An Attempt to Model the Main and Normal Cutting Forces Dependencies between Cutting Parameters and Some Strength Properties of Wood by Flat Longitudinal Cutting

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Abstract: This paper evaluates multi-factor, non-linear dependencies of main (F_C) and normal (trust) (F_N) cutting forces, from chip thickness (a_p), rake angle (γ_F) and some strength properties, namely the compression strength parallel to wood fibers ($R_{C\|}$), tensile strength perpendicular to wood fibers ($R_{T\perp}$), shearing strength parallel to wood fibers ($R_{S\|}$) and bending strength (R_B), for three North American wood species (analyzed altogether), by opened, flat, longitudinal cutting and very low cutting speed, based on work [1]. In the analyzed relations, several strong interactions were evidenced, graphically illustrated and discussed. Although a wood density (D) and a moisture content (m_c), were not taken into account in this study, very good fit was obtained. The lowest influence of the $R_{C\|}$ on the F_C and the F_N cutting forces was observed.

Key words: Yellow Birch, Sugar Pine, White Ash, strength properties, main cutting force, normal cutting force, chip thickness, rake angle, opened cutting, longitudinal cutting, orthogonal cutting, multi-variable non-linear formulas

I. Introduction

Fundamental studies on opened, flat, longitudinal and orthogonal cutting ($\|$) ($\varphi_E = 90°$, $\varphi_V = 0°$, $\varphi_C = 0°$), were started for oak, beech, birch, pine and poplar [2], and were continued over the years by many authors [1, 3–5]. The theoretical study [3] aimed at understanding and modeling dependencies of the F_C, and the F_N, cutting forces from chip thickness (a_P), and cutting angle (δ_F). This study was conducted for air dry state of wood and all main and intermediate cutting cases, in directions perpendicular ($\perp$), parallel ($\|$) and transversal ($\nparallel$) to wood fibers. Strength properties like (1) the $R_{C\|}$, and compression strength perpendicular to wood fibers ($R_{C\perp}$); (2) the $R_{S\|}$ and shearing strength perpendicular to wood fibers ($R_{S\perp}$); (3) tensile strength parallel to wood fibers ($R_{T\|}$) and the $R_{T\perp}$ were taken into account in this work. Cutting speed (v_C) was not examined herein.

The work [1] was done by opened, linear, low v_C, orthogonal and parallel cutting ($\|$) (Fig. 1), which in precise notation can be written as: $\varphi_E = 90°$, $\varphi_V = 0°$, $\varphi_C = 0°$. In the notation proposed for the first time by [4], this cutting situation can be expressed as: B, 90-0. This imprecise notation is contemporarily in use in many countries. Although work [1] evaluated mechanical properties for wood of Sugar Pine (*Pinus lambertiana* Dougl.), Yellow Birch (*Betula alleghansis* Britt.) and White Ash (*Fraxinus americana* L), by three different m_C, further analysis of the cutting forces was conducted without the m_C. It has to be noted that in work [1], the D of examined wood specimens was not evaluated. In work [1], all attention was put on evaluating 2D relations between F_C and F_N and two cutting parameters, namely a_P, and γ_F, without statistical analysis. This work also explains chip formation and shearing angle, with application of a_P and γ_F, as well as strength properties (R_B, $R_{C\|}$, $R_{T\perp}$,

$R_{S\|}$) and moduli of elasticity by bending, stretching and compression parallel to wood fibers (E_B, $E_{T\perp}$, $E_{C\|}$). A significant relation between independent variables and F_C and F_N as well as many interactions between a_P, γ_F and m_C can be seen on the 2D graphs, but they were not evaluated or specified. The choice of low v_C was supported by lack of influence $F_C = f(v_C)$ and $F_N = f(v_C)$, found in preliminary tests.

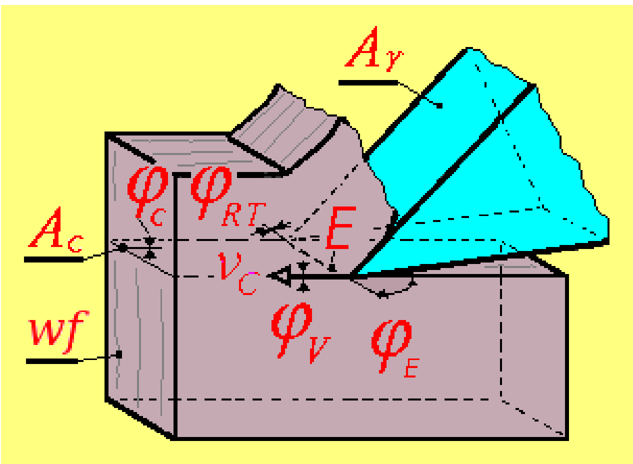

Fig. 1. Defining the orientation angles for longitudinal cutting case ($\|$) ($\varphi_E = 90°$, $\varphi_V = 0°$, $\varphi_C = 0°$, $\varphi_{RT} = 0°$), between the wood fibers *wf* and: φ_V – the vector of cutting speed v_C, φ_E – the cutting edge E, φ_c – the cutting plane A_C; φ_{RT} – is the angle between the wood growth rings and the cutting edge E; A_γ – is the rake face

The lack of influence $F_C = f(v_C)$ and $F_N = f(v_C)$, was also observed in the earlier experiment [4]. However, in more modern literature [5–8], the influence $F_C = f(v_C)$ and $F_N = f(v_C)$, starting from $v_C = 15\ \mathrm{m \cdot s^{-1}}$ has been found out. In recent works a significant influence $F_C = f(v_C)$ was confirmed [9, 10].

The wood species coefficient (C_R) appeared in formulas for calculation of cutting forces in works [5–8]. According to the author, this was a step backwards in the precise consideration of the influence of wood properties on cutting forces. The strength properties of wood were used to achieve this goal in newer works. Work [11] attempts to include the influence of several mechanical properties of wood (different than those analyzed in previously cited works) on C_R, which has to be understood a small step forward. However, in work [11], F_C was confused with the resultant cutting force (F_R). This made it impossible to separate F_C, and F_N from the resultant F_R. In most recent work [12], during longitudinal milling of oak wood (*Quercus robra*), the relation between F_C and four moduli of elasticity parallel and perpendicular to wood fibers, respectively: stretching $E_{S\|}$ and $E_{S\perp}$, and compression $E_{C\|}$ and $E_{C\perp}$, $F_C = f(c_D, f_Z, \gamma_F, E_{S\|}, E_{S\perp}, E_{C\|}, E_{C\perp})$ was successfully developed. The present study also took into account three machining parameters: (1) cutting depth (c_D); (2) feed rate per edge (f_Z); and (3) γ_F. D was not included herein.

The present work attempts to evaluate statistical, nonlinear, and multi-variable dependencies $F_C = f(a_P, \gamma_F, R_B, R_{C\|}, R_{R\perp}, R_{S\|})$ and $F_N = f(a_P, \gamma_F, R_B, R_{C\|}, R_{R\perp}, R_{S\|})$, basing the results of the experiment performed in work [1]. In the present study, a modern elaboration of the research results published in work [1] was undertaken due to the lack of similar, extensive research in the literature, in which the effects of cutting parameters and strength properties of many types of wood (at different humidity) on both F_C and F_N cutting forces are analyzed. According to the present author, the work shows how to break stagnation in this area of interest that has been observed for decades.

II. Materials and Methods

Two axes (1.2 kN), strain gauge, connected to a 2 channel oscilloscope via electronic bridge (Fig. 2), having natural frequency of 1.5 kHz, was used for cutting forces measurements in work [1].

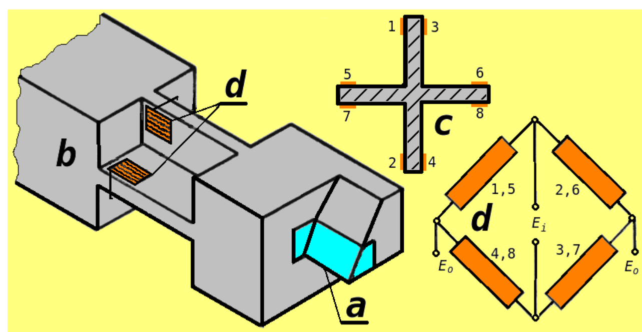

Fig. 2. Sketch of a two component strain gauge dynamometer; a) cutting edge; b) tool holder; c) cross-section of the tool holder with positions of strain gauge resistors; d) measuring bridge; $1, \ldots, 8$ – numbers of strain gauge resistors; Ac. to [1]

In work [1], 368 tests with 3 repetitions were performed for the following machining conditions:

Machining parameters:

- chip thickness, $a_P = 0.051$, 0.127, 0.254, 0.381, 0.508, 0.762 mm (0.002, 0.005, 0.01, 0.015, 0.02, 0.03 in),
- cutting speed, $v_C = 0.00108\ \mathrm{m \cdot s^{-1}}$ ($3.5\ \mathrm{in \cdot min^{-1}}$),
- width of cut, $w_C = 6.35$ mm (0.25 in),
- angle of the cutting edge direction towards wood fibers orientation, $\varphi_E = 90°$,
- angle of the direction of the v_C vector towards wood fibers orientation, $\varphi_V = 0°$,
- angle of the cutting plain direction against wood fibers orientation, $\varphi_C = 0°$,
- angle of the wood season's rings orientation towards cutting edge, $\varphi_{RT} = 0°$.

Parameters of the cutting edge:

- rake angle, $\gamma_F = 5°, 10°, 15°, 20°, 25°, 30°$,
- clearance angle, $\alpha_F = 15°$,
- bevel angle, $\lambda_P = 0°$,
- number of cutting edges, $z = 1$,
- roughness of rake and clearance surfaces (R_a) measured perpendicular to chip flow, 0.18–0.3 μm (7–12 μin),
- sharp cutting edge, radius of the cutting edge $\rho \sim \sim 2$ μm (78.4 μin),
- material of the cutting edge, high speed steel (HSS).

Mechanical properties of wood specimens:

- bending strength, R_B $\langle 36.54; 160.65\rangle$ MPa, $\langle 5300; 23300\rangle$ psi,
- modulus of elasticity by bending, E_B $\langle 6.48; 12.89\rangle$ MPa, $\langle 940; 1850\rangle$ psi,
- compression strength parallel to wood fibers, $R_{C\|}$ $\langle 15.93; 85.7\rangle$ MPa, $\langle 2310; 12430\rangle$ psi,
- modulus of elasticity by compression parallel to wood fibers, $E_{C\|}$ $\langle 8.62; 15.51\rangle$ MPa, $\langle 27400; 134000\rangle$ psi,
- cleavage, R_C $\langle 24.52; 105.08\rangle$ N·mm^{-1}, $\langle 140; 600\rangle$ lb·in^{-1},
- tensile strength perpendicular to wood fibers, $R_{T\perp}$ $\langle 1.62; 6.89\rangle$ MPa, $\langle 235; 1000\rangle$ psi,
- modulus of elasticity by compression perpendicular to wood fibers, $E_{T\perp}$ $\langle 188.92; 926.66\rangle$ MPa, $\langle 27000; 134400\rangle$ psi,
- shearing strength parallel to wood fibers, $R_{S\|}$ $\langle 3.65; 13.65\rangle$ MPa, $\langle 530; 1980\rangle$ psi,
- moisture content, $m_C = 1.5\%, 8\%, 30\%$,
- wood temperature while cutting: 20°C.

In preliminary calculations, linear, multinomial and exponential functions were examined. The present author evaluated estimators of combined, for three examined wood species, statistical dependencies between F_C and F_N cutting forces and machining parameters (a_P, γ_F) and mechanical properties of wood (R_B, $R_{C\|}$, $R_{T\perp}$, $R_{S\|}$), of power type functions with interactions of power type

$$F_C = b_{35} + b_1 \cdot {a_P}^{b11} \cdot {\gamma_F}^{b12} \cdot {R_B}^{b13} \cdot {R_{C\|}}^{c14} \cdot {R_{T\perp}}^{b15} \cdot {R_{S\|}}^{b16} + A + B + C \ \ (\mathrm{N \cdot mm^{-1}}), \tag{1}$$

$$A = b_2 \cdot {a_P}^{b17} \cdot {\gamma_F}^{b26} + b_3 \cdot {a_P}^{b18} \cdot {R_B}^{b27} + b_4 \cdot {a_P}^{b19} \cdot {R_{C\|}}^{b28}\ , \tag{2}$$

$$B = b_5 \cdot {a_P}^{b20} \cdot {R_{TP}}^{b29} + b_6 \cdot {a_P}^{b21} \cdot {R_{S\|}}^{b30} + b_7 \cdot {\gamma_F}^{b22} \cdot {R_B}^{b31}\ , \tag{3}$$

$$C = b_8 \cdot \gamma F^{b23} \cdot {R_{C\|}}^{b32} + b_9 \cdot {\gamma_F}^{b24} \cdot {R_{C\|}}^{b33} + b_{10} \cdot \gamma F^{b25} \cdot {R_{C\|}}^{b34}\ , \tag{4}$$

$$F_N = c_{35} + c_1 \cdot {a_P}^{c11} \cdot {\gamma_F}^{c12} \cdot {R_B}^{c13} \cdot {R_{C\|}}^{c14} \cdot {R_{T\perp}}^{c15} \cdot {R_{S\|}}^{c16} + D + E + F \ \ (\mathrm{N \cdot mm^{-1}}), \tag{5}$$

$$D = c_2 \cdot {a_P}^{c17} \cdot \gamma F^{c26} + c_3 \cdot {a_P}^{c18} \cdot {R_B}^{c27} + c_4 \cdot {a_P}^{b19} \cdot {R_{C\|}}^{c28}\ , \tag{6}$$

$$E = c_5 \cdot {a_P}^{c20} \cdot {R_{TP}}^{c29} + c_6 \cdot {a_P}^{c21} \cdot {R_{S\|}}^{c30} + c_7 \cdot \gamma F^{c22} \cdot {R_B}^{c31}\ , \tag{7}$$

$$F = c_8 \cdot \gamma F^{c23} \cdot {R_{C\|}}^{c32} + c_9 \cdot \gamma F^{c24} \cdot {R_{C\|}}^{c33} + c_{10} \cdot \gamma F^{c25} \cdot {R_{C\|}}^{c34}\ , \tag{8}$$

where: b_1 to b_{31}; c_1 to c_{35} – estimators.

Estimators $b_1, \ldots, b_{35}$ and $c_1, \ldots, c_{35}$, also referred to as coefficients of the regression equation, were evaluated iteratively from the experimental matrix having 368 data points (incomplete for pine wood). Unimportant or low important estimators were eliminated during the evaluation process of selected mathematical models by means of a coefficient of relative importance (C_{RI}) defined by Eq. (9), by assumption $C_{\mathrm{RI}} \gtrsim 0.01$

$$C_{\mathrm{RI}} = (S_K - S_{K0k}) \cdot {S_K}^{-1} \cdot 100\%. \tag{9}$$

In Eq. (9) the new terms are:

- s_K – Summation of square of residuals,
- S_{K0k} – Summation of square of residuals, by estimator $a_k = 0$,
- a_k – Estimator of number k in the empirical formula evaluated.

The calculation was done at Poznań Networking & Supercomputing Center (PCSS) on an Eagle Cluster (CPU E5-2697, Haswell), using an optimization program, based on a method of least square developed by the author. This program is searching for a defined statistical formula and chosen initial approximation. The best solution was the one with the minimum of S_K. Initial estimators can be any small numbers. The gradient and Monte Carlo methods, as well as some combinations of these methods, were used to evaluate corrected estimators from a range of search for every independent variable in an iterative way. The range of search was large in the beginning and reduced as the correlation coefficient increased. This program allows adding or removing parts of the statistical formula due to lack of their importance. Recent works have shown that in order to increase calculation efficiency it should be started from a simplified equation, for example, by keeping as constant estimators related to power, estimators related to interactions and keeping as constant estimator not related to any independent variable. After achieving a high enough correlation coefficient, constant estimators should be included stepwise to the searching process. Unfortunately, for this purposethe optimal value of the correlation coefficient for all calculation cases cannot be determined. Calculation efficiency can be slowed down dramatically in case of exponential formulas by tendency of some estimators to get extremal values. To solve this problem, limits for these estimators should be inserted. In cases with very strong local minimums, calculation ought to be started from a new set of initial approximation. For simple equations, the final solution can be reached much easier. In case of complicated equations, for a large number of independent variables as well as a large number of measuring points, the necessary number of iterations can be much higher. For characterization of approximation quality of the fit, S_K, the standard deviation of residuals (S_R) and the coefficient of determination (R^2) were used.

III. Results and Discussion

The final shape of approximated dependence, Eqs. (1) through (4), obtained after $2.4 \cdot 10^8$ iterations is:

$$\begin{gathered}F_C = 32.32829 + 0.0030251 \cdot a_P{}^{-0.06012} \cdot \gamma F^{01.2597} \cdot R_B{}^{00.82791} \cdot R_{C\|}{}^{0.033382} \cdot R_{T\perp}{}^{0.049921} \cdot R_{S\|}{}^{0.07237} + \\ -0.0092277 \cdot a_P{}^{1.2597} \cdot \gamma F^{2.117325} + 0.28882 \cdot a_P{}^{1.1664} \cdot R_B{}^{1.16548} + 0.29381 \cdot a_P{}^{1.6185} \cdot R_{C\|}{}^{1.09479} + \\ -15.63449 \cdot a_P{}^{-0.1052} \cdot R_{TP}{}^{0.073035} + 0.013993 \cdot a_P{}^{0.75379} \cdot R_{S\|}{}^{2.6983} - 0.0023797 \cdot \gamma F^{2.07723} \cdot R_B{}^{0.74367} + \\ -5.60208 \cdot \gamma F^{0.55257} \cdot R_{C\|}{}^{0.0000000010002} + 0.040711 \cdot \gamma F^{1.96993} \cdot R_{C\|}{}^{-0.0210645} + \\ +4.61728 \cdot \gamma F^{0.38085} \cdot R_{C\|}{}^{-0.25963} \ (\mathrm{N \cdot mm^{-1}}) \quad (0.98 < F_C < 74.46 \ (\mathrm{N \cdot mm^{-1}})).\end{gathered} \tag{10}$$

Approximation quality for formula (10): $S_K = 1300.1$; $S_K = 1.88$ N·mm^{-1}; $R^2 = 0.983$. The final shape of approximated dependence, Eqs. (5) through (8), obtained after $1.5 \cdot 10^8$ iterations is:

$$\begin{gathered}F_N = -5.44685 + 0.0044091 \cdot a_P{}^{-0.02354} \cdot \gamma F^{1.58574} \cdot R_B{}^{-0.10485} \cdot R_{C\|}{}^{0.0055553} \cdot R_{TP}{}^{0.010433} \cdot R_{S\|}{}^{2.16199} + \\ -7.85589 \cdot a_P{}^{1.24498} \cdot \gamma F^{0.32915} + 0.0010042 \cdot a_P{}^{0.12336} \cdot R_B{}^{1.88237} + 328.01665 \cdot a_P{}^{0.15535} \cdot R_{C\|}{}^{-1.19423} + \\ +16.90688 \cdot a_P{}^{1.50751} \cdot R_{TP}{}^{0.0041312} + 0.059885 \cdot a_P{}^{0.49946} \cdot R_{S\|}{}^{2.1559} - 2.4478 \cdot 10^{-10} \cdot \gamma F^{-0.15184} \cdot R_B{}^{4.1793} + \\ -4.15311 \cdot \gamma F^{0.70274} \cdot R_{C\|}{}^{-0.73031} - 7.68755 \cdot \gamma F^{0.0054246} \cdot R_{TP}{}^{-2.19705} + \\ -0.0032064 \cdot \gamma F^{1.5499} \cdot R_{TP}{}^{2.17173} \ (\mathrm{N \cdot mm^{-1}}) \quad (-3.4 < F_N < 19.4 \ (\mathrm{N \cdot mm^{-1}})).\end{gathered} \tag{11}$$

Approximation quality for formula (11): $S_K = 123.3$; $S_R = = 0.58$ N·mm^{-1}; $R^2 = 0.976$.

According to predictions from the 2D plots shown in work [1], for F_C, and F_N, five interactions for a_P and four interactions for γ_F were found in the present study.

Fig. 3 shows asymmetric distribution of measuring points of F_C and F_N in the experimental matrix. There are more lower values than higher ones. On the diagram in Fig. 3b, starting from $F_N^O \gtrsim 6$ N·mm^{-1}, more values laying above the red line can be seen, which suggests the presence of an uncontrolled variation of properties of the examined wood specimens. It is also possible that the reason for the mentioned uncontrolled variation was caused by not taking into account the compression strength perpendicular to wood fibers $R_{C\perp}$.

In Fig. 4, a very strong influence of a_P on F_C and F_N, cutting forces can be seen. The relation $F_C = f(a_P)$ is different than $F_N = f(a_P)$. With an increase of a_P, F_C increases more strongly for lower rake angle γ_F. The relation $F_C = f(a_P)$ has a form of slightly parabolic decreasing manner. The relation $F_C = f(\gamma_F)$ is very strong for the large a_P. This relation has a form of a parabolic decreasing manner for a large rake angle. For small rake angle, and especially for low chip thickness, the form of the relation $F_C = f(\gamma_F)$ changes to the parabolic increasing manner. The ratio between the maximum and minimum value of F_C (Fig. 4a), in relation $F_C = f(a_P)$, (by $\gamma_F = 5°$ and $\gamma_F = 30°$), for the largest and smallest a_P ($a_P = 0.76$ mm and $a_P = 0.05$ mm), was as large as 16.8 and 9.71, respectively. The ratio between the maximum and minimum value of F_C, in the relation $F_C = f(\gamma_F)$, (by $a_P = 0.76$ mm and $a_P = 0.05$ mm) for the smallest and largest γ_F ($\gamma_F = 5°$ and $\gamma_F = 30°$), was as large as 3.59 and 2.07, respectively.

The relations $F_N = f(a_P)$ and $F_N = f(\gamma_F)$ (Fig. 4b) are very complex. With an increaseof a_P, for the largest γ_F, F_N decreases. This tendency stops for $\gamma_F \sim 22.5°$. With an increase of a_P, starting from $\gamma_F \lesssim 22.5°$, F_N increases with an increasing tendency for the lower rake angle. With a decrease of γ_F, for the largest a_P, F_N strongly increases with a decreasing tendency as a_P drops down. For the lowest a_P, the impact of γ_F on F_N is very small. In the relation $F_N = f(a_P)$, for the minimum value of γ_F, there are no negative values. All negative values of F_N start from $\gamma_F \gtrsim 22.5°$, for maximum a_P ($a_P = 0.76$ mm) and from $\gamma_F \gtrsim 10.5°$, for minimum a_P ($a_P = 0.05$ mm). The ratio between the maximum and minimum value of F_N, in the

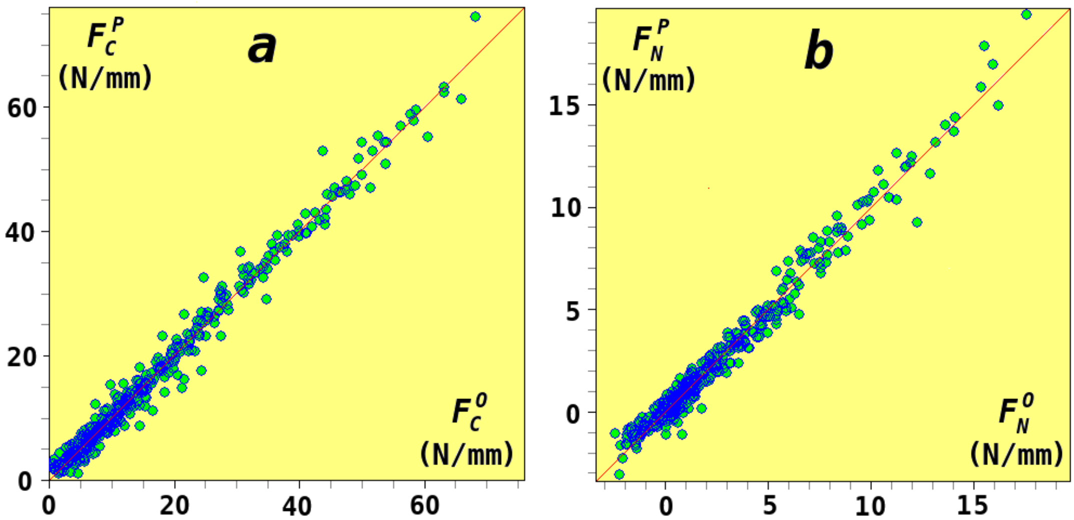

Fig. 3. Plot of observed (^{O}) versus predicted (^{P}) cutting forces: a) main, F_C; b) normal (thrust), F_N; Ac. to (10) and (11)

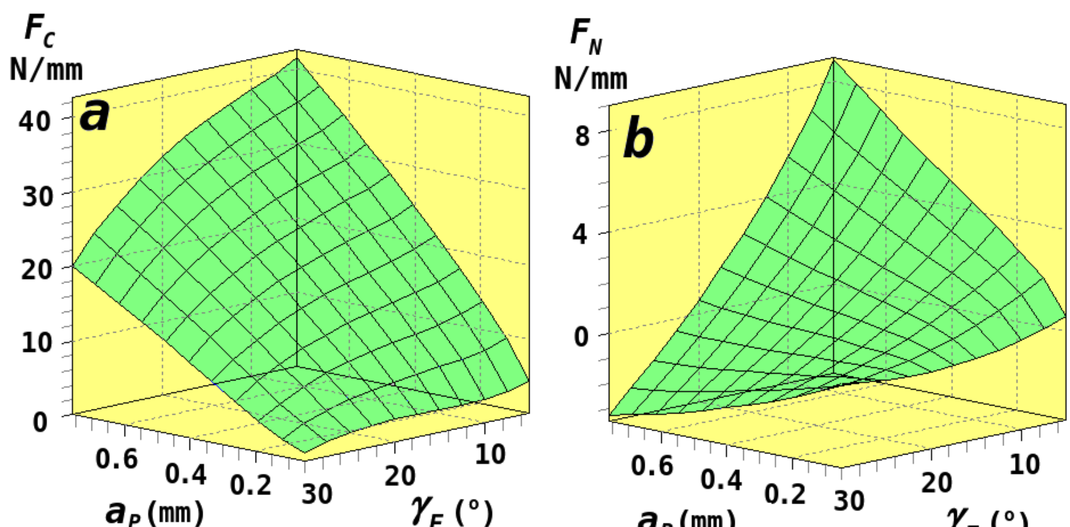

Fig. 4. Dependence of: a) the main force, F_C from chip thickness, a_P and the rake angle, γ_F; b) the normal force, F_N from the chip thickness, a_P and the rake angle, γ_F; for: bending strength, $R_B = 98.6$ MPa; compression strength, $R_{C\|} = 50.81$ MPa; tensile strength, $R_{T\perp} = 4.26$ MPa; shearing strength, $R_{S\|} = 8.65$ MPa; for Sugar Pine; Ac. to (10) and (11)

relation $F_N = f(a_P)$, (by $\gamma_F = 5°$) for the smallest and largest a_P ($a_P = 0.05$ mm and $a_P = 0.76$ mm), was as large as 14.35.

The impact of R_B and $R_{C\|}$ on F_C and F_N cutting forces, illustrated in Fig. 5a, according to Eqs. (10) and (11), is very strong. The relation $F_C = f(R_B)$ increases with an increase of R_B, having a form of a slightly parabolic increasing manner. The relation $F_C = f(R_{C\|})$, less intensive than the previous one, decreases with an increase of $R_{C\|}$ and has a form of a slightly parabolic manner. The ratio between the largest and smallest values of F_C in the relation $F_C = f(R_{C\|})$ (Fig. 5a), (by $R_B = 160.54$ MPa and $R_B = 36.54$ MPa), for the largest and smallest values of $R_{C\|}$ ($R_{C\|} = 85.7$ MPa and $R_{C\|} = 15.9$ MPa), was as large as 1.66 and 4.97, respectively. The ratio between the largest and smallest values of F_C in the relation $F_C = f(R_B)$, (by $R_{C\|} = 15.93$ MPa and $R_{C\|} = 85.7$ MPa), for the smallest and largest of R_B ($R_B = 36.5$ MPa and $R_B = 160.65$ MPa), was as large as 4.4 and 18.84, respectively.

In the relation $F_N = f(R_{C\|})$ (Fig. 5b) negative values start for $R_{C\|} > 43.8$ MPa and $R_B < 98.6$ MPa. A small minimum can be seen in the relation $F_N = f(R_B)$ by $R_B = 61.4$ MPa. Positive values of F_N start from $R_B > 98.6$ MPa and from $R_{C\|} < 43.8$ MPa. The ratio between the largest and smallest values of F_N in the relation $F_N =$

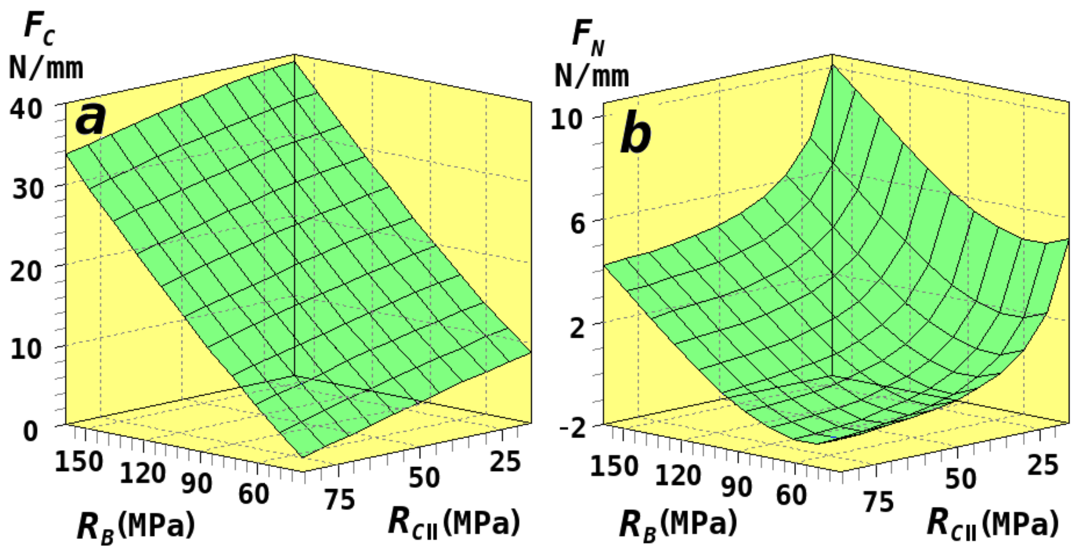

Fig. 5. Dependence of: a) the main cutting force F_C from the compression strength perpendicular to wood fibers, $R_{C\|}$ and the bending strength, R_B; b) the normal cutting force, F_N from the compression strength, parallel to wood fibers, $R_{C\|}$, and the bending strength, R_B; for: chip thickness, $a_P = 0.41$ mm; rake angle, $\gamma_F = 17.5°$; tensile strength, $R_{T\perp} = 4.26$ MPa; shearing strength, $R_{S\|} = 8.65$ MPa, for Sugar Pine; Ac. to (10) and (11)

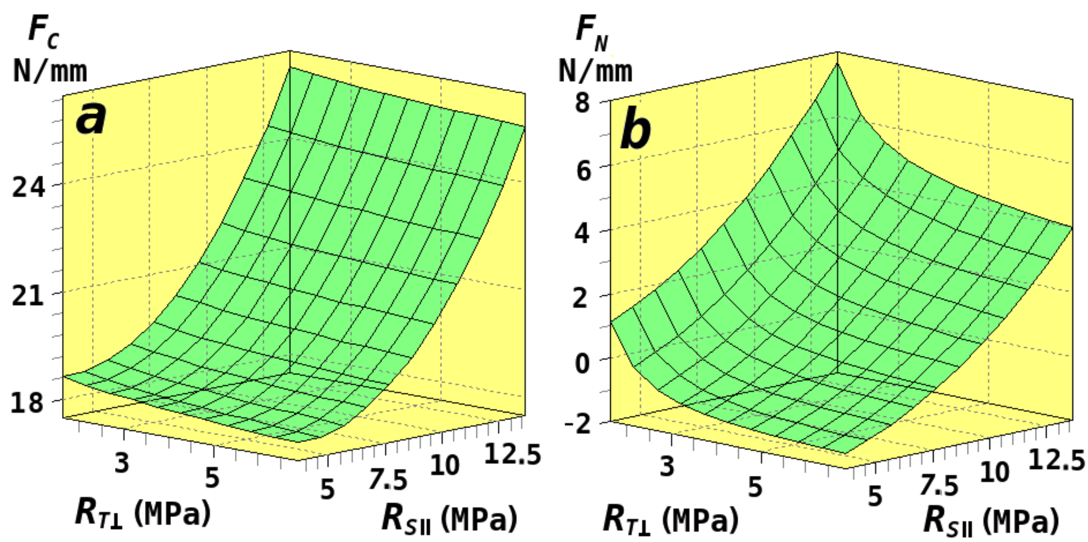

Fig. 6. Dependence of: a) the main cutting force, F_C from tensile strength perpendicular to wood fibers, $R_{T\perp}$; and the sharing strength, $R_{S\|}$; b) the normal cutting force, F_N from tensile strength perpendicular to wood fibers, $R_{T\perp}$ and the sharing strength, $R_{S\|}$; for: chip thickness, $a_P = 0.41$ mm; rake angle, $\gamma_F = 17.5°$; bending strength, $R_B = 98.6$ MPa; compression strength, $R_{C\|} = 50.81$ MPa; shearing strength, $R_{S\|} = 8.65$ MPa; for Sugar Pine; Ac. to (10) and (11)

$= f(R_B)$ (Fig. 5b) (by $R_{C\|} = 15.9$ MPa), for the largest and smallest values of R_B, ($R_B = 160.54$ MPa and $R_B = 36.5$ MPa) was as large as 2.41. The ratio between the largest and smallest values of the F_N in the relation $F_N = f(R_{C\|})$ (Fig. 5b) (by $R_B = 160.65$ MPa), for the smallest and largest values of $R_{C\|}$ ($R_{C\|} = 15.9$ MPa and $R_{C\|} = 85.7$ MPa), was as large as 1.96.

From Fig. 6, according to Eq. (10), a very strong influence of $R_{S\|}$ on F_C can be seen. The relation $F_C = f(R_{S\|})$ increases with an increase of $R_{S\|}$ in a form of a parabolic increasing manner. The relation $F_C = f(R_{T\perp})$ is much less intense than $F_C = f(R_{S\|})$ and decreases with an increase of $R_{T\perp}$, in a form of an almost linear manner. The ratio between the largest and smallest values of F_C in the relation $F_C = f(R_{T\perp})$ (Fig. 6a) (by $R_{S\|} = 3.65$ MPa and $R_{S\|} = 13.65$ MPa) for the smallest and largest values of $R_{T\perp}$ ($R_{T\perp} = 1.63$ MPa and $R_{T\perp} = 6.89$ MPa), was as large as 1.04 and 1.02, respectively. The ratio between the smallest and largest val-

Tab. 1. Comparison of F_C evaluated for pine wood for three formulas: (10), (O_1982) and (P_2011)

no.	a_P [mm]	γ_F [°]	m_C [%]	ρ [μm]	$F_C^{(10)}$ [N/mm]	$F_C^{O\_1982}$ [N/mm]	$F_C^{P\_2011}$ [N/mm]
1	0.05	5	8	5	5.0	3.78	5.63
2	0.76	5	8	5	30.83	16.0	57.7
3	0.05	30	8	5	0.68	1.19	5.17
4	0.76	30	8	5	10.69	8.41	28.03

ues of F_C in the relation $F_C = f(R_{S\|})$, (by $R_{T\perp} = 1.62$ MPa and $R_{T\perp} = 6.89$ MPa) for the largest and smallest values of $R_{S\|}$ ($R_{S\|} = 13.65$ MPa and $R_{S\|} = 3.65$ MPa), was as large as 1.4 in both cases.

The relation $F_N = f(R_{T\perp})$ (Fig. 6b), according to Eq. (11), is strong in part of larger values of $R_{T\perp}$, having a form of a parabolic decreasing manner. The relation $F_N = f(R_{S\|})$ is also strong and has a form of a parabolic, slightly increasing manner. In the relation $F_N = f(R_{S\|})$ for $R_{T\perp} = 1.62$ MPa, there are no negative values. Negative values of F_N appear for $R_{S\|} \lesssim 7.65$ MPa and $R_{T\perp} \gtrsim 2.15$ MPa. The ratio between the largest and smallest values of F_N in the relation $F_N = f(R_{T\perp})$ (Fig. 6b) (by $R_{S\|} = 13.65$ MPa) for the largest and smallest values of $R_{T\perp}$ ($R_{T\perp} = 6.89$ MPa and $R_{T\perp} = 1.62$ MPa), was as large as 1.93. The ratio between the largest and smallest values of F_N in the relation $F_N = f(R_{S\|})$, (by $R_{T\perp} = 1.62$ MPa), for the smallest and largest values of $R_{S\|}$ ($R_{S\|} = 3.65$ MPa and $R_{S\|} = 13.65$ MPa), was as large as 4.58.

Despite the lack of D, in the relations (10) and (11) the fit of approximation was very good. In connection with that it can be stated that D does not play any clear and dominant role in the cutting forces. It must also be pointed out that strength properties taken into account in this study were evaluated for wood originating from the same board which was cut. In comparison to the theoretical studies (on values of the tabular strength properties) performed by [3] in the analyzed experiment $R_{T\|}$ and $R_{S\|}$ were not taken into account but R_B, and $R_{C\|}$, which were not analyzed in the cited work, were applied instead. Looking at Fig. 5a, it seems that in the examined case of the parallel cutting (‖) (Mode B) $R_{C\perp}$ might be more suitable than $R_{C\|}$ used.

The values for formula (10) were evaluated for Sugar Pine, for which average values of strength properties are as follows: $R_B = 75.8$ MPa; $R_{C\|} = 43.99$ MPa; $R_{T\perp} = 1.7$ MPa; $R_{S\|} = 6.3$ MPa (by average $D = 400$ kg·m^{-3}), for $m_C = 8\%$ and for $v_C = 0.001$ m·s^{-1}. The values for formulas (O_1982) and (P_2011) were evaluated for Scots Pine, for which average values of strength properties, according to work [13], are as follows: $R_B = 102.8$ MPa; $R_{C\|} = 54$ MPa; $R_{S\|} = 9.8$ MPa. Values of F_C, for formula (P_2011) were calculated for $D = 520$ kg·m^{-3}. Values of F_C, for formula (O_1982) and (P_2011) were calculated for $v_C = 36.9$ m·s^{-1}. Comparing values of F_C evaluated for pine wood (Tab. 1), for model (P_2011), published in work [10] with F_C evaluated from formula (10) and with F_C evaluated from model (O_1982), prepared in the Wood_Cutting program [14], it can be seen that the values of F_C for model (P_2011) are the largest, for position no. 1: 1.1 times and 1.5 times; for position no. 2: 2 times and 3.6 times; for position no. 3: 7.6 times and 4.3 times; and for position no. 4: 2.6 times and 3.3 times. The lowest values of the F_C can be seen for positions no. 1 and no. 2 and no. 4, evaluated from model (O_1982), as well as for position no. 3 evaluated for model (10). The values evaluated for model (10) are not particularly small or large, despite very low $v_C = 0.001$ m·s^{-1}, at which they were measured, and despite that the analyzed strength properties (including the D) of the Sugar Pine are clearly lower than for the Scots Pine.

Comparing the relation $F_C = f(\gamma_F)$ for large values of a_P, with the same relation evaluated from the Wood_Cutting program [14] a difference can be seen. The first relation has a form of a parabolic decreasing manner (Fig. 4a) while the second relation has a form of a parabolic increasing manner. The explanation for this difference could be the v_C used in these two cases, respectively very small ($v_C = 0.001$ ms^{-1}) and much higher ($v_C = 15$ ms^{-1}) because, as it was mentioned earlier, contemporary experiments done with the use of more sophisticated apparatus, confirm the existence relation $F_C = f(v_C)$.

It must also be pointed out that for developing models (P_2011) and (O_1982), strength properties of wood were not taken into account.

The results of the current work have shown that instead of using the imprecise C_R to determine cutting forces for different wood species [5–8, 11], in the future it would be better to develop the mentioned relation in which strength properties (R_B, $R_{C\|}$, $R_{T\perp}$, $R_{S\|}$) or moduli of elasticity (E_B, $E_{C\|}$, $E_{T\perp}$, $E_{S\|}$) of examined wood species are used.

IV. Conclusions

The analysis of results obtained in this study reveals the following conclusions in the case of orthogonal, parallel and flat cutting of solid wood of Yellow Birch, Sugar Pine and White Ash by very low cutting speed:

1. The application of strengths properties: the shear parallel to fibers, $R_{S\|}$, the compression parallel to fibers, $R_{C\|}$, the tensile perpendicular to fibers, $R_{T\perp}$, and the bending, R_B, made it possible to get very good fit for relations $F_C = f(a_P, \gamma_F, R_{S\|}, R_{C\|}, R_{T\perp}, R_B)$ and $F_N = f(a_P, \gamma_F, R_B, R_{C\|}, R_{T\perp}, R_{S\|})$, by coefficient of determination $R^2 = 0.983$ and $R^2 = 0.976$, respectively.
2. The application of tensile strength $R_{T\perp}$, compression strength $R_{C\|}$, bending strength R_B, and shearing strength $R_{S\|}$, (evaluated by three different moisture contents, m_C) made it possible to get good fit of relations mentioned in conclusion 1, instead of using the moisture content m_C itself.
3. An increase of chip thickness a_P strongly increases the main cutting force F_C in a form of a parabolic decreasing manner in a whole range of variation.
4. An increase of rake angle γ_F strongly increases the main cutting force F_C in a form of a parabolic decreasing manner only for large chip thickness.
5. An increase of chip thickness a_P by low rake angle γ_F strongly increases normal (thrust) cutting force F_N in a form of a parabolic decreasing manner.
6. An increase of chip thickness a_P by large rake angle γ_F strongly decreases normal (thrust) cutting force F_N in a form of a parabolic increasing manner.
7. An increase of bending strength R_B causes a strong increase of the main cutting force F_C in a form of a parabolic, slightly increasing manner in the whole range of variation.
8. An increase of compression strength parallel to wood fibers $R_{C\|}$ causes a weak decrease of the main cutting force F_C.
9. An increase of bending strength R_B and the compression strength parallel to wood fibers $R_{C\|}$ causes a decrease of normal (thrust) cutting force, F_N.
10. In the relation $F_N = f(R_B, R_{C\|})$, negative values of normal (thrust) cutting force F_N can be seen for $R_{C\|} > 43.8$ MPa and $R_B < 98.6$ MPa.
11. An increase of shearing strength $R_{S\|}$ causes an intense increase of main cutting force F_C, in a form of a parabolic increasing manner in the whole range of variation.
12. The relation $F_C = f(R_{T\perp})$ is weak. An increase of tensile strength $R_{T\perp}$ causes a decrease of main cutting force F_C in a form of an almost linear manner.
13. An increase of tensile strength $R_{T\perp}$ causes a decrease of normal (thrust) cutting force F_N in a form of a parabolic decreasing manner, more intensively for the smallest $R_{T\perp}$.
14. An increase of shearing strength $R_{S\|}$ causes an increase of normal (thrust) cutting force F_N in a form of a parabolic increasing manner in the whole range of variation.
15. In the relation $F_N = f(R_{T\perp}, R_{T\perp})$, negative values of normal (thrust) cutting force F_N appear for $R_{S\|} \lesssim 7.65$ MPa and for $R_{T\perp} \gtrsim 2.15$ MPa.
16. The lack of wood density D did not disturb getting good fit of relations $F_C = f(a_P, \gamma_F, R_B, R_{C\|}, R_{T\perp}, R_{S\|})$ and $F_N = f(a_P, \gamma_F, R_B, R_{C\|}, R_{T\perp}, R_{S\|})$.

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References

[1] N.C. Franz, *An Analysis of the Wood-cutting Process*, The University of Michigan Press, Ann Arbor (1958).

[2] J. Time, *Soprotivlene metallov i dereva rezaniû (Cutting resistance of metals and wood)*, Dermacov Press House, St. Petersburg, Russia (1870).

[3] M.A. Deševoj, *Mehaničeskaâ tehnologâ dereva (Mechanical technology of wood)*, LTA (1939).

[4] E. Kivimaa, *The cutting force in woodworking*, Rep. No. 18, The State Institute for Technical Research, Helsinki (1950).

[5] T. Orlicz, *Obróbka drewna narzędziami tnącymi* (*Machining of wood with use of cutting tools*), Study book SGGW-AR, Warsaw (1982).

[6] P.S. Afanasev, *Derevoobrabatyvaûŝie stanki (Woodworking machinery)*, Moskva (1961).

[7] V.V. Amalitskij, V.I. Lûbčenko, *Stanki i instrumenty derevoobrabatyvaûŝih predpriâtij (Machinery and tools of woodworking factories)*, Moskva (1977).

[8] A.L. Beršadskij, *Razčet režimov rezaniâ devesiny (Resolution of modes of wood machining)*, Moskva (1967).

[9] G. Goli, *Measurement of cutting forces in high speed cutting with simultaneous ultra-high-speed photography of chip formation patterns*, COST-STSM-E35-1782 (2007).

[10] B. Porankiewicz, B. Axelsson, A.Grönlund, B. Marklund, *Main and normal cutting forces by machining of wood of Pinus Sylvestris*, BioResources **6**(4), 3687–3713 (2011).

[11] F. Eyma, *Study of the properties of thirteen tropical wood species to improve the prediction of cutting forces in mode B*, Ann. For. Sci. **61**, 55–64 (2004).

[12] B. Porankiewicz, D. Wieczorek, M. Durkovic, I. Idzikowski, Z. Węgrzyn, *Modelling Cutting Forces using the Moduli of Elasticity in Oak Peripheral Milling*, BioResources **16**(1), 1424–1437 (2021).

[13] R. Wagenfür, C. Scheiber, *Holzatlas*, VEB Fachbuchverlag, Leipzig, Germany (1974).

[14] B. Porankiewicz, *Wood_Cutting*, Program in Delphy for calculation wood cutting forces, not published (2021).

Appendix

The following estimators for were evaluated for formula (1)–(4): $b_1 = 0.0030251$; $b_2 = -0.0092277$; $b_3 = 0.28882$; $b_4 = -0.29381$; $b_5 = -15.63449$; $b_6 = 0.013993$; $b_7 = -0.0023797$; $b_8 = -5.60208$; $b_9 = 0.040711$; $b_{10} = 4.61728$; $b_{11} = -0.06012$; $b_{12} = 1.68017$; $b_{13} = 0.82791$; $b_{14} = 0.033382$; $b_{15} = 0.049921$; $b_{16} = 0.07237$; $b_{17} = 1.2597$; $b_{18} = 1.16639$; $b_{19} = 1.6185$; $b_{20} = -0.1052$; $b_{21} = 0.75379$; $b_{22} = 2.07723$; $b_{23} = 0.55257$; $b_{24} = 1.96993$; $b_{25} = 0.38085$; $b_{26} = 2.117325$; $b_{27} = 1.16548$; $b_{28} = 1.09479$; $b_{29} = 0.073035$; $b_{30} = 2.6983$; $b_{31} = 0.74367$; $b_{32} = 1.00022 \cdot 10^{-10}$; $b_{33} = -0.0210645$; $b_{34} = -0.25963$; $b_{35} = 32.32829$.

A rounding value of estimators b_1–b_{35} to 6 and 5 decimal place produced an acceptable deterioration of the fit of less than $7.9 \cdot 10^{-4}\%$. Reducing the number of rounded decimal digits to 4, 3, 2 and 1 would cause deterioration of the fit as much as 0.0034%, 0.125%, 14.4% and 5926%, respectively. It was decided to reduce the number of rounded decimal digits to 5.

The coefficients of relative importance, C_{RI}, for estimators b_1–b_{35} have the following values: $C_{\mathrm{RI1}} = 3.3 \cdot 10^4$; $C_{\mathrm{RI2}} = 190$; $C_{\mathrm{RI3}} = 1.7 \cdot 10^4$; $C_{\mathrm{RI4}} = 1.6 \cdot 10^3$; $C_{\mathrm{RI5}} = 1.1 \cdot 10^4$; $C_{\mathrm{RI6}} = 513$; $C_{\mathrm{RI7}} = 5.1 \cdot 10^4$; $C_{\mathrm{RI8}} = 2 \cdot 10^4$; $C_{\mathrm{RI9}} = 8 \cdot 10^3$; $C_{\mathrm{RI10}} = 1.7 \cdot 10^3$; $C_{\mathrm{RI11}} = 255$; $C_{\mathrm{RI12}} = 3.3 \cdot 10^4$; $C_{\mathrm{RI13}} = 3.2 \cdot 10^4$; $C_{\mathrm{RI14}} = 491$; $C_{\mathrm{RI15}} = 153$; $C_{\mathrm{RI16}} = 745$; $C_{\mathrm{RI17}} = 581$; $C_{\mathrm{RI18}} = 5.3 \cdot 10^4$; $C_{\mathrm{RI19}} = 8.1 \cdot 10^3$; $C_{\mathrm{RI20}} = 330$; $C_{\mathrm{RI21}} = 727$; $C_{\mathrm{RI22}} = 5.1 \cdot 10^4$; $C_{\mathrm{RI23}} = 1.3 \cdot 10^4$; $C_{\mathrm{RI24}} = 7.9 \cdot 10^3$; $C_{\mathrm{RI25}} = 722$; $C_{\mathrm{RI26}} = 190$; $C_{\mathrm{RI27}} = 1.7 \cdot 10^4$; $C_{\mathrm{RI28}} = 1.5 \cdot 10^3$; $C_{\mathrm{RI29}} = 119$; $C_{\mathrm{RI30}} = 512$; $C_{\mathrm{RI31}} = 4.8 \cdot 10^4$; $C_{\mathrm{RI32}} = 0.01$; $C_{\mathrm{RI33}} = 13$; $C_{\mathrm{RI34}} = 924$; $C_{\mathrm{RI35}} = 2.8 \cdot 10^4$.

The following estimators for were evaluated for formula (5)–(8): $c_1 = 0.0044091$; $c_2 = -7.85589$; $c_3 = 0.0010042$; $c_4 = 328.01665$; $c_5 = 16.90688$; $c_6 = 0.059885$; $c_7 = -2.4478 \cdot 10^{-10}$; $c_8 = -4.15311$; $c_9 = 7.68755$; $c_{10} = -0.0032064$; $c_{11} = -0.02354$; $c_{12} = 1.58574$; $c_{13} = -0.10485$; $c_{14} = 0.0055553$; $c_{15} = -0.010433$; $c_{16} = 2.16199$; $c_{17} = 1.24498$; $c_{18} = 0.12336$; $c_{19} = 0.15535$; $c_{20} = 1.50751$; $c_{21} = 0.49946$; $c_{22} = -0.15184$; $c_{23} = 0.70274$; $c_{24} = 0.0054246$; $c_{25} = 1.5499$; $c_{26} = 0.32915$; $c_{27} = 1.88237$; $c_{28} = -1.19423$; $c_{29} = 0.0041312$; $c_{30} = 2.1559$; $c_{31} = 4.17926$; $c_{32} = -0.73031$; $c_{33} = -2.19705$; $c_{34} = 2.17173$; $c_{35} = -5.44685$.

A rounding value of estimators c_1–c_{35} to 6 and 5 decimal place, produced an acceptable deterioration of the fit of less than $6.8 \cdot 10^{-4}\%$. Reducing the number of rounded decimal digits to 4, 3, 2 and 1 would cause deterioration of the fit as much as $5.7 \cdot 10^{-3}$ %, 4.4 %, 83.6% and 9873 %, respectively. It was decided to reduce the number of rounded decimal digits to 5.

The coefficients of relative importance, C_{RI}, for the estimators c_1–c_{35} have the following values: $C_{\mathrm{RI1}} = 8.7 \cdot 10^5$; $C_{\mathrm{RI2}} = 1.8 \cdot 10^4$; $C_{\mathrm{RI3}} = 1.1 \cdot 10^4$; $C_{\mathrm{RI4}} = 9.4 \cdot 10^3$; $C_{\mathrm{RI5}} = 1.1 \cdot 10^4$; $C_{\mathrm{RI6}} = 9.8 \cdot 10^3$; $C_{\mathrm{RI7}} = 324$; $C_{\mathrm{RI8}} = 2.2 \cdot 10^3$; $C_{\mathrm{RI9}} = 655$; $C_{\mathrm{RI10}} = 9.7 \cdot 10^5$; $C_{\mathrm{RI11}} = 1.2 \cdot 10^3$; $C_{\mathrm{RI12}} = 8.7 \cdot 10^5$; $C_{\mathrm{RI13}} = 3.6 \cdot 10^5$; $C_{\mathrm{RI14}} = 438$; $C_{\mathrm{RI15}} = 285$; $C_{\mathrm{RI16}} = 8.7 \cdot 10^5$; $C_{\mathrm{RI17}} = 6.1 \cdot 10^4$; $C_{\mathrm{RI18}} = 405$; $C_{\mathrm{RI19}} = 561$; $C_{\mathrm{RI20}} = 5 \cdot 10^4$; $C_{\mathrm{RI21}} = 5.8 \cdot 10^3$; $C_{\mathrm{RI22}} = 83$; $C_{\mathrm{RI23}} = 1.7 \cdot 10^3$; $C_{\mathrm{RI24}} = 0.1$; $C_{\mathrm{RI25}} = 9.5 \cdot 10^5$; $C_{\mathrm{RI26}} = 6.8 \cdot 10^3$; $C_{\mathrm{RI27}} = 1.1 \cdot 10^4$; $C_{\mathrm{RI28}} = 2.2 \cdot 10^7$; $C_{\mathrm{RI29}} = 0.3$; $C_{\mathrm{RI30}} = 9.7 \cdot 10^3$; $C_{\mathrm{RI31}} = 324$; $C_{\mathrm{RI32}} = 2.6 \cdot 10^5$; $C_{\mathrm{RI33}} = 1.4 \cdot 10^4$; $C_{\mathrm{RI34}} = 9.6 \cdot 10^5$; $C_{\mathrm{RI35}} = 8.9 \cdot 10^3$.

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