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Fundamental Solutions in the Theory of Micromorphic Thermoelastic Diffusion Materials with Microtemperatures and Microconcentrations

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Abstract: The main purpose of this paper is to construct the fundamental solutions of a system of equations of isotropic micromorphic thermoelastic diffusion materials with microtemperatures and microconcentrations in case of steady oscillations in terms of elementary functions. In a particular case, the fundamental solutions of the system of equations of equilibrium theory of isotropic micromorphic thermoelastic diffusion materials with microtemperatures and microconcentrations are also established.

Key words: thermoelasticity, diffusion, microtemperatures, microconcentrations, microstretch

I. Introduction

Diffusion is defined as the mass transfer of a substance from high concentration regions to low concentration regions. Various authors [1–7] have established different theories of thermoelastic diffusion to describe the coupled mechanical behavior among temperature, concentration, and strain fields in elastic solids.

The theory of thermodynamics of elastic bodies with microstructure was extended by Grot [8] with the assumption that the microelements have different temperatures. He modified Clausius-Duhem inequality to include microtemperatures and added first-order moment of energy equations to the basic balance laws for determining the microtemperatures of a continuum. Iesan and Quintanilla [9] constructed a linear theory for elastic materials with inner structure whose particles, in addition to the classical displacement and temperature fields, possess microtemperatures. They proved an existence theorem for initial boundary value problems via the semigroup theory and established the continuous dependence of solutions of the initial data and body loads. The field equations of a theory of microstretch thermoelastic bodies with microtemperatures were established by Iesan [10]. He proved a uniqueness theorem in the dynamic theory of anisotropic materials. Iesan [11] derived a linear theory of microstretch elastic solids with microtemperatures in which a microelement of a continuum is equipped with the mechanical degrees of freedom for rigid rotations and microdilatation in addition to the classical translation degrees of freedom. He also presented uniqueness and continuous dependence results. Svandaze [12, 13] constructed fundamental solutions in the theories of thermoelasticity with microtemperatures and micromorphic elastic solids with microtemperatures, respectively. Aouadi et al. [14] developed the nonlinear theory of thermoelastic diffusion materials with microtemperatures and microconcentrations. They also obtained a linear theory of thermoelastic diffusion materials with microtemperatures and microconcentrations. They proved the well-posedness of the linear anisotropic problem with the help of the semigroup theory of linear operators and studied the asymptotic behavior of the solutions. Chirilă and Marin [15] derived the field equations and consecutive equations of the linear theory of microstretch thermoelasticity for materials whose particles have microelements that are equipped with microtemperatures and microconcentrations.

The constitutive relations, field equations for isotropic micromorphic thermoelastic diffusion materials with microtemperatures and microconcentrations are established in Sec. 2. In Sec. 3, the system of linearized equations of steady oscillations and equilibrium in the theory of micromorphic thermoelastic diffusion solids with microtemperatures and microconcentrations are obtained. In Sec. 4, in terms of elementary functions, the fundamental solution of basic governing equations in case of steady oscillations are constructed. Some basic properties of the fundamental matrix in case of steady oscillations are discussed in Sec. 5. In Sec. 6, the fundamental solution of basic governing equations in case of equilibrium are constructed.

II. Basic Equations

We assume that the body occupies at time t_0 the bounded regular region B of three-dimensional space. We confine our attention to the linear theory of elastic bodies. The balance of linear momentum can be written in the form

$$t_{ji,j} + \rho f_i = \rho \ddot{u}_i \, , \tag{1}$$

where t_{ij} are the components of stress tensor, u_i are the components of displacement vector $\mathbf{u}$, ρ is the reference mass density, and f_i is the body force.

The balance of first stress moments is given by

$$m_{pij,p} + t_{ji} - s_{ji} + \rho \, l_{ij} = \dot{\sigma}_{ij} \, , \tag{2}$$

where m_{pij} are the components of first stress moment tensor, s_{ij} are the components of microstress tensor, σ_{ij} are the components of inertial spin tensor, and l_{ij} is the first body moment density.

Let ϵ denote the internal energy density and let ε_i, Ω_i denote the first moments of energy vector and mass diffusion respectively. Then the balance of energy, the balance of the first moment of energy and mass diffusion are respectively given by

$$\rho\dot{\epsilon} = t_{ij}\dot{u}_{j,i} + (s_{ij} - t_{ij})\dot{\phi}_{ji} + m_{pij}\dot{\phi}_{ij,p} + q_{i,i} + \rho\eta, \tag{3}$$

$$\rho\dot{\varepsilon}_i = q_{ji,j} + q_i - \tilde{\varsigma}_i + \rho M_i \, , \tag{4}$$

$$\rho\dot{\Omega}_i = \eta_{ji,j} + \eta_i - \sigma_i \, . \tag{5}$$

Here q_{ij}, η_{ij} are the first moment of heat flux and mass diffusion flux tensors, respectively, $\tilde{\varsigma}_i$ is the microheat flux average, σ_i is the micromass diffusion flux average, q_i are the components of heat flux vector , η_i are the components of mass diffusion flux vector, η is the heat supply, M_i is the first heat source moment vector, and ϕ_{ij} are the components of microdeformation tensor.

The local form of the principle of entropy can be expressed as

$$\rho\dot{S} - \left(\frac{1}{T}q_p + \frac{1}{T}q_{pg}T_g\right)_{,p} + \left(\frac{P\eta_p}{T} + \frac{P}{T}T_g\eta_{pg}\right)_{,p} + \\ -\frac{1}{T}\rho(\eta + M_iT_i) \geq 0, \tag{6}$$

where S is the entropy density, T is the absolute temperature, and T_i is the microtemperature vector.

The local form of the mass concentration law is

$$\eta_{j,j} = \dot{C}, \tag{7}$$

where C is the concentration of diffusion material. For each micro element, the mass conservation law becomes

$$\dot{C} = \left(\eta_g + C_p\eta_{gp}\right)_{,g} \, , \tag{8}$$

where C_p is the microconcentartion vector.

The spin inertia is given by

$$\sigma_{ij} = \dot{\tilde{n}}_{gj}\dot{\phi}_{ip}\dot{\phi}_{pg} \, , \tag{9}$$

where $\dot{\tilde{n}}_{gj}$ is the microinertia tensor.

Eringen [16] introduced a special kind of micromorphic solids called microstretch solids. In this case, for all motions, we have

$$\phi_{ij} = \phi\delta_{ij}, \; m_{ijg} = \frac{1}{3}\tilde{h}_i\delta_{jg}, \; \dot{\tilde{n}}_{ij} = \frac{1}{3}\tau'\delta_{ij}, \; l_{ij} = \frac{1}{3}\tilde{L}\delta_{ij} \, , \tag{10}$$

where ϕ is the dilatation function, $\tilde{h}_i$ is the microstress vector, $\tilde{L}$ is the generalized external body load, and τ' is a given constant.

Eqs. (2), (3) and (9) become

$$\tilde{h}_{i,i} - s + \rho\tilde{L} = \tau' \, \ddot{\phi}, \tag{11}$$

$$\rho\dot{\epsilon} = t_{ij}\dot{e}_{ij} + \tilde{h}_i\dot{\phi}_{,i} + s\dot{\phi} + q_{i,i} + \rho\eta, \tag{12}$$

$$\sigma_{ij} = \tau'\dot{\phi}, \tag{13}$$

where $s = s_{ii} - t_{ii}$ is the intrinsic body load, and $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ are the components of strain tensor.

From Eqs. (4)–(6), (8), (12), we get

$$\rho\dot{S}T - \rho\dot{\epsilon} + t_{ij}\dot{e}_{ij} + s\dot{\phi} + \tilde{h}_i\dot{\phi}_{,i} + \frac{1}{T}T_{,i}q_i - \rho\dot{\varepsilon}_iT_i + \\ +(q_i - \tilde{\varsigma}_i)T_i + \frac{1}{T}q_{pg}T_{,p}T_g - q_{pg}T_{g,p} - \frac{1}{T}P\eta_pT_{,p} + \\ +P_{,p}\eta_p + P\dot{C} - PC_i\eta_{ji,j} - PC_{i,j}\eta_{ji} + T\left(\frac{P}{T}T_i\eta_{ji}\right)_{,j} + \\ +\eta_{ji,j}C_i + (\eta_j - \sigma_j)C_j - \rho\, C_i\dot{\Omega}_i \geq 0. \tag{14}$$

If we introduce function ψ by

$$\psi = \epsilon + T_i\varepsilon_i + C_i\Omega_i - TS, \tag{15}$$

then, the relation (14) becomes

$$\begin{aligned} &-\rho[\dot\psi + \dot T S - \dot T_i\varepsilon_i - \dot C_i\Omega_i] + t_{ij}\dot e_{ij} + s\dot\phi + \\ &+\tilde h_i\dot\phi_{,i} + \frac{1}{T}T_{,i}q_i + (q_i - \tilde\varsigma_i)T_i + \\ &+\frac{1}{T}q_{pg}T_{,p}T_g - q_{pg}T_{g,p} - \frac{1}{T}P\eta_pT_{,p} + \\ &+P_{,p}\eta_p + P\dot C - PC_i\eta_{ji,j} - PC_{i,j}\eta_{ji} + \\ &+T\left(\frac{P}{T}T_i\eta_{ji}\right)_{,j} + \eta_{ji,j}C_i + (\eta_j - \sigma_j)C_j \geq 0. \end{aligned} \tag{16}$$

Function ψ can be expressed in terms of independent variables $e_{ij}, \phi, \phi_{,i}, T, T_{,i}, T_i, T_{i,j}, C, C_{,i}, C_i$ and $C_{i,j}$. Therefore, we have

$$\begin{aligned} \dot\psi = &\frac{\partial\psi}{\partial e_{ij}}\dot e_{ij} + \frac{\partial\psi}{\partial\phi}\dot\phi + \frac{\partial\psi}{\partial\phi_{,i}}\dot\phi_{,i} + \frac{\partial\psi}{\partial T}\dot T + \frac{\partial\psi}{\partial T_{,i}}\dot T_{,i} + \frac{\partial\psi}{\partial T_i}\dot T_i + \\ &+\frac{\partial\psi}{\partial T_{i,j}}\dot T_{i,j} + \frac{\partial\psi}{\partial C}\dot C + \frac{\partial\psi}{\partial C_{,i}}\dot C_{,i} + \frac{\partial\psi}{\partial C_i}\dot C_i + \frac{\partial\psi}{\partial C_{i,j}}\dot C_{i,j}\ . \end{aligned} \tag{17}$$

Eq. (16) with the help of Eq. (17) becomes

$$\begin{aligned} &\left[t_{ij} - \rho\frac{\partial\psi}{\partial e_{ij}}\right]\dot e_{ij} + \rho\left[\Omega_i - \frac{\partial\psi}{\partial C_i}\right]\dot C_i + \rho\left[\varepsilon_i - \frac{\partial\psi}{\partial T_i}\right]\dot T_i + \\ &+\left[s - \rho\frac{\partial\psi}{\partial\phi}\right]\dot\phi + \left[\tilde h_i - \rho\frac{\partial\psi}{\partial\phi_{,i}}\right]\dot\phi_{,i} - \rho\left[S + \frac{\partial\psi}{\partial T}\right]\dot T + \\ &+\left[P - \rho\frac{\partial\psi}{\partial C}\right]\dot C - \rho\frac{\partial\psi}{\partial T_{,i}}\dot T_{,i} - \rho\frac{\partial\psi}{\partial T_{i,j}}\dot T_{i,j} - \rho\frac{\partial\psi}{\partial C_{,i}}\dot C_{,i} + \\ &-\rho\frac{\partial\psi}{\partial C_{i,j}}\dot C_{i,j} + \frac{1}{T}T_{,i}q_i + (q_i - \tilde\varsigma_i)T_i + \frac{1}{T}q_{pg}T_{,p}T_g + \\ &-q_{pg}T_{g,p} - \frac{1}{T}P\eta_pT_{,p} + P_{,p}\eta_p - PC_i\eta_{ji,j} - PC_{i,j}\eta_{ji} + \\ &+T\left(\frac{P}{T}T_i\eta_{ji}\right)_{,j} + \eta_{ji,j}C_i + (\eta_j - \sigma_j)C_j \geq 0. \end{aligned} \tag{18}$$

The inequality should be convinced for all rates $\dot e_{ij}, \dot\phi, \dot\phi_{,i}, \dot T, \dot T_{,i}, \dot T_i, \dot T_{i,j}, \dot C, \dot C_{,i}, \dot C_i$ and $\dot C_{i,j}$. Hence the coefficients of the above variables must vanish, that is

$$\begin{aligned} t_{ij} = \rho\frac{\partial\psi}{\partial e_{ij}}, \Omega_i = \frac{\partial\psi}{\partial C_i}, \varepsilon_i = \frac{\partial\psi}{\partial T_i}, s = \rho\frac{\partial\psi}{\partial\phi}, \\ \tilde h_i = \rho\frac{\partial\psi}{\partial\phi_{,i}}, S = -\frac{\partial\psi}{\partial T}, P = \rho\frac{\partial\psi}{\partial C}\ , \end{aligned} \tag{19}$$

$$\frac{\partial\psi}{\partial T_{,i}} = 0, \frac{\partial\psi}{\partial T_{i,j}} = 0, \frac{\partial\psi}{\partial C_{,i}} = 0, \frac{\partial\psi}{\partial C_{i,j}} = 0, \tag{20}$$

$$\begin{aligned} &T_{,i}q_i + T(q_i - \tilde\varsigma_i)T_i + q_{pg}T_{,p}T_g - Tq_{pg}T_{g,p} - P\eta_pT_{,p} + \\ &+TP_{,p}\eta_p - TPC_i\eta_{ji,j} - TPC_{i,j}\eta_{ji} + T^2\left(\frac{P}{T}T_i\eta_{ji}\right)_{,j} + \\ &+T\eta_{ji,j}C_i + T(\eta_j - \sigma_j)C_j \geq 0. \end{aligned} \tag{21}$$

Let us introduce the notation

$$\theta = T - T_0, \tag{22}$$

where T_0 is the reference temperature of the body chosen such that $|\frac{\theta}{T_0}| \ll 1$.

In the linear theory of materials possessing a centre of symmetry, we can take ψ in the form

$$\begin{aligned} 2\rho\psi = &c_{ijpl}e_{ij}e_{pl} + 2a_{ij}e_{ij}\theta + 2b_{ij}e_{ij}C + 2f_{ij}e_{ij}\phi + \\ &+A_{ij}\phi_{,i}\phi_{,j} + \vartheta\phi^2 - 2n\theta\phi - 2\nu C\phi - 2d_{ij}\phi_{,i}T_j + \\ &-2\vartheta_{ij}\phi_{,i}C_j - 2\varpi\theta C - \alpha_{ij}T_iT_j - \beta_{ij}C_iC_j + \\ &-2E_{ij}T_iC_j + \chi C^2 - \frac{\rho C_E\theta^2}{T_0}\ . \end{aligned} \tag{23}$$

From Eq. (19), it follows that

$$\begin{aligned} t_{ij} &= c_{ijpl}e_{pl} + a_{ij}\theta + b_{ij}C + f_{ij}\phi, \\ \tilde h_i &= A_{ij}\phi_{,j} - d_{ij}T_j - \vartheta_{ij}C_j\ , \\ \rho S &= -a_{ij}e_{ij} + n\phi + \frac{\rho C_E\theta}{T} + \varpi C, \\ \rho\varepsilon_i &= -d_{ji}\phi_{,j} - \alpha_{ij}T_j - E_{ij}C_j\ , \\ P &= b_{ij}e_{ij} - \nu\phi - \varpi\theta + \chi C, \\ s &= f_{ij}e_{ij} - n\theta - \nu C + \vartheta\phi, \\ \Omega_i &= -\beta_{ij}C_j - E_{ji}T_j - \vartheta_{ij}\phi_{,j}\ . \end{aligned} \tag{24}$$

The linear expressions for $q_i, q_{ij}, \tilde\varsigma_i, \eta_{ij}, \sigma_i, \eta_i$ are

$$\begin{aligned} q_i &= k_{ij}\theta_{,j} + \kappa_{ij}T_j\ , \\ q_{ij} &= -m_{ijpg}T_{g,p}\ , \\ \tilde\varsigma_i &= (k_{ij} - K_{ij})\theta_{,j} + (\kappa_{ij} - L_{ij})T_j\ , \\ \eta_{ij} &= -n_{ijpg}C_{g,p}\ , \\ \sigma_i &= (h_{ij} - H_{ij})P_{,j} + (\tilde h_{ij} - \tilde H_{ij})C_j\ , \\ \eta_i &= h_{ij}P_{,j} + \tilde h_{ij}C_j\ . \end{aligned} \tag{25}$$

The linearlized form of Eq. (6) is

$$\rho T_0\dot S = q_{i,i}\ . \tag{26}$$

In view of Eqs. (13), (24) and (25), Eqs. (1), (4), (5), (7), (11) and (26) become

$$\begin{aligned} &c_{ijpl}e_{pl,j} + a_{ij}\theta_{,j} + b_{ij}C_{,j} + f_{ij}\phi_{,j} + \rho f_i = \rho\ddot u_i\ , \\ &-d_{ji}\dot\phi_{,j} - \alpha_{ij}\dot T_j - E_{ij}\dot C_j + m_{jipg}T_{g,pj} = K_{ij}\theta_{,j} + L_{ij}T_j + \rho M_i\ , \\ &-\beta_{ij}\dot C_j - E_{ji}\dot C_j - \vartheta_{ij}\dot\phi_{,j} + n_{jipg}C_{g,pj} = H_{ij}P_{,j} + \tilde H_{ij}C_j\ , \\ &T_0[-a_{ij}\dot e_{ij} + n\dot\phi + \frac{\rho C_E}{T_0}\dot\theta + \varpi\dot C] = k_{ij}\theta_{,ij} + \kappa_{ij}T_{j,i}\ , \\ &h_{ij}[b_{ij}e_{ij} - \nu\phi - \varpi\theta + \chi C]_{,ji} + \tilde h_{ij}C_{j,i} = \dot C, \\ &A_{ij}\phi_{,ij} - d_{ij}T_{j,i} - \vartheta_{ij}C_{j,i} - f_{ij}e_{ij} + n\theta + \nu C - \vartheta\phi + \rho\tilde L = \tau'\ddot\phi. \end{aligned} \tag{27}$$

In case of an isotropic and homogeneous material, the consecutive equations become

$$\begin{aligned}
t_{ij} &= \lambda\, e_{ll}\delta_{ij} + 2\mu e_{ij} - \beta_1\theta\delta_{ij} - \beta_2 C\delta_{ij} + b\,\phi\,\delta_{ij}\ ,\\
\rho S &= \beta_1 e_{ll} + \frac{\rho C_E}{T_0}\theta + \varpi C + n\,\phi,\\
P &= -\beta_2 e_{ll} - \varpi\,\theta + \chi C - \nu\,\phi,\\
\rho\varepsilon_i &= -c_1 T_i - \kappa_1 C_i - \varrho\,\phi_{,i}\ ,\\
\rho\Omega_i &= -m_1 C_i - \kappa_1 T_i - w\,\phi_{,i}\ ,\\
\tilde{\varsigma}_i &= (k-k_3)\theta_{,i} + (k_1-k_2)T_i\ ,\\
q_{ij} &= -k_4 T_{l,l}\delta_{ij} - k_5 T_{i,j} - k_6 T_{j,i}\ ,\\
\sigma_i &= (h-h_3)P_{,i} + (h_1-h_2)C_i\ ,\\
\eta_{ij} &= -h_4 C_{l,l}\delta_{ij} - h_5 C_{i,j} - h_6 C_{j,i}\ ,\\
\tilde{h}_i &= \gamma\,\phi_{,i} - \varrho\,T_i - w\,C_i\ ,\\
s &= b\,e_{ll} - n\,\theta - \nu\,C + \vartheta\,\phi,\\
q_i &= k\theta_{,i} + k_1 T_i\\
\eta_i &= hP_{,i} + h_1 C_i\ ,
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
&c_{ijpl} = \lambda\delta_{ij}\delta_{pl} + \mu\delta_{ip}\delta_{jl} + \mu\delta_{il}\delta_{jp}, a_{ij} = -\beta_1\delta_{ij},\\
&b_{ij} = -\beta_2\delta_{ij}, d_{ij} = \varrho\delta_{ij}, E_{ij} = \kappa_1\delta_{ij}, \alpha_{ij} = c_1\delta_{ij},\\
&f_{ij} = b\delta_{ij}, \vartheta_{ij} = w\delta_{ij}, \beta_{ij} = m_1\delta_{ij}, A_{ij} = \gamma\delta_{ij},\\
&k_{ij} = k\delta_{ij}, \kappa_{ij} = k_1\delta_{ij}, L_{ij} = k_2\delta_{ij}, K_{ij} = k_3\delta_{ij},\\
&m_{ijpl} = k_4\delta_{ij}\delta_{pl} + k_6\delta_{ip}\delta_{jl} + k_5\delta_{il}\delta_{jp},\\
&h_{ij} = h\delta_{ij}, \tilde{h}_{ij} = h_1\delta_{ij}, \tilde{H}_{ij} = h_2\delta_{ij}, H_{ij} = h_3\delta_{ij},\\
&n_{ijpl} = h_4\delta_{ij}\delta_{pl} + h_6\delta_{ip}\delta_{jl} + h_5\delta_{il}\delta_{jp}.
\end{aligned} \tag{29}$$

Here λ, μ, β_1, β_2, ϱ, c_1, κ_1, b, w, m_1, γ, k, $k_1, \ldots, k_6$, h, $h_1, \ldots, h_6$ are material constants.

Therefore, from equation (27) we obtain the governing equations for homogeneous isotropic micromorphic thermoelastic diffusion solid with microtemperatures and microconcentrations in the absence of heat, mass diffusion sources and loads as:

$$\begin{aligned}
&\mu\Delta\mathbf{u} + (\lambda+\mu)\,\mathrm{grad\,div}\,\mathbf{u} - \beta_1\,\mathrm{grad}\,\theta +\\
&\quad -\beta_2\,\mathrm{grad}\,C + b\,\mathrm{grad}\,\phi = \rho\ddot{\mathbf{u}},\\
&k_6\Delta\mathbf{v} + (k_4+k_5)\,\mathrm{grad\,div}\,\mathbf{v} - k_2\mathbf{v} - k_3\,\mathrm{grad}\,\theta =\\
&\quad = c_1\dot{\mathbf{v}} + \kappa_1\dot{\mathbf{w}} + \varrho\,\mathrm{grad}\,\dot{\phi},\\
&h_6\Delta\mathbf{w} + (h_4+h_5)\,\mathrm{grad\,div}\,\mathbf{w} - h_2\mathbf{w} - h_3\,\mathrm{grad}\,P =\\
&\quad = \kappa_1\dot{\mathbf{v}} + m_1\dot{\mathbf{w}} + w\,\mathrm{grad}\,\dot{\phi},\\
&\beta_1 T_0\,\mathrm{div}\,\dot{\mathbf{u}} + \rho C_E\dot{\theta} + \varpi T_0\dot{C} + nT_0\,\dot{\phi} =\\
&\quad = k\,\Delta\,\theta + k_1\,\mathrm{div}\,\mathbf{v},\\
&h\Delta[-\beta_2\,\mathrm{div}\,\mathbf{u} - \varpi\,\theta + \chi\,C - \nu\,\phi] + h_1\,\mathrm{div}\,\mathbf{w} = \dot{C},\\
&-b\,\mathrm{div}\,\mathbf{u} - \varrho\,\mathrm{div}\,\mathbf{v} - w\,\mathrm{div}\,\mathbf{w} + n\,\theta + \nu\,C +\\
&\quad +(\gamma\Delta - \vartheta)\phi = \tau'\ddot{\phi},
\end{aligned} \tag{30}$$

where Δ is Laplacian operator, $\mathbf{v} = (T_1, T_2, T_3)$ and $\mathbf{w} = (C_1, C_2, C_3)$

In the upcoming sections, the chemical potential has been used as a state variable rather than concentration. Therefore, the system of equations (30) with the help of Eq. $(28)_3$ becomes

$$\begin{aligned}
&[\mu\Delta + (\lambda_0+\mu)\,\mathrm{grad\,div}]\mathbf{u} - \rho\,\ddot{\mathbf{u}} - \gamma_1\,\mathrm{grad}\,\theta +\\
&\quad -\gamma_2\,\mathrm{grad}\,P + \gamma_3\,\mathrm{grad}\,\phi = \mathbf{0},\\
&k_6\Delta\mathbf{v} + (k_4+k_5)\,\mathrm{grad\,div}\,\mathbf{v} - k_2\mathbf{v} - k_3\,\mathrm{grad}\,\theta =\\
&\quad = c_1\dot{\mathbf{v}} + \kappa_1\dot{\mathbf{w}} + \varrho\,\mathrm{grad}\,\dot{\phi},\\
&h_6\Delta\mathbf{w} + (h_4+h_5)\,\mathrm{grad\,div}\,\mathbf{w} - h_2\mathbf{w} - h_3\,\mathrm{grad}\,P =\\
&\quad = \kappa_1\dot{\mathbf{v}} + m_1\dot{\mathbf{w}} + w\,\mathrm{grad}\,\dot{\phi},\\
&-\gamma_1 T_0\,\mathrm{div}\,\dot{\mathbf{u}} + k_1\,\mathrm{div}\,\mathbf{v} + k\,\Delta\,\theta - cT_0\,\dot{\theta} +\\
&\quad -\kappa T_0\dot{P} - \beta\,T_0\,\dot{\phi} = 0,\\
&-\gamma_2\,\mathrm{div}\,\dot{\mathbf{u}} + h_1\,\mathrm{div}\,\mathbf{w} - \kappa\,\dot{\theta} + h\Delta\,P - m\,\dot{P} - \alpha\,\dot{\phi} = 0,\\
&-\gamma_3\,\mathrm{div}\,\mathbf{u} - \varrho\,\mathrm{div}\,\mathbf{v} - w\,\mathrm{div}\,\mathbf{w} + \beta\,\theta + \alpha\,P +\\
&\quad +(\gamma\Delta - \upsilon)\phi = \tau'\ddot{\phi},
\end{aligned} \tag{31}$$

where

$$\begin{aligned}
&m = \frac{1}{\chi},\ \kappa = m\varpi,\ \alpha = \nu\,m,\ \gamma_1 = \beta_1 + \beta_2\kappa,\ \gamma_2 = \beta_2 m,\\
&\lambda_0 = \lambda - \beta_2\gamma_2, \gamma_3 = b - \beta_2\,\alpha, c = \frac{\rho C_E}{T_0} + \varpi\kappa,\\
&\beta = n + \varpi\,\alpha,\ \upsilon = \vartheta - \nu\,\alpha.
\end{aligned}$$

III. Steady Oscillations

The displacement vector, microtemperature, microconcentration, temperature change, chemical potential and microstretch functions are assumed as:

$$\begin{aligned}
&\Big[\mathbf{u}(\mathbf{x},t), \mathbf{v}(\mathbf{x},t), \mathbf{w}(\mathbf{x},t), \theta(\mathbf{x},t), P(\mathbf{x},t), \phi(\mathbf{x},t)\Big] =\\
&\quad = \mathrm{Re}\Big[(\mathbf{u}^*, \mathbf{v}^*, \mathbf{w}^*, \theta^*, P^*, \phi^*)e^{-\iota\omega t}\Big],
\end{aligned} \tag{32}$$

where ω is oscillation frequency.

Using Eq. (32) in the system of equations (31) and omitting asterisk (*) for simplicity, the system of equations of steady oscillations are obtained as

$$\begin{aligned}
&[\mu\Delta + (\lambda_0+\mu)\,\mathrm{grad\,div} + \rho\omega^2]\mathbf{u} - \gamma_1\,\mathrm{grad}\,\theta +\\
&\quad -\gamma_2\,\mathrm{grad}\,P + \gamma_3\,\mathrm{grad}\,\phi = \mathbf{0},\\
&[k_6\Delta + (k_4+k_5)\,\mathrm{grad\,div} - k_2 + \iota\omega c_1]\mathbf{v} + \iota\omega\kappa_1\mathbf{w} +\\
&\quad -k_3\,\mathrm{grad}\,\theta + \iota\omega\varrho\,\mathrm{grad}\,\phi = \mathbf{0},
\end{aligned}$$

$$\iota\omega\kappa_1\mathbf{v} + [h_6\Delta + (h_4 + h_5)\,\mathrm{grad\,div} - h_2 + \iota\omega m_1]\mathbf{w} + \\ -h_3\,\mathrm{grad}\,P + \iota\omega\,w\,\mathrm{grad}\,\phi = \mathbf{0},$$

$$\iota\omega\gamma_1 T_0\,\mathrm{div}\,\mathbf{u} + k_1\,\mathrm{div}\,\mathbf{v} + [k\,\Delta + \\ +\iota\omega c T_0]\theta + \iota\omega\kappa T_0 P + \iota\omega\beta T_0\,\phi = 0,$$

$$\iota\omega\gamma_2\,\mathrm{div}\,\mathbf{u} + h_1\,\mathrm{div}\,\mathbf{w} + \iota\omega\kappa\,\theta + [h\,\Delta + \\ +\iota\omega m]P + \iota\omega\alpha\,\phi = 0,$$

$$-\gamma_3\,\mathrm{div}\,\mathbf{u} - \varrho\,\mathrm{div}\,\mathbf{v} - w\,\mathrm{div}\,\mathbf{w} + \beta\,\theta + \alpha\,P + \\ +(\gamma\Delta - \upsilon + \tau'\omega^2)\phi = 0. \tag{33}$$

We introduce the second order matrix differential operators with constant coefficients

$$\mathbf{F}(\mathbf{D_x}) = \Big(F_{gl}(\mathbf{D_x})\Big)_{12\times12},$$

where

$$F_{pq}(\mathbf{D_x}) = [\mu\Delta + \rho\omega^2]\delta_{pq} + (\lambda_0 + \mu)\frac{\partial^2}{\partial x_p \partial x_q}, F_{p;q+3}(\mathbf{D_x}) = F_{p+3;q}(\mathbf{D_x}) = 0,$$
$$F_{p;q+6}(\mathbf{D_x}) = F_{p+6;q}(\mathbf{D_x}) = 0, F_{p;10}(\mathbf{D_x}) = -\gamma_1\frac{\partial}{\partial x_p}, F_{p;11}(\mathbf{D_x}) = -\gamma_2\frac{\partial}{\partial x_p},$$
$$F_{p;12}(\mathbf{D_x}) = \gamma_3\frac{\partial}{\partial x_p}, F_{p+3;q+3}(\mathbf{D_x}) = [k_6\Delta - k_2 + \iota\omega c_1]\delta_{pq} + (k_4 + k_5)\frac{\partial^2}{\partial x_p \partial x_q},$$
$$F_{p+3;q+6}(\mathbf{D_x}) = F_{p+6;q+3}(\mathbf{D_x}) = \iota\omega\kappa_1\delta_{pq}, F_{p+3;10}(\mathbf{D_x}) = -k_3\frac{\partial}{\partial x_p},$$
$$F_{p+3;11}(\mathbf{D_x}) = F_{11;p+3}(\mathbf{D_x}) = 0, F_{p+3;12}(\mathbf{D_x}) = \iota\omega\varrho\frac{\partial}{\partial x_p},$$
$$F_{p+6;q+6}(\mathbf{D_x}) = [h_6\Delta - h_2 + \iota\omega m_1]\delta_{pq} + (h_4 + h_5)\frac{\partial^2}{\partial x_p \partial x_q}, \tag{34}$$
$$F_{p+6;10}(\mathbf{D_x}) = F_{10;p+6}(\mathbf{D_x}) = 0, F_{p+6;11}(\mathbf{D_x}) = -h_3\frac{\partial}{\partial x_p}, F_{p+6;12}(\mathbf{D_x}) = \iota\omega w\frac{\partial}{\partial x_p},$$
$$F_{10;q}(\mathbf{D_x}) = \iota\omega\gamma_1 T_0\frac{\partial}{\partial x_q}, F_{10;q+3}(\mathbf{D_x}) = k_1\frac{\partial}{\partial x_q}, F_{10;10}(\mathbf{D_x}) = k\Delta + \iota\omega c T_0, F_{10;11}(\mathbf{D_x}) = \iota\omega\kappa T_0,$$
$$F_{10;12}(\mathbf{D_x}) = \iota\omega\beta T_0, F_{11;q} = \iota\omega\gamma_2\frac{\partial}{\partial x_q}, F_{11;q+6}(\mathbf{D_x}) = h_1\frac{\partial}{\partial x_q}, F_{11;10}(\mathbf{D_x}) = \iota\omega\kappa,$$
$$F_{11;11}(\mathbf{D_x}) = h\Delta + \iota\omega m, F_{11;12}(\mathbf{D_x}) = \iota\omega\alpha, F_{12;q}(\mathbf{D_x}) = -\gamma_3\frac{\partial}{\partial x_q},$$
$$F_{12;q+3}(\mathbf{D_x}) = -\varrho\frac{\partial}{\partial x_q}, F_{12;q+6}(\mathbf{D_x}) = -w\frac{\partial}{\partial x_q},$$
$$F_{12;10}(\mathbf{D_x}) = \beta, F_{12;11}(\mathbf{D_x}) = \alpha, F_{12;12}(\mathbf{D_x}) = \gamma\Delta - \upsilon + \tau'\omega^2\ \ p, q = 1, 2, 3,$$

and

$$\tilde{\mathbf{F}}(\mathbf{D_x}) = \Big(\tilde{F}_{gl}(\mathbf{D_x})\Big)_{12\times12},$$

where

$$\tilde{F}_{pq}(\mathbf{D_x}) = \mu\Delta\delta_{pq} + (\lambda_0 + \mu)\frac{\partial^2}{\partial x_p \partial x_q},$$
$$\tilde{F}_{p+3;q+3}(\mathbf{D_x}) = k_6\Delta\delta_{pq} + (k_4 + k_5)\frac{\partial^2}{\partial x_p \partial x_q},$$
$$\tilde{F}_{p+6;q+6}(\mathbf{D_x}) = h_6\Delta\delta_{pq} + (h_4 + h_5)\frac{\partial^2}{\partial x_p \partial x_q},$$
$$\tilde{F}_{10;10}(\mathbf{D_x}) = k\Delta, \tilde{F}_{11;11}(\mathbf{D_x}) = h\Delta,$$
$$\tilde{F}_{12;12}(\mathbf{D_x}) = \gamma\Delta, \tilde{F}_{p;q+3}(\mathbf{D_x}) = \tilde{F}_{p;q+6}(\mathbf{D_x}) = \\ = \tilde{F}_{p+3;q}(\mathbf{D_x}) = \tilde{F}_{p+6;q}(\mathbf{D_x}) = 0, \\ \tilde{F}_{p+3;q+6}(\mathbf{D_x}) = \tilde{F}_{p+6;q+3}(\mathbf{D_x}) = \tilde{F}_{ie}(\mathbf{D_x}) = \tilde{F}_{ei}(\mathbf{D_x}) = 0, \\ \tilde{F}_{er}(\mathbf{D_x}) = 0;\ p, q = 1, 2, 3;\ e, r = 10, 11, 12; \\ e \neq r;\ i = 1, \dots, 9. \tag{35}$$

The system of equations (33) can be represented as

$$\mathbf{F}(\mathbf{D_x})\mathbf{U}(\mathbf{x}) = \mathbf{0}, \tag{36}$$

where $\mathbf{U} = (\mathbf{u}, \mathbf{v}, \mathbf{w}, \theta, P, \phi)$ is a twelve-component vector function on E^3. The matrix $\tilde{\mathbf{F}}(\mathbf{D_x})$ is called the principal part of operator $\mathbf{F}(\mathbf{D_x})$.

Definition 1. The operator $\mathbf{F}(\mathbf{D_x})$ is said to be elliptic if $|\tilde{\mathbf{F}}(\boldsymbol{k})| \neq 0$, where $\boldsymbol{k} = (\mu_1, \mu_2, \mu_3)$.

Since $|\tilde{\mathbf{F}}(\boldsymbol{k})| = \mu^2\tilde{\lambda}kk_6k_7hh_6h_7\gamma|\boldsymbol{k}|^{24}$, $\tilde{\lambda} = \lambda_0 + 2\mu$, $k_7 = k_4 + k_5 + k_6$, $h_7 = h_4 + h_5 + h_6$. Therefore, operator $\mathbf{F}(\mathbf{D_x})$ is an elliptic differential operator iff

$$\mu\tilde{\lambda}kk_6k_7hh_6h_7\gamma \neq 0. \tag{37}$$

Definition 2. The fundamental solutions of the system of equations (33) (the fundamental matrix of operator $\mathbf{F}$) is the matrix $\mathbf{G}(\mathbf{x}) = \Big(G_{gl}(\mathbf{x})\Big)_{12\times12}$ satisfying condition

$$\mathbf{F}(\mathbf{D_x})\mathbf{G}(\mathbf{x}) = \delta(\mathbf{x})\,\mathbf{I}(\mathbf{x}), \tag{38}$$

where $\delta(\mathbf{x})$ is the Dirac delta, $\mathbf{I} = (\delta_{gl})_{12\times12}$ is the unit matrix and $\mathbf{x} \in \mathrm{E}^3$.

IV. Construction of $\mathbf{G}(\mathbf{x})$ in Terms of Elementary Functions

Let us consider the system of non-homogeneous equations

$$[\mu\Delta + (\lambda_0+\mu)\,\mathrm{grad\,div} + \rho\omega^2]\mathbf{u} + \iota\omega\gamma_1T_0\,\mathrm{grad}\,\theta + +\iota\omega\gamma_2\,\mathrm{grad}\,P - \gamma_3\,\mathrm{grad}\,\phi = \mathbf{H},$$

$$[k_6\Delta + (k_4+k_5)\,\mathrm{grad\,div} + k_8]\mathbf{v} + \iota\omega\kappa_1\mathbf{w} + k_1\,\mathrm{grad}\,\theta + -\varrho\,\mathrm{grad}\,\phi = \mathbf{V},$$

$$\iota\omega\kappa_1\mathbf{v} + [h_6\Delta + (h_4+h_5)\,\mathrm{grad\,div} + h_8]\mathbf{w} + h_1\,\mathrm{grad}\,P + -w\,\mathrm{grad}\,\phi = \mathbf{W},$$

$$-\gamma_1\,\mathrm{div}\,\mathbf{u} - k_3\,\mathrm{div}\,\mathbf{v} + [k\,\Delta + \iota\omega cT_0]\theta + \iota\omega\kappa P + \beta\,\phi = Z,$$

$$-\gamma_2\,\mathrm{div}\,\mathbf{u} - h_3\,\mathrm{div}\,\mathbf{w} + \iota\omega\kappa T_0\,\theta + [h\,\Delta + \iota\omega m]P + \alpha\,\phi = X,$$

$$\gamma_3\,\mathrm{div}\,\mathbf{u} + \iota\omega\varrho\,\mathrm{div}\,\mathbf{v} + \iota\omega\,w\,\mathrm{div}\,\mathbf{w} + \iota\omega\beta\,T_0\,\theta + \iota\omega\alpha\,P + +[\gamma\Delta + \zeta]\phi = Y, \tag{39}$$

where $k_8 = -k_2 + \iota\omega c_1, h_8 = -h_2 + \iota\omega m_1, \zeta = \tau'\omega^2 + -\upsilon, \mathbf{H}, \mathbf{V}, \mathbf{W}$ are three-component vector functions on E^3; Z and X are scalar functions on E^3.

The system of equations (39) may also be written in the form

$$\mathbf{F}^{tr}(\mathbf{D_x})\mathbf{U}(\mathbf{x}) = \mathbf{Q}(\mathbf{x}), \tag{40}$$

where $\mathbf{F}^{tr}$ is the transpose of matrix $\mathbf{F}$, $\mathbf{Q} = (\mathbf{H}, \mathbf{V}, \mathbf{W}, Z, X, Y)$ and $\mathbf{x} \in \mathrm{E}^3$.

Applying operator div to the Eqs. $(39)_{1-3}$, we obtain

$$[\tilde{\lambda}\Delta + \rho\omega^2]\,\mathrm{div}\,\mathbf{u} + \iota\omega\gamma_1T_0\Delta\theta + \iota\omega\gamma_2\Delta P - \gamma_3\Delta\,\phi = \mathrm{div}\,\mathbf{H}, \tag{41}$$

$$(k_7\Delta + k_8)\,\mathrm{div}\,\mathbf{v} + \iota\omega\kappa_1\,\mathrm{div}\,\mathbf{w} + k_1\Delta\theta - \varrho\,\Delta\,\phi = \mathrm{div}\,\mathbf{V}, \tag{42}$$

$$\iota\omega\kappa_1\,\mathrm{div}\,\mathbf{v} + (h_7\Delta + h_8)\,\mathrm{div}\,\mathbf{w} + h_1\Delta P - w\,\Delta\,\phi = \mathrm{div}\,\mathbf{W}. \tag{43}$$

The Eqs. $(39)_{4-6}$ and (41)–(43) may be expressed in the form

$$\mathbf{N}(\Delta)\mathbf{S} = \tilde{\mathbf{Q}}, \tag{44}$$

where $\mathbf{S} = (\mathrm{div}\,\mathbf{u}, \mathrm{div}\,\mathbf{v}, \mathrm{div}\,\mathbf{w}, \theta, P, \phi)$, $\tilde{\mathbf{Q}} = (w_1, \ldots, w_6) = (\mathrm{div}\,\mathbf{H}, \mathrm{div}\,\mathbf{V}, \mathrm{div}\,\mathbf{W}, Z, X, Y)$ and

$$\mathbf{N}(\Delta) = \Big(N_{gl}(\Delta)\Big)_{6\times6} = \\ = \begin{pmatrix} \tilde{\lambda}\Delta + \rho\omega^2 & 0 & 0 & \iota\omega\gamma_1T_0\Delta & \iota\omega\gamma_2\Delta & -\gamma_3\Delta \\ 0 & k_7\Delta + k_8 & \iota\omega\kappa_1 & k_1\Delta & 0 & -\varrho\Delta \\ 0 & \iota\omega\kappa_1 & h_7\Delta + h_8 & 0 & h_1\Delta & -w\Delta \\ -\gamma_1 & -k_3 & 0 & k\Delta + \iota\omega cT_0 & \iota\omega\kappa & \beta \\ -\gamma_2 & 0 & -h_3 & \iota\omega\kappa T_0 & h\Delta + \iota\omega m & \alpha \\ \gamma_3 & \iota\omega\varrho & \iota\omega\,w & \iota\omega\beta\,T_0 & \iota\omega\alpha & \gamma\Delta + \zeta \end{pmatrix}_{6\times6}. \tag{45}$$

The Eqs. $(39)_{4-6}$ and (41)–(43) may also be written as

$$\Gamma_1(\Delta)\mathbf{S} = \boldsymbol{\Psi}, \tag{46}$$

where

$$\boldsymbol{\Psi} = (\Psi_1, \ldots, \Psi_6), \Psi_p = \frac{1}{M^*}\sum_{i=1}^{6} N_{ip}^* w_i,$$
$$\Gamma_1(\Delta) = \frac{1}{M^*}\,|\mathbf{N}(\Delta)|, M^* = \tilde{\lambda}kk_7hh_7\gamma, p = 1, \ldots, 6, \tag{47}$$

and N_{ip}^* is the cofactor of element N_{ip} of matrix $\mathbf{N}$. From Eqs. (45) and (47), we see that

$$\Gamma_1(\Delta) = \prod_{i=1}^{6}(\Delta + \lambda_i^2),$$

where λ_i^2, $i = 1, \ldots, 6$ are the roots of the equation $\Gamma_1(-\xi) = 0$ (with respect to ξ).

Applying operator $\Gamma_1(\Delta)$ to the Eq. $(39)_1$, we get

$$\Gamma_1(\Delta)(\Delta+\lambda_7^2)\mathbf{u}=\mathbf{\Psi}', \tag{48}$$

where

$$\lambda_7^2=\frac{\rho\omega^2}{\mu},\mathbf{\Psi}'=\frac{1}{\mu}\Big[\Gamma_1(\Delta)\mathbf{H}-\operatorname{grad}[(\lambda_0+\mu)\Psi_1+ +\iota\omega\gamma_1 T_0\Psi_4+\iota\omega\gamma_2\Psi_5-\gamma_3\Psi_6]\Big].$$

Multiplying Eqs. $(39)_2$ and $(39)_3$ by $h_6\Delta+h_8$ and $\iota\omega\kappa_1$, respectively, we obtain

$$(h_6\Delta+h_8)[k_6\Delta+(k_4+k_5)\operatorname{grad}\operatorname{div}+k_8]\mathbf{v}+(h_6\Delta+ +h_8)\iota\omega\kappa_1\mathbf{w}=(h_6\Delta+h_8)[\mathbf{V}-k_1\operatorname{grad}\theta+\varrho\operatorname{grad}\phi], \tag{49}$$

and

$$(\iota\omega\kappa_1)^2\mathbf{v}+\iota\omega\kappa_1[h_6\Delta+(h_4+h_5)\operatorname{grad}\operatorname{div}+h_8]\mathbf{w}= =\iota\omega\kappa_1[\mathbf{W}-h_1\operatorname{grad}P+w\operatorname{grad}\phi]. \tag{50}$$

Using Eq. (50) in (49), we obtain

$$[(h_6\Delta+h_8)(k_6\Delta+k_8)-(\iota\omega\kappa_1)^2]\mathbf{v}=\iota\omega\kappa_1(h_4+ +h_5)\operatorname{grad}\operatorname{div}\mathbf{w}+(h_6\Delta+h_8)[\mathbf{V}-k_1\operatorname{grad}\theta+\varrho\operatorname{grad}\phi+ -(k_4+k_5)\operatorname{grad}\operatorname{div}\mathbf{v}]-\iota\omega\kappa_1[\mathbf{W}-h_1\operatorname{grad}P+w\operatorname{grad}\phi]. \tag{51}$$

Applying operator $\Gamma_1(\Delta)$ to the Eq. (51) and using Eq. (46), we get

$$\Gamma_1(\Delta)\Gamma_2(\Delta)\mathbf{v}=\mathbf{\Psi}'', \tag{52}$$

where

$$\Gamma_2(\Delta)=\frac{1}{N^*}\begin{vmatrix} k_6\Delta+k_8 & \iota\omega\kappa_1\\ \iota\omega\kappa_1 & h_6\Delta+h_8\end{vmatrix}, N^*=k_6h_6,$$

and

$$\mathbf{\Psi}''=\frac{1}{N^*}\Big[(h_6\Delta+h_8)[\Gamma_1(\Delta)\mathbf{V}-k_1\operatorname{grad}\Psi_4+\varrho\operatorname{grad}\Psi_6+ -(k_4+k_5)\operatorname{grad}\Psi_2]-\iota\omega\kappa_1[\Gamma_1(\Delta)\mathbf{W}-h_1\operatorname{grad}\Psi_5+ +w\operatorname{grad}\Psi_6-(h_4+h_5)\operatorname{grad}\Psi_3]\Big]. \tag{53}$$

It can be seen that

$$\Gamma_2(\Delta)=(\Delta+\lambda_8^2)(\Delta+\lambda_9^2),$$

where λ_8^2,λ_9^2 are the roots of the equation $\Gamma_2(-\xi)=0$ (with respect to ξ).

Multiplying Eqs. $(39)_2$ and $(39)_3$ by $\iota\omega\kappa_1$ and $k_6\Delta+k_8$, respectively, we obtain

$$(\iota\omega\kappa_1)[k_6\Delta+(k_4+k_5)\operatorname{grad}\operatorname{div}+k_8]\mathbf{v}+(\iota\omega\kappa_1)^2\mathbf{w}= =(\iota\omega\kappa_1)[\mathbf{V}-k_1\operatorname{grad}\theta+\varrho\operatorname{grad}\phi], \tag{54}$$

and

$$(\iota\omega\kappa_1)(k_6\Delta+k_8)\mathbf{v}+(k_6\Delta+k_8)[h_6\Delta+ +(h_4+h_5)\operatorname{grad}\operatorname{div}+h_8]\mathbf{w}= =(k_6\Delta+k_8)[\mathbf{W}-h_1\operatorname{grad}P+w\operatorname{grad}\phi]. \tag{55}$$

Using Eq. (54) in (55), we obtain

$$[(h_6\Delta+h_8)(k_6\Delta+k_8)-(\iota\omega\kappa_1)^2]\mathbf{w}= =\iota\omega\kappa_1(k_4+k_5)\operatorname{grad}\operatorname{div}\mathbf{v}+(\kappa_6\Delta+\kappa_8)\times \times[\mathbf{W}-h_1\operatorname{grad}P+w\operatorname{grad}\phi-(h_4+h_5)\operatorname{grad}\operatorname{div}\mathbf{w}]+ -\iota\omega\kappa_1[\mathbf{V}-k_1\operatorname{grad}\theta+\varrho\operatorname{grad}\phi]. \tag{56}$$

Applying operator $\Gamma_1(\Delta)$ to the Eq. (56) and using Eq. (46), we get

$$\Gamma_1(\Delta)\Gamma_2(\Delta)\mathbf{w}=\mathbf{\Psi}''', \tag{57}$$

where

$$\mathbf{\Psi}'''=\frac{1}{N^*}\Big[(k_6\Delta+k_8)[\Gamma_1(\Delta)\mathbf{W}-h_1\operatorname{grad}\Psi_5+ +w\operatorname{grad}\Psi_6-(h_4+h_5)\operatorname{grad}\Psi_3]+ -\iota\omega\kappa_1[\Gamma_1(\Delta)\mathbf{V}-k_1\operatorname{grad}\Psi_4+\varrho\operatorname{grad}\Psi_6+ -(k_4+k_5)\operatorname{grad}\Psi_2]\Big]. \tag{58}$$

From Eqs. (46), (48), (52) and (57), we obtain

$$\mathbf{\Theta}(\Delta)\mathbf{U}(\mathbf{x})=\hat{\mathbf{\Psi}}(\mathbf{x}), \tag{59}$$

where $\hat{\mathbf{\Psi}}=(\mathbf{\Psi}',\mathbf{\Psi}'',\mathbf{\Psi}''',\Psi_4,\Psi_5,\Psi_6)$ and

$$\mathbf{\Theta}(\Delta)=\Big(\Theta_{gq}(\Delta)\Big)_{12\times12},$$

$$\Theta_{pp}(\Delta)=\Gamma_1(\Delta)(\Delta+\lambda_7^2)=\prod_{i=1}^{7}(\Delta+\lambda_i^2),$$

$$\Theta_{p+3;p+3}(\Delta)=\Theta_{p+6;p+6}(\Delta)=\Gamma_1(\Delta)\Gamma_2(\Delta)= =\prod_{i=1,i\neq7}^{9}(\Delta+\lambda_i^2),$$

$$\Theta_{p+9;p+9}(\Delta)=\Gamma_1(\Delta)=\prod_{i=1}^{6}(\Delta+\lambda_i^2),\ \Theta_{gq}(\Delta)=0,$$

$$p=1,2,3;\ g,q=1,\dots,12;\ g\neq q.$$

The Eqs. (47), (48), (53) and (58) can be rewritten in the form

$$
\begin{aligned}
\mathbf{\Psi}' &= \frac{1}{\mu}\Big[\Gamma_1(\Delta)\mathbf{J} + w_{11}(\Delta)\,\mathrm{grad\,div}\Big]\mathbf{H} + \sum_{i=2}^{6} w_{i1}(\Delta)\,\mathrm{grad}\,w_i\ ,\\
\mathbf{\Psi}'' &= \Big[\frac{1}{N^*}(h_6\Delta + h_8)\Gamma_1(\Delta)\mathbf{J} + w_{22}(\Delta)\,\mathrm{grad\,div}\Big]\mathbf{V} + w_{12}(\Delta)\,\mathrm{grad\,div}\,\mathbf{H} + w_{42}(\Delta)\,\mathrm{grad}\,Z+\\
&\quad + w_{52}(\Delta)\,\mathrm{grad}\,X + w_{62}(\Delta)\,\mathrm{grad}\,Y + \Big[-\frac{1}{N^*}\iota\omega\kappa_1\Gamma_1(\Delta)\mathbf{J} + w_{32}(\Delta)\,\mathrm{grad\,div}\Big]\mathbf{W},\\
\mathbf{\Psi}''' &= \Big[\frac{1}{N^*}(k_6\Delta + k_8)\Gamma_1(\Delta)\mathbf{J} + w_{33}(\Delta)\,\mathrm{grad\,div}\Big]\mathbf{W} + w_{13}(\Delta)\,\mathrm{grad\,div}\,\mathbf{H} + w_{43}(\Delta)\,\mathrm{grad}\,Z+\\
&\quad + w_{53}(\Delta)\,\mathrm{grad}\,X + w_{63}(\Delta)\,\mathrm{grad}\,Y + \Big[-\frac{1}{N^*}\iota\omega\kappa_1\Gamma_1(\Delta)\mathbf{J} + w_{23}(\Delta)\,\mathrm{grad\,div}\Big]\mathbf{V},\\
\Psi_p &= w_{1p}(\Delta)\,\mathrm{div}\,\mathbf{H} + w_{2p}(\Delta)\,\mathrm{div}\,\mathbf{V} + w_{3p}(\Delta)\,\mathrm{div}\,\mathbf{W} + w_{4p}(\Delta)\,Z+\\
&\quad + w_{5p}(\Delta)\,X + w_{6p}(\Delta)\,Y;\ p = 4,5,6,
\end{aligned}
\tag{60}
$$

where $\mathbf{J} = (\delta_{gh})_{3\times3}$ is the unit matrix.

In the Eqs. (60), the following notations have been used:

$$
\begin{aligned}
w_{p1}(\Delta) &= -\frac{1}{M^*\mu}\Big[(\lambda_0+\mu)N^*_{p1}(\Delta) + \iota\omega\gamma_1 T_0 N^*_{p4}(\Delta) + \iota\omega\gamma_2 N^*_{p5}(\Delta) - \gamma_3 N^*_{p6}(\Delta)\Big],\\
w_{p2}(\Delta) &= -\frac{1}{M^*N^*}\Big[(h_6\Delta+h_8)[(k_4+k_5)N^*_{p2} + k_1N^*_{p4} - \varrho N^*_{p6}] - \iota\omega\kappa_1[h_1N^*_{p5} + (h_4+h_5)N^*_{p3} - wN^*_{p6}]\Big],\\
w_{p3}(\Delta) &= -\frac{1}{M^*N^*}\Big[(k_6\Delta+k_8)[(h_4+h_5)N^*_{p3} + h_1N^*_{p5} - wN^*_{p6}] - \iota\omega\kappa_1[k_1N^*_{p4} + (k_4+k_5)N^*_{p2} - \varrho N^*_{p6}]\Big],\\
w_{p4}(\Delta) &= \frac{N^*_{p4}}{M^*},\ w_{p5}(\Delta) = \frac{N^*_{p5}}{M^*},\ w_{p6}(\Delta) = \frac{N^*_{p6}}{M^*}\ \ p = 1,\ldots,6.
\end{aligned}
$$

From Eqs. (60), we have

$$
\hat{\mathbf{\Psi}}(\mathbf{x}) = \mathbf{R}^{tr}(\mathbf{D_x})\mathbf{Q}(\mathbf{x}),
\tag{61}
$$

where

$$
\begin{aligned}
\mathbf{R}(\mathbf{D_x}) &= \Big(R_{gq}(\mathbf{D_x})\Big)_{12\times12}\ ,\\
R_{ij}(\mathbf{D_x}) &= \frac{1}{\mu}\Gamma_1(\Delta)\delta_{ij} + w_{11}(\Delta)\frac{\partial^2}{\partial x_i\partial x_j},\\
R_{i+3;j+3}(\mathbf{D_x}) &= \frac{1}{N^*}(h_6\Delta+h_8)\Gamma_1(\Delta)\delta_{ij} + w_{22}(\Delta)\frac{\partial^2}{\partial x_i\partial x_j},\\
R_{i+6;j+6}(\mathbf{D_x}) &= \frac{1}{N^*}(k_6\Delta+k_8)\Gamma_1(\Delta)\delta_{ij} + w_{33}(\Delta)\frac{\partial^2}{\partial x_i\partial x_j},\\
R_{i;j+3}(\mathbf{D_x}) &= w_{12}(\Delta)\frac{\partial^2}{\partial x_i\partial x_j}, R_{i;j+6}(\mathbf{D_x}) = w_{13}(\Delta)\frac{\partial^2}{\partial x_i\partial x_j},\\
R_{i;p+6}(\mathbf{D_x}) &= w_{1p}(\Delta)\frac{\partial}{\partial x_i}, R_{i+3;j}(\mathbf{D_x}) = w_{21}(\Delta)\frac{\partial^2}{\partial x_i\partial x_j},\\
R_{i+3;j+6}(\mathbf{D_x}) &= w_{23}(\Delta)\frac{\partial^2}{\partial x_i\partial x_j} - \frac{1}{N^*}\iota\omega\kappa_1\Gamma_1(\Delta)\delta_{ij},\\
R_{i+3;p+6}(\mathbf{D_x}) &= w_{2p}(\Delta)\frac{\partial}{\partial x_i}, R_{i+6;j} = w_{31}(\Delta)\frac{\partial^2}{\partial x_i\partial x_j},\\
R_{i+6;j+3}(\mathbf{D_x}) &= w_{32}(\Delta)\frac{\partial^2}{\partial x_i\partial x_j} - \frac{1}{N^*}\iota\omega\kappa_1\Gamma_1(\Delta)\delta_{ij},
\end{aligned}
$$

$$R_{i+6;p+6}(\mathbf{D_x}) = w_{3p}(\Delta)\frac{\partial}{\partial x_i}, R_{p+6;i}(\mathbf{D_x}) = w_{p1}(\Delta)\frac{\partial}{\partial x_i},$$
$$R_{p+6;i+3}(\mathbf{D_x}) = w_{p2}(\Delta)\frac{\partial}{\partial x_i}, R_{p+6;i+6}(\mathbf{D_x}) = w_{p3}(\Delta)\frac{\partial}{\partial x_i}, \tag{62}$$
$$R_{p+6;l+6} = w_{pl}(\Delta)\ i,j=1,2,3\ p,l=4,5,6.$$

From Eqs. (40), (59) and (61), we obtain

$$\mathbf{\Theta U} = \mathbf{R}^{tr}\mathbf{F}^{tr}\mathbf{U}.$$

The above relation implies

$$\mathbf{R}^{tr}\mathbf{F}^{tr} = \mathbf{\Theta}.$$

Therefore, we obtain

$$\mathbf{F}(\mathbf{D_x})\mathbf{R}(\mathbf{D_x}) = \mathbf{\Theta}(\Delta). \tag{63}$$

We assume that

$$\lambda_p^2 \neq \lambda_q^2 \neq 0\ p,q=1,\dots,9\ p\neq q.$$

Let

$$\mathbf{Y}(\mathbf{x}) = \Big(Y_{ij}(\mathbf{x})\Big)_{12\times 12},\ Y_{pp}(\mathbf{x}) = \sum_{g=1}^{7} r_{1g}\varsigma_g(\mathbf{x}),$$
$$Y_{p+3;p+3}(\mathbf{x}) = Y_{p+6;p+6}(\mathbf{x}) = \sum_{g=1,g\neq 7}^{9} r_{2g}\varsigma_g(\mathbf{x}),$$
$$Y_{p+9;p+9}(\mathbf{x}) = \sum_{g=1}^{6} r_{3g}\varsigma_g(\mathbf{x}), Y_{qz}(\mathbf{x}) = 0,$$
$$p=1,2,3;\ q,z=1,\dots,12;\ q\neq z,$$

where

$$\varsigma_g(\mathbf{x}) = -\frac{e^{\iota\lambda_g|\mathbf{x}|}}{4\pi|\mathbf{x}|}, r_{1p} = \prod_{i=1,i\neq p}^{7}(\lambda_i^2-\lambda_p^2)^{-1},$$
$$r_{2l} = \prod_{i=1,i\neq 7,i\neq l}^{9}(\lambda_i^2-\lambda_l^2)^{-1}, r_{3q} = \prod_{i=1,i\neq q}^{6}(\lambda_i^2-\lambda_q^2)^{-1},$$
$$p=1,\dots,7;\ g=1,\dots,9;\ l=1,\dots,6,8,9;\ q=1,\dots,6. \tag{64}$$

Lemma 1. Matrix $\mathbf{Y}$ defined above is the fundamental matrix of operator $\mathbf{\Theta}(\Delta)$, i.e.

$$\mathbf{\Theta}(\Delta)\mathbf{Y}(\mathbf{x}) = \delta(\mathbf{x})\,\mathbf{I}(\mathbf{x}). \tag{65}$$

Proof: To prove the lemma, it is sufficient to prove that

$$\Gamma_1(\Delta)(\Delta+\lambda_7^2)Y_{11}(\mathbf{x}) = \delta(\mathbf{x}),$$
$$\Gamma_1(\Delta)\Gamma_2(\Delta)Y_{44}(\mathbf{x}) = \delta(\mathbf{x}), \tag{66}$$
$$\Gamma_1(\Delta)Y_{10;10}(\mathbf{x}) = \delta(\mathbf{x}).$$

Consider

$$\sum_{i=1}^{7} r_{1i} = \frac{\sum_{j=1}^{7}(-1)^{j+1}z_j}{z_8},$$

where

$$z_1 = \prod_{i=3}^{7}(\lambda_2^2-\lambda_i^2)\prod_{j=4}^{7}(\lambda_3^2-\lambda_j^2)\prod_{l=5}^{7}(\lambda_4^2-\lambda_l^2)\prod_{p=6}^{7}(\lambda_5^2-\lambda_p^2)(\lambda_6^2-\lambda_7^2),$$
$$z_2 = \prod_{i=3}^{7}(\lambda_1^2-\lambda_i^2)\prod_{j=4}^{7}(\lambda_3^2-\lambda_j^2)\prod_{l=5}^{7}(\lambda_4^2-\lambda_l^2)\prod_{p=6}^{7}(\lambda_5^2-\lambda_p^2)(\lambda_6^2-\lambda_7^2),$$
$$z_3 = \prod_{i=2,i\neq 3}^{7}(\lambda_1^2-\lambda_i^2)\prod_{j=4}^{7}(\lambda_2^2-\lambda_j^2)\prod_{l=5}^{7}(\lambda_4^2-\lambda_l^2)\prod_{p=6}^{7}(\lambda_5^2-\lambda_p^2)(\lambda_6^2-\lambda_7^2),$$
$$z_4 = \prod_{i=2,i\neq 4}^{7}(\lambda_1^2-\lambda_i^2)\prod_{j=3,j\neq 4}^{7}(\lambda_2^2-\lambda_j^2)\prod_{l=5}^{7}(\lambda_3^2-\lambda_l^2)\prod_{p=6}^{7}(\lambda_5^2-\lambda_p^2)(\lambda_6^2-\lambda_7^2),$$
$$z_5 = \prod_{i=2,i\neq 5}^{7}(\lambda_1^2-\lambda_i^2)\prod_{j=3,j\neq 5}^{7}(\lambda_2^2-\lambda_j^2)\prod_{l=4,l\neq 5}^{7}(\lambda_3^2-\lambda_l^2)\prod_{p=6}^{7}(\lambda_4^2-\lambda_p^2)(\lambda_6^2-\lambda_7^2),$$

$$z_6=\prod_{i=2,i\neq 6}^{7}(\lambda_1^2-\lambda_i^2)\prod_{j=3,j\neq 6}^{7}(\lambda_2^2-\lambda_j^2)\prod_{l=4,l\neq 6}^{7}(\lambda_3^2-\lambda_l^2)\prod_{p=5,p\neq 6}^{7}(\lambda_4^2-\lambda_p^2)(\lambda_5^2-\lambda_7^2),$$

$$z_7=\prod_{i=2}^{6}(\lambda_1^2-\lambda_i^2)\prod_{j=3}^{6}(\lambda_2^2-\lambda_j^2)\prod_{l=4}^{6}(\lambda_3^2-\lambda_l^2)\prod_{p=5}^{6}(\lambda_4^2-\lambda_p^2)(\lambda_5^2-\lambda_6^2), \tag{67}$$

$$z_8=\prod_{i=2}^{7}(\lambda_1^2-\lambda_i^2)\prod_{j=3}^{7}(\lambda_2^2-\lambda_j^2)\prod_{l=4}^{7}(\lambda_3^2-\lambda_l^2)\prod_{p=5}^{7}(\lambda_4^2-\lambda_p^2)\prod_{q=6}^{7}(\lambda_5^2-\lambda_q^2)(\lambda_6^2-\lambda_7^2).$$

On simplifying the right hand side of above relation, we obtain

$$\sum_{i=1}^{7} r_{1i}=0. \tag{68}$$

Similarly, we find that

$$\sum_{i=2}^{7} r_{1i}(\lambda_1^2-\lambda_i^2)=0,\sum_{i=3}^{7} r_{1i}\left[\prod_{j=1}^{2}(\lambda_j^2-\lambda_i^2)\right]=0,$$
$$\sum_{i=4}^{7} r_{1i}\left[\prod_{j=1}^{3}(\lambda_j^2-\lambda_i^2)\right]=0,\sum_{i=5}^{7} r_{1i}\left[\prod_{j=1}^{4}(\lambda_j^2-\lambda_i^2)\right]=0,$$
$$\sum_{i=6}^{7} r_{1i}\left[\prod_{j=1}^{5}(\lambda_j^2-\lambda_i^2)\right]=0,\prod_{j=1}^{6} r_{17}(\lambda_j^2-\lambda_7^2)=1. \tag{69}$$

Also,

$$(\Delta+\lambda_p^2)\varsigma_g(\mathbf{x})=\delta(\mathbf{x})+(\lambda_p^2-\lambda_g^2)\varsigma_g(\mathbf{x})\ p,g=1,\dots,9. \tag{70}$$

Now consider

$$\Gamma_1(\Delta)(\Delta+\lambda_7^2)Y_{11}(\mathbf{x})=\prod_{i=1}^{7}(\Delta+\lambda_i^2)\sum_{g=1}^{7} r_{1g}\varsigma_g(\mathbf{x})=$$
$$=\prod_{i=2}^{7}(\Delta+\lambda_i^2)\sum_{g=1}^{7} r_{1g}\Big[\delta(\mathbf{x})+(\lambda_1^2-\lambda_g^2)\varsigma_g(\mathbf{x})\Big]=$$
$$=\prod_{i=2}^{7}(\Delta+\lambda_i^2)\Big[\delta(\mathbf{x})\sum_{g=1}^{7} r_{1g}+\sum_{g=2}^{7} r_{1g}(\lambda_1^2-\lambda_g^2)\varsigma_g(\mathbf{x})\Big].$$

Using Eqs. (68)–(70) in the above relation, we obtain

$$\Gamma_1(\Delta)(\Delta+\lambda_7^2)Y_{11}(\mathbf{x})=\prod_{i=2}^{7}(\Delta+\lambda_i^2)\Big[\sum_{g=2}^{7} r_{1g}(\lambda_1^2-\lambda_g^2)\varsigma_g(\mathbf{x})\Big]=$$
$$=\prod_{i=3}^{7}(\Delta+\lambda_i^2)\Big[\sum_{g=2}^{7} r_{1g}(\lambda_1^2-\lambda_g^2)\Big[\delta(\mathbf{x})+(\lambda_2^2-\lambda_g^2)\varsigma_g(\mathbf{x})\Big]\Big]=\prod_{i=3}^{7}(\Delta+\lambda_i^2)\Big[\sum_{g=3}^{7} r_{1g}\Big[\prod_{j=1}^{2}(\lambda_j^2-\lambda_g^2)\Big]\varsigma_g(\mathbf{x})\Big]=$$
$$=\prod_{i=4}^{7}(\Delta+\lambda_i^2)\Big[\sum_{g=3}^{7} r_{1g}\Big[\prod_{j=1}^{2}(\lambda_j^2-\lambda_g^2)\Big]\Big[\delta(\mathbf{x})+(\lambda_3^2-\lambda_g^2)\varsigma_g(\mathbf{x})\Big]\Big]=\prod_{i=4}^{7}(\Delta+\lambda_i^2)\Big[\sum_{g=4}^{7} r_{1g}\Big[\prod_{j=1}^{3}(\lambda_j^2-\lambda_g^2)\Big]\varsigma_g(\mathbf{x})\Big]=$$
$$=\prod_{i=5}^{7}(\Delta+\lambda_i^2)\Big[\sum_{g=4}^{7} r_{1g}\Big[\prod_{j=1}^{3}(\lambda_j^2-\lambda_g^2)\Big]\Big[\delta(\mathbf{x})+(\lambda_4^2-\lambda_g^2)\varsigma_g(\mathbf{x})\Big]\Big]=\prod_{i=5}^{7}(\Delta+\lambda_i^2)\Big[\sum_{g=5}^{7} r_{1g}\Big[\prod_{j=1}^{4}(\lambda_j^2-\lambda_g^2)\Big]\varsigma_g(\mathbf{x})\Big]=$$
$$=\prod_{i=6}^{7}(\Delta+\lambda_i^2)\Big[\sum_{g=5}^{7} r_{1g}\Big[\prod_{j=1}^{4}(\lambda_j^2-\lambda_g^2)\Big]\Big[\delta(\mathbf{x})+(\lambda_5^2-\lambda_g^2)\varsigma_g(\mathbf{x})\Big]\Big]=\prod_{i=6}^{7}(\Delta+\lambda_i^2)\Big[\sum_{g=6}^{7} r_{1g}\Big[\prod_{j=1}^{5}(\lambda_j^2-\lambda_g^2)\Big]\varsigma_g(\mathbf{x})\Big]=$$
$$=(\Delta+\lambda_7^2)\Big[\sum_{g=6}^{7} r_{1g}\Big[\prod_{j=1}^{5}(\lambda_j^2-\lambda_g^2)\Big]\Big[\delta(\mathbf{x})+(\lambda_6^2-\lambda_g^2)\varsigma_g(\mathbf{x})\Big]\Big]=(\Delta+\lambda_7^2)\varsigma_7(\mathbf{x})=\delta(\mathbf{x}).$$

The Eqs. $(66)_2$ and $(66)_3$ can be proved in the similar way.

We introduce the matrix

$$\mathbf{G}(\mathbf{x})=\mathbf{R}(\mathbf{D_x})\mathbf{Y}(\mathbf{x}). \tag{71}$$

From Eqs. (63), (65) and (71), we obtain

$$\mathbf{F}(\mathbf{D_x})\mathbf{G}(\mathbf{x})=\mathbf{F}(\mathbf{D_x})\mathbf{R}(\mathbf{D_x})\mathbf{Y}(\mathbf{x})=$$
$$=\mathbf{\Theta}(\Delta)\mathbf{Y}(\mathbf{x})=\delta(\mathbf{x})\,\mathbf{I}(\mathbf{x}).$$

Hence, $\mathbf{G}(\mathbf{x})$ is a solution to Eq. (38).

Theorem 1. If the condition (37) is satisfied, then the matrix $\mathbf{G}(\mathbf{x})$ defined by the Eq. (71) is the fundamental solution of the system of equations (33) and the matrix $\mathbf{G}(\mathbf{x})$ is represented in the following form:

$$G_{gl}(\mathbf{x}) = R_{gl}(\mathbf{D_x})Y_{11}(\mathbf{x}),\\ G_{gq}(\mathbf{x}) = R_{gq}(\mathbf{D_x})Y_{44}(\mathbf{x}),\\ G_{gj}(\mathbf{x}) = R_{gj}(\mathbf{D_x})Y_{10;10}(\mathbf{x}),\\ g = 1,\dots,12;\ l = 1,2,3;\ q = 4,\dots,9;\ j = 10,11,12.$$

V. Basic Properties of Matrix $\mathbf{G}(\mathbf{x})$

Theorem 2. Each column of matrix $\mathbf{G}(\mathbf{x})$ is a solution of the system of equations (33) at every point $\mathbf{x} \in \mathrm{E}^3$ except at the origin.

Theorem 3. If the condition (37) is satisfied, then the fundamental solution of the system $\tilde{\mathbf{F}}(\mathbf{D_x})\mathbf{U}(\mathbf{x}) = \mathbf{0}$ is the matrix

$$\mathbf{B}(\mathbf{x}) = \Big(B_{rz}(\mathbf{x})\Big)_{12\times 12},$$
$$B_{ij}(\mathbf{x}) = \left[\frac{1}{\tilde{\lambda}}\frac{\partial^2}{\partial x_i \partial x_j} - \frac{1}{\mu}\tilde{R}_{ij}\right]\varsigma_2^*(\mathbf{x}),$$
$$B_{i+3;j+3}(\mathbf{x}) = \left[\frac{1}{k_7}\frac{\partial^2}{\partial x_i \partial x_j} - \frac{1}{k_6}\tilde{R}_{ij}\right]\varsigma_2^*(\mathbf{x}),$$
$$B_{i+6;j+6}(\mathbf{x}) = \left[\frac{1}{h_7}\frac{\partial^2}{\partial x_i \partial x_j} - \frac{1}{h_6}\tilde{R}_{ij}\right]\varsigma_2^*(\mathbf{x}),$$
$$B_{10;10} = \frac{\varsigma_1^*(\mathbf{x})}{k}, B_{11;11} = \frac{\varsigma_1^*(\mathbf{x})}{h}, B_{12;12} = \frac{\varsigma_1^*(\mathbf{x})}{\gamma},$$
$$B_{iq} = B_{qi} = 0, B_{i+3;l} = B_{l;i+3} = 0,$$
$$B_{i+6;d} = B_{d;i+6} = 0, B_{dp} = 0, \varsigma_1^* = -\frac{1}{4\pi|\mathbf{x}|}, \varsigma_2^* = -\frac{|\mathbf{x}|}{8\pi},$$
$$\tilde{R}_{ij} = \frac{\partial^2}{\partial x_i \partial x_j} - \Delta\delta_{ij};\ i,j = 1,2,3;\ q = 4,\dots,12;\\ l = 7,\dots,12;\ d,p = 10,11,12;\ d \neq p. \tag{72}$$

VI. Fundamental Solutions of System of Equations in Equilibrium Theory

If we put $\omega = 0$ in the system of equations (33), we obtain the system of equations in the equilibrium theory of micromorphic thermoelastic diffusion with microtemperatures and microconcentrations as:

$$[\mu\Delta + (\lambda_0 + \mu)\operatorname{grad}\operatorname{div}]\mathbf{u} - \gamma_1 \operatorname{grad}\theta +\\ -\gamma_2 \operatorname{grad} P + \gamma_3 \operatorname{grad}\phi = \mathbf{0},$$
$$[k_6\Delta + (k_4 + k_5)\operatorname{grad}\operatorname{div} - k_2]\mathbf{v} - k_3 \operatorname{grad}\theta = \mathbf{0},$$
$$[h_6\Delta + (h_4 + h_5)\operatorname{grad}\operatorname{div} - h_2]\mathbf{w} - h_3 \operatorname{grad} P = \mathbf{0},$$
$$k_1 \operatorname{div}\mathbf{v} + k\,\Delta\,\theta = 0,$$
$$h_1 \operatorname{div}\mathbf{w} + h\,\Delta\,P = 0,$$
$$-\gamma_3 \operatorname{div}\mathbf{u} - \varrho \operatorname{div}\mathbf{v} - w \operatorname{div}\mathbf{w} + \beta\,\theta + \alpha\,C +\\ +(\gamma\Delta - \upsilon)\phi = 0. \tag{73}$$

We introduce the second order matrix differential operators with constant coefficients

$$\mathbf{E}(\mathbf{D_x}) = \Big(E_{gl}(\mathbf{D_x})\Big)_{12\times 12},$$

where matrix $\mathbf{E}(\mathbf{D_x})$ can be obtained from $\mathbf{F}(\mathbf{D_x})$ by taking $\omega = 0$.

The system of equations (73) can be represented as

$$\mathbf{E}(\mathbf{D_x})\mathbf{U}(\mathbf{x}) = \mathbf{0}. \tag{74}$$

Definition 3. Operator $\mathbf{E}(\mathbf{D_x})$ is said to be elliptic differential operator iff Eq. (37) is satisfied.

Definition 4. The fundamental solution of the system of equations (73) (the fundamental matrix of operator $\mathbf{E}$) is matrix $\mathbf{G}'(\mathbf{x}) = \Big(G'_{gl}(\mathbf{x})\Big)_{12\times 12}$ satisfying condition

$$\mathbf{E}(\mathbf{D_x})\mathbf{G}'(\mathbf{x}) = \delta(\mathbf{x})\,\mathbf{I}(\mathbf{x}). \tag{75}$$

We consider the system of non-homogeneous equations

$$[\mu\Delta + (\lambda_0 + \mu)\operatorname{grad}\operatorname{div}]\mathbf{u} - \gamma_3 \operatorname{grad}\phi = \mathbf{H}',$$
$$[k_6\Delta + (k_4 + k_5)\operatorname{grad}\operatorname{div} - k_2]\mathbf{v} +\\ +k_1 \operatorname{grad}\theta - \varrho \operatorname{grad}\phi = \mathbf{V}',$$
$$[h_6\Delta + (h_4 + h_5)\operatorname{grad}\operatorname{div} - h_2]\mathbf{w} +\\ +h_1 \operatorname{grad} P - w \operatorname{grad}\phi = \mathbf{W}', \tag{76}$$
$$-\gamma_1 \operatorname{div}\mathbf{u} - k_3 \operatorname{div}\mathbf{v} + k\,\Delta\,\theta + \beta\phi = Z',$$
$$-\gamma_2 \operatorname{div}\mathbf{u} - h_3 \operatorname{div}\mathbf{w} + h\,\Delta\,P + \alpha\phi = X',$$
$$\gamma_3 \operatorname{div}\mathbf{u} + (\gamma\Delta - \upsilon)\phi = Y',$$

where $\mathbf{H}', \mathbf{V}', \mathbf{W}'$ are three-component vector functions on E^3; Z', X' and Y' are scalar functions on E^3.

The system of equations (76) may also be written in the form

$$\mathbf{E}^{tr}(\mathbf{D_x})\mathbf{U}(\mathbf{x}) = \mathbf{Q}'(\mathbf{x}), \tag{77}$$

where $\mathbf{E}^{tr}$ is the transpose of matrix $\mathbf{E}$ and $\mathbf{Q}'(\mathbf{x}) = (\mathbf{H}', \mathbf{V}', \mathbf{W}', Z', X', Y')$.

Applying operator div to the Eqs. $(76)_{1-3}$, we obtain

$$\tilde{\lambda}\,\Delta \operatorname{div}\mathbf{u} - \gamma_3\Delta\,\phi = \operatorname{div}\mathbf{H}',\\ (k_7\Delta - k_2)\operatorname{div}\mathbf{v} + k_1\,\Delta\,\theta - \varrho\Delta\,\phi = \operatorname{div}\mathbf{V}',\\ (h_7\Delta - h_2)\operatorname{div}\mathbf{w} + h_1\,\Delta\,P - w\Delta\,\phi = \operatorname{div}\mathbf{W}'. \tag{78}$$

Multiplying Eq. $(76)_6$ by $-\gamma_3\Delta$ and Eq. $(78)_1$ by $\gamma\Delta - \upsilon$ and then subtracting, we get

$$\Delta(\Delta-\tau^2)\,\mathrm{div}\,\mathbf{u} = \Phi_1\,, \tag{79}$$

where

$$\tau^2 = \frac{1}{\gamma\,\tilde\lambda}(\tilde\lambda\,\upsilon - \gamma_3^2), \Phi_1 = \frac{1}{\gamma\,\tilde\lambda}[\gamma_3\,\Delta\,Y' + (\gamma\Delta-\upsilon)\,\mathrm{div}\,\mathbf{H}'].$$

Multiplying Eq. $(76)_6$ by $\tilde\lambda\Delta$ and Eq. $(78)_1$ by γ_3 and then subtracting, we get

$$\Delta(\Delta-\tau^2)\phi = \Phi_6, \quad \Phi_6 = \frac{1}{\gamma\,\tilde\lambda}[\tilde\lambda\,\Delta\,Y' - \gamma_3\,\mathrm{div}\,\mathbf{H}']. \tag{80}$$

Using Eq. $(76)_4$ in Eq. $(78)_2$ and then applying $\Delta(\Delta + -\tau^2)$, we get

$$\Delta(\Delta-\tau^2)(\Delta-D^2)\,\mathrm{div}\,\mathbf{v} = \Phi_2\,, \tag{81}$$

where

$$D^2 = \frac{1}{k\,k_7}(k\,k_2 - k_3k_1), \Phi_2 = \frac{1}{k\,k_7}[k\,\Delta(\varrho\Phi_6 + \\ +(\Delta-\tau^2)\,\mathrm{div}\,\mathbf{V}')] + \frac{k_1}{k\,k_7}[-\gamma_1\Phi_1 + \beta\Phi_6 - \Delta(\Delta-\tau^2)Z'].$$

Using Eq. $(76)_5$ in Eq. $(78)_3$ and then applying $\Delta(\Delta + -\tau^2)$, we get

$$\Delta(\Delta-\tau^2)(\Delta-L^2)\,\mathrm{div}\,\mathbf{w} = \Phi_3, \tag{82}$$

where

$$L^2 = \frac{1}{h\,h_7}(h\,h_2 - h_3h_1), \Phi_3 = \frac{1}{h\,h_7}[h\,\Delta(w\Phi_6 + \\ +(\Delta-\tau^2)\,\mathrm{div}\,\mathbf{W}')] + \frac{h_1}{h\,h_7}[-\gamma_2\Phi_1 + \alpha\Phi_6 - \Delta(\Delta-\tau^2)\,X'].$$

Applying operators $\Delta(\Delta-\tau^2)(\Delta-D^2)$ and $\Delta(\Delta-\tau^2)(\Delta + -L^2)$ to the equations $(76)_4$ and $(76)_5$, respectively, and using equations (81) and (82), we get

$$\Delta^2(\Delta-\tau^2)(\Delta-D^2)\,\theta = \Phi_4, \tag{83}$$
$$\Delta^2(\Delta-\tau^2)(\Delta-L^2)\,P = \Phi_5, \tag{84}$$

where

$$\begin{aligned}\Phi_4 &= \frac{1}{k}[k_3\,\Phi_2 + (\Delta-D^2)(\Delta(\Delta-\tau^2)\,Z' + \gamma_1\Phi_1 - \beta\Phi_6)],\\ \Phi_5 &= \frac{1}{h}[h_3\,\Phi_3 + (\Delta-L^2)(\Delta(\Delta-\tau^2)\,X' + \gamma_2\Phi_1 - \alpha\Phi_6)].\end{aligned} \tag{85}$$

Applying operators $\Delta(\Delta - \tau^2), \Delta^2(\Delta - \tau^2)(\Delta + -D^2), \Delta^2(\Delta-\tau^2)(\Delta-L^2)$ to equations $(76)_1$,$(76)_2$ and $(76)_3$, respectively, and using Eqs. (79)–(84), we obtain

$$\begin{aligned}&\Delta^2(\Delta-\tau^2)\,\mathbf{u} = \mathbf{\Phi}',\\ &\Delta^2(\Delta-\tau^2)(\Delta-D^2)\left(\Delta - \frac{k_2}{k_6}\right)\mathbf{v} = \mathbf{\Phi}'',\\ &\Delta^2(\Delta-\tau^2)(\Delta-L^2)\left(\Delta - \frac{h_2}{h_6}\right)\mathbf{w} = \mathbf{\Phi}''',\end{aligned} \tag{86}$$

where

$$\begin{aligned}\mathbf{\Phi}' &= \frac{1}{\mu}[\Delta(\Delta-\tau^2)\,\mathbf{H}' - (\lambda_0+\mu)\,\mathrm{grad}\,\Phi_1 + \gamma_3\,\mathrm{grad}\,\Phi_6],\\ \mathbf{\Phi}'' &= \frac{1}{k_6}[\Delta^2(\Delta-\tau^2)(\Delta-D^2)\mathbf{V}' - (k_4+k_5)\Delta\,\mathrm{grad}\,\Phi_2 + \\ &\quad -k_1\,\mathrm{grad}\,\Phi_4 + \varrho\,\Delta(\Delta-D^2)\,\mathrm{grad}\,\Phi_6],\\ \mathbf{\Phi}''' &= \frac{1}{h_6}[\Delta^2(\Delta-\tau^2)(\Delta-L^2)\mathbf{W}' - (h_4+h_5)\Delta\,\mathrm{grad}\,\Phi_3 + \\ &\quad -h_1\,\mathrm{grad}\,\Phi_5 + w\,\Delta(\Delta-L^2)\,\mathrm{grad}\,\Phi_6].\end{aligned} \tag{87}$$

From Eqs. (80), (83), (84) and (86), we get

$$\mathbf{\Lambda}(\Delta)\,\mathbf{U}(\mathbf{x}) = \hat{\mathbf{\Phi}}(\mathbf{x}), \tag{88}$$

where

$$\begin{gathered}\hat{\mathbf{\Phi}}(\mathbf{x}) = (\mathbf{\Phi}', \mathbf{\Phi}'', \mathbf{\Phi}''', \Phi_4, \Phi_5, \Phi_6), \mathbf{\Lambda}(\Delta) = \Big(\Lambda_{pq}(\Delta)\Big)_{12\times 12},\\ \Lambda_{ii}(\Delta) = \Delta^2(\Delta-\tau^2), \Lambda_{i+3;i+3}(\Delta) = \Delta^2(\Delta-\tau^2)(\Delta-D^2)\left(\Delta - \frac{k_2}{k_6}\right),\\ \Lambda_{i+6;i+6}(\Delta) = \Delta^2(\Delta-\tau^2)(\Delta-L^2)\left(\Delta - \frac{h_2}{h_6}\right), \Lambda_{10;10} = \Delta^2(\Delta-\tau^2)(\Delta-D^2),\\ \Lambda_{11;11} = \Delta^2(\Delta-\tau^2)(\Delta-L^2), \Lambda_{12;12} = \Delta^2(\Delta-\tau^2),\\ \Lambda_{lj} = 0;\ i,j = 1,2,3;\ l,j = 1,\dots,12;\ l \neq j.\end{gathered}$$

Eqs. (80), (85) and (87) can be rewritten as

$$\hat{\mathbf{\Phi}}(\mathbf{x}) = \mathbf{T}^{tr}(\mathbf{D}_\mathbf{x})\mathbf{Q}'(\mathbf{x}), \tag{89}$$

where

$$\mathbf{T}(\mathbf{D_x}) = \Big(T_{gl}(\mathbf{D_x})\Big)_{12\times 12},$$

$$T_{ij}(\mathbf{D_x}) = \frac{1}{\mu}\,\Delta(\Delta-\tau^2)\,\delta_{ij} + m_{11}(\Delta)\frac{\partial^2}{\partial x_i \partial x_j},$$

$$T_{i+3;j+3}(\mathbf{D_x}) = \frac{1}{k_6}\,\Delta^2(\Delta-\tau^2)(\Delta-D^2)\,\delta_{ij} + m_{22}(\Delta)\frac{\partial^2}{\partial x_i \partial x_j},$$

$$T_{i+6;j+6}(\mathbf{D_x}) = \frac{1}{h_6}\,\Delta^2(\Delta-\tau^2)(\Delta-L^2)\,\delta_{ij} + m_{22}(\Delta)\frac{\partial^2}{\partial x_i \partial x_j},$$

$$T_{i+9;i+9}(\mathbf{D_x}) = m_{i+3;i+3}(\Delta), T_{i;j+3}(\mathbf{D_x}) = m_{12}(\Delta)\frac{\partial^2}{\partial x_i \partial x_j},$$

$$T_{i;j+6}(\mathbf{D_x}) = m_{13}(\Delta)\frac{\partial^2}{\partial x_i \partial x_j}, T_{i;j+9}(\mathbf{D_x}) = m_{1;j+3}(\Delta)\frac{\partial}{\partial x_i},$$

$$T_{i+3;j}(\mathbf{D_x}) = T_{i+3;j+6}(\mathbf{D_x}) = T_{i+3;11}(\mathbf{D_x}) = T_{i+3;12}(\mathbf{D_x}) = 0,$$

$$T_{i+3;10}(\mathbf{D_x}) = m_{24}(\Delta)\frac{\partial}{\partial x_i}, T_{i+6;j}(\mathbf{D_x}) = T_{i+6;j+3}(\mathbf{D_x}) = 0,$$

$$T_{i+6;10}(\mathbf{D_x}) = T_{i+6;12}(\mathbf{D_x}) = 0, T_{i+6;11}(\mathbf{D_x}) = m_{35}(\Delta)\frac{\partial}{\partial x_i},$$

$$T_{10;i}(\mathbf{D_x}) = T_{10;i+6}(\mathbf{D_x}) = T_{10;11}(\mathbf{D_x}) = T_{10;12}(\mathbf{D_x}) = 0,$$

$$T_{10;i+3}(\mathbf{D_x}) = m_{42}(\Delta)\frac{\partial}{\partial x_i}, T_{11;i}(\mathbf{D_x}) = T_{11;i+3}(\mathbf{D_x}) = 0,$$

$$T_{11;10}(\mathbf{D_x}) = T_{11;12}(\mathbf{D_x}) = 0, T_{11;i+6}(\mathbf{D_x}) = m_{53}(\Delta)\frac{\partial}{\partial x_i},$$

$$T_{12;i}(\mathbf{D_x}) = m_{61}(\Delta)\frac{\partial}{\partial x_i}, T_{12;i+3}(\mathbf{D_x}) = m_{62}(\Delta)\frac{\partial}{\partial x_i}, T_{12;i+6}(\mathbf{D_x}) = m_{63}(\Delta)\frac{\partial}{\partial x_i},$$

$$T_{12;10}(\mathbf{D_x}) = m_{64}(\Delta), T_{12;11}(\mathbf{D_x}) = m_{65}(\Delta),$$

$$m_{11}(\Delta) = -\frac{(\lambda_0+\mu)(\gamma\Delta-\upsilon)+\gamma_3^2}{\gamma\mu\tilde{\lambda}}, m_{22}(\Delta) = -\frac{\Delta(\Delta-\tau^2)[k(k_4+k_5)\Delta+k_1k_3]}{k\,k_6\,k_7},$$

$$m_{33}(\Delta) = -\frac{\Delta(\Delta-\tau^2)[h(h_4+h_5)\Delta+h_1h_3]}{h\,h_6\,h_7}, m_{44}(\Delta) = \frac{\Delta(\Delta-\tau^2)(k_7\Delta-k_2)}{k\,k_7},$$

$$m_{55}(\Delta) = \frac{\Delta(\Delta-\tau^2)(h_7\Delta-h_2)}{h\,h_7}, m_{66}(\Delta) = \frac{\Delta}{\gamma},$$

$$m_{12}(\Delta) = -\frac{(\Delta-\frac{k_2}{k_6})[k_1\gamma_1(\gamma\Delta-\upsilon)+\gamma_3(k\varrho\,\Delta+k_1\beta)]}{\gamma\,\tilde{\lambda}\,k\,k_7},$$

$$m_{13}(\Delta) = -\frac{(\Delta-\frac{h_2}{h_6})[h_1\gamma_2(\gamma\Delta-\upsilon)+\gamma_3(h\,w\,\Delta+h_1\alpha)]}{\gamma\,\tilde{\lambda}\,h\,h_7},$$

$$m_{14}(\Delta) = \frac{\gamma_1(k_7\,\Delta-k_2)(\gamma\Delta-\upsilon)-\gamma_3\Delta(\varrho\,k_3-\beta\,k_7)-\gamma_3\beta\,k_2}{\gamma\,\tilde{\lambda}\,k\,k_7},$$

$$m_{15}(\Delta) = \frac{\gamma_2(h_7\,\Delta-h_2)(\gamma\Delta-\upsilon)-\gamma_3\Delta(w\,h_3-\alpha\,h_7)-\gamma_3\alpha\,h_2}{\gamma\,\tilde{\lambda}\,h\,h_7},$$

$$m_{16}(\Delta) = -\frac{\gamma_3}{\gamma\,\tilde{\lambda}}, m_{24}(\Delta) = \frac{k_3\,\Delta(\Delta-\tau^2)}{k\,k_7}, m_{35}(\Delta) = \frac{h_3\,\Delta(\Delta-\tau^2)}{h\,h_7},$$

$$m_{42}(\Delta) = -\frac{k_1\,\Delta(\Delta-\tau^2)(\Delta-\frac{k_2}{k_6})}{k\,k_7}, m_{53}(\Delta) = -\frac{h_1\,\Delta(\Delta-\tau^2)(\Delta-\frac{h_2}{h_6})}{h\,h_7},$$

$$m_{61}(\Delta) = \frac{\gamma_3\Delta}{\gamma\,\tilde{\lambda}}, m_{62}(\Delta) = \frac{\Delta(\Delta-\frac{k_2}{k_6})[\tilde{\lambda}(\varrho\,k\,\Delta+\beta\,k_1)-k_1\gamma_1\gamma_3]}{\gamma\,\tilde{\lambda}\,k\,k_7},$$

$$m_{63}(\Delta)=\frac{\Delta(\Delta-\frac{h_2}{h_6})[\tilde{\lambda}(w\,h\,\Delta+\alpha\,h_1)-h_1\gamma_2\gamma_3]}{\gamma\,\tilde{\lambda}\,h\,h_7},$$
$$m_{64}(\Delta)=\frac{\Delta[\gamma_1\gamma_3(k_7\,\Delta-k_2)+\tilde{\lambda}[\Delta(\varrho\,k_3-\beta\,k_7)+\beta\,k_2]]}{\gamma\,\tilde{\lambda}\,k\,k_7}, \tag{90}$$
$$m_{65}(\Delta)=\frac{\Delta[\gamma_2\gamma_3(h_7\,\Delta-h_2)+\tilde{\lambda}[\Delta(w\,h_3-\alpha\,h_7)+\alpha\,h_2]]}{\gamma\,\tilde{\lambda}\,h\,h_7};\ i,j=1,2,3.$$

From Eqs. (77), (88) and (89), we get

$$\mathbf{E}(\mathbf{D_x})\mathbf{T}(\mathbf{D_x})=\mathbf{\Lambda}(\Delta). \tag{91}$$

Let

$$\mathbf{Y}'(\mathbf{x})=\Big(Y'_{ij}(\mathbf{x})\Big)_{12\times12},\ Y'_{pp}(\mathbf{x})=r'_{11}\varsigma_2^*(\mathbf{x})+r'_{12}\varsigma_1^*(\mathbf{x})+r'_{13}\varsigma_3^*(\mathbf{x}),$$
$$Y'_{p+3;p+3}(\mathbf{x})=r'_{21}\varsigma_2^*(\mathbf{x})+r'_{22}\varsigma_1^*(\mathbf{x})+r'_{23}\varsigma_3^*(\mathbf{x})+r'_{24}\varsigma_4^*(\mathbf{x})+r'_{26}\varsigma_6^*(\mathbf{x}),$$
$$Y'_{p+6;p+6}(\mathbf{x})=r'_{31}\varsigma_2^*(\mathbf{x})+r'_{32}\varsigma_1^*(\mathbf{x})+r'_{33}\varsigma_3^*(\mathbf{x})+r'_{35}\varsigma_5^*(\mathbf{x})+r'_{37}\varsigma_7^*(\mathbf{x}),$$
$$Y'_{10;10}(\mathbf{x})=r'_{41}\varsigma_2^*(\mathbf{x})+r'_{42}\varsigma_1^*(\mathbf{x})+r'_{43}\varsigma_3^*(\mathbf{x})+r'_{44}\varsigma_4^*(\mathbf{x}),$$
$$Y'_{11;11}(\mathbf{x})=r'_{51}\varsigma_2^*(\mathbf{x})+r'_{52}\varsigma_1^*(\mathbf{x})+r'_{53}\varsigma_3^*(\mathbf{x})+r'_{55}\varsigma_5^*(\mathbf{x}),$$
$$Y'_{12;12}(\mathbf{x})=r'_{62}\varsigma_1^*(\mathbf{x})+r'_{63}\varsigma_3^*(\mathbf{x}),$$
$$Y'_{qz}(\mathbf{x})=0;\ p=1,2,3;\ q,z=1,\dots,12;\ q\neq z,$$

where

$$\varsigma_3^*(\mathbf{x})=-\frac{e^{-\tau|\mathbf{x}|}}{4\pi|\mathbf{x}|},\varsigma_4^*(\mathbf{x})=-\frac{e^{-D|\mathbf{x}|}}{4\pi|\mathbf{x}|},\varsigma_5^*(\mathbf{x})=-\frac{e^{-L|\mathbf{x}|}}{4\pi|\mathbf{x}|},$$
$$\varsigma_6^*(\mathbf{x})=-\frac{e^{-\tau_1|\mathbf{x}|}}{4\pi|\mathbf{x}|},\varsigma_7^*(\mathbf{x})=-\frac{e^{-\tau_2|\mathbf{x}|}}{4\pi|\mathbf{x}|},$$
$$r'_{11}=-\frac{1}{\tau^2},r'_{12}=-r'_{13}=-\frac{1}{\tau^4},$$
$$r'_{21}=-(\tau\,D\,\tau_1)^{-2},r'_{22}=-(\tau^2D^2+D^2\tau_1^2+\tau^2\tau_1^2)(\tau\,D\,\tau_1)^{-4},$$
$$r'_{23}=\frac{1}{\tau^4(\tau^2-D^2)(\tau^2-\tau_1^2)},r'_{24}=\frac{1}{D^4(D^2-\tau^2)(D^2-\tau_1^2)},$$
$$r'_{26}=\frac{1}{\tau_1^4(\tau_1^2-\tau^2)(\tau_1^2-D^2)},r'_{31}=-(\tau\,L\,\tau_2)^{-2}, \tag{92}$$
$$r'_{32}=-(\tau^2L^2+L^2\tau_2^2+\tau^2\tau_2^2)(\tau\,L\,\tau_2)^{-4},r'_{33}=\frac{1}{\tau^4(\tau^2-L^2)(\tau^2-\tau_2^2)},$$
$$r'_{35}=\frac{1}{L^4(L^2-\tau^2)(L^2-\tau_2^2)},r'_{37}=\frac{1}{\tau_2^4(\tau_2^2-\tau^2)(\tau_2^2-L^2)},$$
$$r'_{41}=\frac{1}{\tau^2D^2},r'_{42}=\frac{\tau^2+D^2}{\tau^4D^4},r'_{43}=\frac{1}{\tau^4(\tau^2-D^2)},r'_{44}=\frac{1}{D^4(D^2-\tau^2)},$$
$$r'_{51}=\frac{1}{\tau^2L^2},r'_{52}=\frac{\tau^2+L^2}{\tau^4L^4},r'_{53}=\frac{1}{\tau^4(\tau^2-L^2)},r'_{55}=\frac{1}{L^4(L^2-\tau^2)},$$
$$r'_{62}=-r'_{63}=-\frac{1}{\tau^4},\tau_1^2=\frac{k_2}{k_6},\tau_2^2=\frac{h_2}{h_6}\ .$$

Lemma 2. Matrix $\mathbf{Y}'$ defined above is the fundamental matrix of operator $\mathbf{\Lambda}(\Delta)$, i.e.

$$\mathbf{\Lambda}(\Delta)\mathbf{Y}'(\mathbf{x}) = \delta(\mathbf{x})\,\mathbf{I}(\mathbf{x}). \tag{93}$$

Proof: To prove the lemma, it is sufficient to prove that

$$\begin{gathered}\Delta^2(\Delta-\tau^2)Y'_{11}(\mathbf{x}) = \delta(\mathbf{x}),\\ \Delta^2(\Delta-\tau^2)(\Delta-D^2)(\Delta-\tau_1^2)Y'_{44}(\mathbf{x}) = \delta(\mathbf{x}),\\ \Delta^2(\Delta-\tau^2)(\Delta-L^2)(\Delta-\tau_2^2)Y'_{77}(\mathbf{x}) = \delta(\mathbf{x}),\\ \Delta^2(\Delta-\tau^2)(\Delta-D^2)Y'_{10;10}(\mathbf{x}) = \delta(\mathbf{x}),\\ \Delta^2(\Delta-\tau^2)(\Delta-L^2)Y'_{11;11}(\mathbf{x}) = \delta(\mathbf{x}),\\ \Delta^2(\Delta-\tau^2)Y'_{12;12}(\mathbf{x}) = \delta(\mathbf{x}).\end{gathered} \tag{94}$$

It is much easier to prove Eqs. (94). It has been left for the reader.

We introduce the matrix

$$\mathbf{G}'(\mathbf{x}) = \mathbf{T}(\mathbf{D_x})\mathbf{Y}'(\mathbf{x}). \tag{95}$$

From Eqs. (91), (93) and (95), we obtain

$$\mathbf{E}(\mathbf{D_x})\mathbf{G}'(\mathbf{x}) = \delta(\mathbf{x})\mathbf{I}(\mathbf{x}).$$

Hence $\mathbf{G}'(\mathbf{x})$ is a solution to Eq. (75).

Theorem 4. If condition (37) is satisfied, then matrix $\mathbf{G}'(\mathbf{x})$ defined by Eq. (95) is the fundamental solution of the system of equations (73) and matrix $\mathbf{G}'(\mathbf{x})$ is represented in the following form:

$$\begin{gathered}G'_{gl}(\mathbf{x}) = T_{gl}(\mathbf{D_x})Y'_{11}(\mathbf{x}), G'_{g;l+3}(\mathbf{x}) = T_{g;l+3}(\mathbf{D_x})Y'_{44}(\mathbf{x}),\\ G'_{g;l+6}(\mathbf{x}) = T_{g;l+6}(\mathbf{D_x})Y'_{77}(\mathbf{x}), G'_{gj}(\mathbf{x}) = T_{gj}(\mathbf{D_x})Y'_{jj}(\mathbf{x}),\\ g = 1,\dots,11;\ l = 1,2,3;\ j = 10,11,12.\end{gathered} \tag{96}$$

VII. Conclusions

The fundamental solution of a system of equations in the theory of micromorphic thermoelastic diffusion materials with microtemperatures and microconcentrations in case of steady oscillations in terms of elementary functions has been constructed. Using the potential method, the fundamental solution of the system of equations makes it possible to investigate three-dimensional boundary value problems of the theory of micromorphic thermoelastic diffusion materials with microtemperatures and microconcentrations. Also some basic properties of the fundamental matrix are discussed.

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