5^2-\lambda_q^2)\prod_{k=7}^{8}(\lambda_6^2-\lambda_k^2)(\lambda_7^2-\lambda_8^2).
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{H}^{(e)}(\mathbf{x}) &= \left(H_{gh}^{(e)}(\mathbf{x})\right)_{3\times 3}, \quad \mathbf{H}^{(y)}(\mathbf{x}) = \left(H_{gh}^{(y)}(\mathbf{x})\right)_{3\times 5},\\
\mathbf{H}^{(z)}(\mathbf{x}) &= \left(H_{gh}^{(z)}(\mathbf{x})\right)_{5\times 3}, \quad \mathbf{H}^{(9)}(\mathbf{x}) = \left(H_{gh}^{(9)}(\mathbf{x})\right)_{5\times 5},\\
\mathbf{H}^{(i)}(\mathbf{x}) &= \mathbf{R}^{(i)}(\mathbf{D_x})Y_{11}^{(1)}(\mathbf{x}), \quad \mathbf{H}^{(j)}(\mathbf{x}) = \mathbf{R}^{(j)}(\mathbf{D_x})Y_{44}^{(1)}(\mathbf{x}),\\
\mathbf{H}^{(l)}(\mathbf{x}) &= \mathbf{R}^{(l)}(\mathbf{D_x})Y_{77}^{(1)}(\mathbf{x}), \quad \mathbf{H}^{(q)}(\mathbf{x}) = \mathbf{0},\\
e &= 1,2,3,4 \quad y=5,6 \quad z=7,8 \quad i=1,3,7 \quad j=2,4 \quad l=5,9 \quad q=6,8.
\end{aligned}
$$

XII. Appendix E

$$
\begin{aligned}
\mathbf{W}^{(i)}(\mathbf{x}) &= \left(W_{gh}^{(i)}(\mathbf{x})\right)_{3\times 3}, \quad \mathbf{W}^{(j)}(\mathbf{x}) = \left(W_{gh}^{(j)}(\mathbf{x})\right)_{3\times 5}, \quad \mathbf{W}^{(l)}(\mathbf{x}) = \left(W_{gh}^{(l)}(\mathbf{x})\right)_{5\times 3},\\
\mathbf{W}^{(9)}(\mathbf{x}) &= \left(W_{gh}^{(9)}(\mathbf{x})\right)_{5\times 5}, \quad W_{gh}^{(1)}(\mathbf{x}) = \left[\frac{1}{\tilde{\lambda}}\nabla\operatorname{div} - \frac{1}{\tilde{\mu}}\operatorname{curl}\operatorname{curl}\right]\varsigma_2^*(\mathbf{x}),\\
W_{gh}^{(q)}(\mathbf{x}) &= 0, \quad W_{gh}^{(4)}(\mathbf{x}) = \left[\frac{1}{\tilde{\alpha}}\nabla\operatorname{div} - \frac{1}{\gamma}\operatorname{curl}\operatorname{curl}\right]\varsigma_2^*(\mathbf{x}),\\
W_{11}^{(9)}(\mathbf{x}) &= \frac{A_2A_3 - A_5^2}{\varrho}\varsigma_1^*(\mathbf{x}),\\
W_{12}^{(9)}(\mathbf{x}) &= W_{21}^{(9)}(\mathbf{x}) = \frac{A_5A_6 - A_4A_3}{\varrho}\varsigma_1^*(\mathbf{x}), \quad W_{22}^{(9)}(\mathbf{x}) = \frac{A_1A_3 - A_6^2}{\varrho}\varsigma_1^*(\mathbf{x}),\\
W_{33}^{(9)}(\mathbf{x}) &= \frac{A_1A_2 - A_4^2}{\varrho}\varsigma_1^*(\mathbf{x}), \quad W_{13}^{(9)}(\mathbf{x}) = W_{31}^{(9)}(\mathbf{x}) = \frac{A_4A_5 - A_2A_6}{\varrho}\varsigma_1^*(\mathbf{x}),\\
W_{23}^{(9)}(\mathbf{x}) &= W_{32}^{(9)}(\mathbf{x}) = \frac{A_4A_6 - A_1A_5}{\varrho}\varsigma_1^*(\mathbf{x}), \quad W_{ey}^{(9)}(\mathbf{x}) = W_{ye}^{(9)}(\mathbf{x}) = 0,\\
W_{44}^{(9)}(\mathbf{x}) &= \frac{1}{K}\varsigma_1^*(\mathbf{x}), \quad W_{55}^{(9)}(\mathbf{x}) = \frac{1}{D}\varsigma_1^*(\mathbf{x}), \quad W_{45}^{(9)}(\mathbf{x}) = W_{54}^{(9)}(\mathbf{x}) = 0,\\
i &= 1,\ldots,4 \quad j=5,6 \quad l=7,8 \quad e=1,2,3 \quad y=4,5 \quad q=2,3,5,6,7,8.
\end{aligned}
$$

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