CMST 23(4) 317–329 (2017) DOI:10.12921/cmst.2016.0000059

Thermo-Mechanical Responses of an Annular Cylinder with Temperature Dependent Material Properties under Thermoelasticity without Energy Dissipation

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Received: 01 December 2016; revised: 06 September 2017; accepted: 07 September 2017; published online: 31 December 2017

Abstract: The present work is concerned with thermoelasticity without the energy dissipation theory for a problem of an infinitely long and isotropic annular cylinder of temperature dependent physical properties. We employ the thermoelasticity theory of GN-II and derive the basic governing equations with variable material properties. The formulation is then applied to solve a boundary value problem of an annular cylinder with its inner boundary assuming to be stress free and subjected to exponential decay in temperature and sinusoidal temperature distribution. The outer boundary is also assumed to be stress free and is maintained at reference temperature in both cases. We solve the non-linear coupled differential equations by applying the finite difference approach efficiently. We analyze the numerical results in a detailed way with the help of different graphs. The effects of temperature dependency of material properties on the thermo-mechanical responses for two different time dependent temperature distributions applied at the inner boundary are highlighted.

Key words: annular cylinder, temperature dependent materials, thermoelasticity without energy dissipation, finite difference method

I. INTRODUCTION

Structural elements are frequently subjected to non-uniform heating along with mechanical loads. A non-uniform heating of elements gives rise to thermal stresses that play a significant role in the analysis of complete strength of materials. Hence, determination of thermal stresses in combination with mechanical stress and temperature fields are of practical importance in the analysis of such type of problems. The coupled theory of thermoelasticity takes into account of the fact that the changes in temperature of a deformable body affect the state of strain and stress of the body. Conversely, any mechanical load and corresponding stress causes a change in temperature in the body. The thermoelasticity theory deals with the mutual interactions between temperature and strain fields in an elastic medium. Therefore, thermoelasticity may be regarded as a fusion of the two independently developed theories: the "theory of heat conduction" and the "theory of elasticity". An extensive research work carried out in the field of thermoelasticity has established that it has various applications to the problems in engineering science and technology. The field of thermoelasticity was first stimulated through a work by Biot [1]. Biot derived the constitutive relations and equations of coupled thermoelasticity by using Duhamel-Neumann relations [2-4] to consider the coupling between strain and temperature fields based on the firm grounds of irreversible thermodynamics. However, this theory of thermoelasticity is derived by employing Fourier law of heat conduction. The heat conduction equation derived for this theory is a parabolic type partial differential equation that describes the fact that this theory involves the wave type equations of motion and diffusion type equation of heat conduction. This indicates that if an elastic medium is subjected to a thermal or mechanical disturbances, the effects in both the temperature and displacement fields are felt instantaneously at an infinite distance far from the source of disturbances. Therefore, Biot's theory removes the drawback of uncoupled theory of thermoelasticity, but it suffers from the paradox of infinite speed for a thermal signal. Moreover, besides this paradox of infinite propagation speed, this theory also shows an unsatisfactory or poor description of a solid's response to fast transient loading, like short laser pulses, and at low temperatures (see Chandrasekharaiah [5], Ignaczak and Ostoja-Starzewski [6]). Such drawbacks in the classical coupled thermoelasticity theory have drawn the attention of researchers in recent years to step out to modify the concept of this theory. The major objective of this is to minimize the shortcomings which are inherent in the classical thermoelasticity theory of Biot [1].Accordingly, various generalized theories are designated to account for the finite speed of propagation of thermal disturbance. Firstly, we would like to recall the theories developed by Lord and Shulman [7] and Green and Lindsay [8]. Two thermal relaxation time parameters are introduced in the theory of Green and Lindsay, where as one relaxation time parameter is introduced in the theory proposed by Lord and Shulman [7]. It is worth being further mentioned in this context that Hetnarski& Ignaczak [9] introduced another model called a low temperature model. In this model, the heat flux and free energy depend on temperature, strain tensor and the heat flux. Subsequently, this model is explained by the system of non linear field equations.

Later on, Green and Naghdi [10-12] proposed a basic fundamental theory of thermoelasticity in an alternative way. Their study reported three categories of models which are subsequently known as thermoelastic models of type GN-I, GN-II and GN-III. The first two models are the special cases of model-III. The specialty in these theories is that the temperature gradient and thermal displacement gradient are taken to be among the constitutive variables. The linearized version of model-I is closely related to the classical thermoelastic model. In GN-II model, there is no dissipation of thermal energy which is caused by no change in internal energy. This model admits undamped thermoelastic waves in a thermoelastic material and is known as the theory of thermoelasticity without energy dissipation (TEWOED). The thermoelasticity theories developed by Green and Naghdi have drawn the attention of several researchers during recent years. Quintanilla and Straughan [13] and Quintanilla [14] have proved the uniqueness theorem and discussed the growth of solutions in the contexts of type II and III theories. Further, Quintanilla [15] proved the impossibility of the localization in time of the solutions of linear thermoelasticity for the theories of Green and Naghdi. Recently, the variational and reciprocity theorems in the contexts of linear theory of thermoelasticity of type II and type III were established by Chirita and Ciarletta [16] and Mukhopadhyay and Prasad [17], respectively.

While studying the problems of thermoelasticity, the material properties of the medium are in general considered to be constant. However, the structural elements are often subjected to thermal loads due to ultra-high temperature, ultra-high temperature gradient, cyclical changes of ultra-high temperature, etc. as reported by Noda [18, 19]. The material parameters in these circumstances remain no longer constant and they depend on temperature. Hence, in order to perform a more accurate analysis of thermoelastic behavior of the structural elements, temperature dependency of material properties needs to be considered. It is to be noted that Suhara [20] studied a thermoelastic problem of hollow cylinder by considering temperature dependent shearing modulus. Subsequently, several investigations have been reported on the thermal stress analysis in elastic and inelastic materials with temperature dependent properties. The survey/review articles by Noda [18, 19] and the references therein may be recalled in this context. We also recall some investigations on thermoelastic deformation of several basic structures like disk, cylinders, tubes etc. with temperature dependent properties as reported by Ezzat *et al.* [21], Othman [22], Ezzat and Othman [23], Othman [24], Eraslan and Kartal [25], Ezzat *et al*. [26], Youssef and Abbas [27], Argeso and Eraslan [28], Mukhopadhyay and Kumar [29], Othman *et al.* [30],Othman and Hilal [31], Kalkal and Deswal [32], Wang *et al.* [33], Abbas [34] etc. In these studies the temperature dependent properties of the medium have been considered. Recently, Abbas and Youssef [35], Zenkour and Abbas [36, 37], He and Shi [38] employed the generalized thermoelasticity theory by Lord and Shulman [7] to investigate the effects of temperature dependent material properties on the numerical solution of thermoelastic problems obtained by the finite element method. Salam *et al.* [39] solved a problem on magneto-thermoelasticity for non-homogeneous cylinder by the finite difference method and discussed the effects of non-homogeneity. Subsequently, Mukhopadhyay and Kumar [29] investigated the effects of temperature dependent material properties on thermoelastic interactions in the context of Lord-Shulman model by applying the finite difference method.

The main objective of the present work is to investigate a problem of an infinitely long annular cylinder, whose material properties like modulus of elasticity and thermal conductivity vary with temperature in the context of the thermoelasticity theory without energy dissipation (TEWOED). By considering temperature dependency of material properties, we formulate the governing equations under the TEWOED theory as introduced by Green and Naghdi [12]. The governing equations in this case are derived as coupled nonlinear partial differential equations because of varying material parameters. The outer boundary of the annulus is assumed to be stress free and is maintained at reference temperature, while the inner surface is subjected to two different types of variation in temperature together with zero stress. Using the finite difference method, the governing equations are transformed into a system of coupled difference equations and the numerical solution of the problem is obtained. The values of the field variables inside the annulus for copper material are simulated directly in the space-time domain. Results are displayed graphically and compared with the results obtained for temperature-independent material properties. A thorough comparison between the results predicted by present model with corresponding results under thermoelasticity with one thermal relaxation parameter reported by Mukhopadhyay and Kumar [29] is presented. This study brings to light several points highlighting the effects of temperature dependency of material properties under thermoelasticity without energy dissipation theory that accounts for the finite speed for the thermal disturbance.

II. BASIC GOVERNING EQUATIONS

We consider an isotropic, homogeneous, linear and thermally conducting elastic medium with temperature dependent mechanical properties. The governing equations for the thermoelastic model without dissipation of energy by Green Naghdi [12] in the absence of external body forces and heat sources can therefore be written as follows:

Stress-strain temperature relation:

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e - \gamma T)\delta_{ij}. \tag{1}$$

Strain-displacement relations:

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{2}$$

Equation of motion in the absence of body forces:

$$\sigma_{ij,j} = \rho\ddot{u}_i. \tag{3}$$

Heat conduction equation in the absence of heat sources:

$$\left(K^*\bar{T}_{,i}\right)_{,i} = K\eta\frac{\partial^2 T}{\partial t^2} + \gamma T_0\frac{\partial^2 e}{\partial t^2}, \tag{4}$$

where u_i are the components of displacement vector, e_{ij} are the components of elastic strain tensor, σ_{ij} are the component of stress tensor, $e = e_{ii}$ is the dilatation, t is the time, T is the temperature variation above the uniform reference temperature, T_0. λ and μ are the Lamé's constants, ρ is the mass density, $\gamma = (3\lambda + 2\mu)\alpha_t$, where α_t is the coefficient of linear thermal expansion, K, K^* are the thermal conductivity, conductivity rate respectively. η is the thermal diffusivity, where $\eta = \frac{\rho c_E}{K}$ and c_E is the specific heat at constant strain.

Our goal is to investigate the effects of temperature dependent material properties on thermoelastic behavior.

Therefore, we assume that $\lambda = \lambda_0 f(T)$, $\mu = \mu_0 f(T)$, $K = K_0 f(T)$, $K^* = K_0 f(T)$, $\gamma = \gamma_0 f(T)$.

where λ_0, μ_0, K_0 and γ_0 are considered to be constant material properties at reference temperature, and $f(T)$ is a given function of temperature. It is to be noted that the case of temperature-independent material property corresponds to the case when $f(T) = 1$, i.e., $\lambda = \lambda_0$, $\mu = \mu_0$, $K = K_0$, $\gamma = \gamma_0$. In general, different material properties vary in different manner with the increase of temperature. For example, Young's modulus, shearing modulus, density, thermal conductivity, etc. usually decrease with the rising of temperature, while the coefficient of Poisson ratio, and linear thermal expansion usually increase with the increase of temperature. However, the dependency of some properties like Poisson ratio is lower than that of other material properties (Noda [18]). Hence, for simplicity of the present problem it is assumed that λ, μ, K and γ vary as per the above law and the other properties are assumed to be independent of temperature for our present analysis.

In view of the above assumptions, equations (1), (3) and (4) yields

$$\sigma_{ij} = \left[2\mu_0 e_{ij} + \left(\lambda_0 e - \gamma_0 T_0\right)\delta_{ij}\right] f\left(T\right), \tag{5}$$

$$\begin{aligned}\rho\ddot{u}_i = {} & \left[2\mu_0 e_{ij} + \left(\lambda_0 e - \gamma_0 T_0\right)\delta_{ij}\right]_{,j} f\left(T\right) \\ & + \left(f\left(T\right)\right)_{,j}\left[2\mu_0 e_{ij} + \left(\lambda_0 e - \gamma_0 T_0\right)\delta_{ij}\right],\end{aligned} \tag{6}$$

$$\left[K_0^* f\left(T\right) T_{,i}\right]_{,i} = K_0\eta f\left(T\right)\frac{\partial^2 T}{\partial t^2} + \gamma_0 T_0 f\left(T\right)\frac{\partial^2 e}{\partial t^2}. \tag{7}$$

III. PROBLEM FORMULATION

We consider an infinitely long annular cylinder of isotropic elastic material. It is assumed that the material properties, except density and specific heat, of the cylinder are temperature dependent. (r, ϕ, z) are taken as cylindrical polar coordinates with the origin at the center of the system and z-axis is taken to be along the axis of the cylinder. We consider axi-symmetric plane strain problem and the physical quantities are assumed to be the functions of radial coordinate r and time t. Since modulus of rigidity and many other properties decrease monotonically with the rise of temperature (see Rishin *et al.* [40]), we assume $f(T) = e^{-\alpha T}$. However, for simplicity and without any loss of generality we approximate the function $f(T)$ as $f(T) = 1 - \alpha T$, where α is an empirical material parameter of the dimension K^{-1} (Noda [18]).

Therefore, with the help of (5) we get the non-zero stress components as

$$\sigma_{rr} = \left[\left(\lambda_0 + 2\mu_0\right)\frac{\partial u}{\partial r} + \lambda_0\frac{u}{r} - \gamma_0 T\right]\left(1 - \alpha T\right) \tag{8}$$

$$\sigma_{\phi\phi}=\left[(\lambda_0+2\mu_0)\frac{u}{r}+\lambda_0\frac{\partial u}{\partial r}-\gamma_0 T\right](1-\alpha T) \tag{9}$$

We use the equation (3) to get the equation of motion in the cylindrical co-ordinates as

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}(\sigma_{rr}-\sigma_{\phi\phi})=\rho\frac{\partial^2 u}{\partial t^2} \tag{10}$$

By using (8)-(10), we obtain

$$\begin{aligned}&(\lambda_0+2\mu_0)\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right](1-\alpha T)\\&-\left[\alpha\left\{(\lambda_0+2\mu_0)\frac{\partial u}{\partial r}+\lambda_0\frac{u}{r}-\gamma_0 T\right\}\right.\\&\left.+\gamma_0(1-\alpha T)\right]\frac{\partial T}{\partial r}=\rho\frac{\partial^2 u}{\partial t^2}\end{aligned} \tag{11}$$

Equation (7) then yields

$$\begin{aligned}&\frac{K_0^*}{K_0}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)-\frac{K_0^*}{K_0}\frac{\alpha}{(1-\alpha T)}\left(\frac{\partial T}{\partial r}\right)^2\\&=\eta\frac{\partial^2 T}{\partial t^2}+\frac{\gamma_0 T_0}{K_0}\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\end{aligned} \tag{12}$$

For our convenience, we now introduce the following non dimensional variables and notations:

$$\begin{aligned}r'&=c_0\eta r,\quad u'=c_0\eta r,\\t'&=c_0^2\eta t,\quad T'=\frac{T-T_0}{T_0},\\\sigma'_{ij}&=\frac{\sigma_{ij}}{(\lambda_0+2\mu_0)},\\\lambda_1&=\frac{\lambda_0}{(\lambda_0+2\mu_0)},\\c_0^2&=\frac{(\lambda_0+2\mu_0)}{\rho},\\a_0&=\frac{K_0^*}{K_0c_0^2\eta},\\a_1&=\frac{\gamma_0T_0}{(\lambda_0+2\mu_0)},\\a_2&=\frac{\gamma_0}{K_0\eta},\quad\beta=\alpha T_0.\end{aligned}$$

Therefore, the dimensionless forms of equations (8)-(12) are obtained as follows (after dropping the primes for convenience):

$$\begin{aligned}&\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right]\{1-\beta(T+1)\}\\&-\left[a_1\{1-2\beta(T+1)\}+\beta\left(\frac{\partial u}{\partial r}+\lambda_1\frac{u}{r}\right)\right]=\frac{\partial^2 u}{\partial t^2}\end{aligned} \tag{13}$$

$$\begin{aligned}&a_0\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)-\frac{\beta a_0}{\{1-\beta(T+1)\}}\left(\frac{\partial T}{\partial r}\right)^2=\\&=\frac{\partial^2 T}{\partial t^2}+a_2\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)\end{aligned} \tag{14}$$

$$\sigma_{rr}=\{1-\beta(T+1)\}\left[\frac{\partial u}{\partial r}+\lambda_1\frac{u}{r}-a_1T\right] \tag{15}$$

$$\sigma_{\phi\phi}=\{1-\beta(T+1)\}\left[\lambda_1\frac{\partial u}{\partial r}+\frac{u}{r}-a_1T\right] \tag{16}$$

III. 1. Initial and Boundary Conditions

We assume that initially the annulus has no deformation and have the reference temperature T_0 and also has the zero rate of change of temperature. Therefore, initial conditions are expressed to be homogeneous. i.e, we have

$$u(r,0)=\frac{\partial u(r,0)}{\partial t}=0,\ T(r,0)=\frac{\partial T(r,0)}{\partial t}=0,\quad a\le r\le b, \tag{17}$$

where a and b are the dimensionless inner and outer radii of the cylinder.

It is assumed that both the inner and outer curved surfaces of the annulus are stress free and the inner surface is subjected to a temperature which is varying as $f(t)$ with time t, whereas the outer surface is maintained at the reference temperature. The boundary conditions are therefore taken to be as follows:

$$\sigma_{rr}=0,\quad T=f(t)\quad \text{at } r=a,\ t>0 \tag{18}$$

$$\sigma_{rr}=0,\quad T=0\quad \text{at } r=b,\ t>0 \tag{19}$$

IV. SOLUTION OF THE PROBLEM (NUMERICAL SCHEME)

The governing equations obtained in the last section are non linear partial differential equations. For the solution of the problem we therefore use the finite difference method. We assume that the solution domain $a\le r\le b$, $0\le t\le t_0$ is replaced by a grid described by the set of node points (r_m,t_n) , in which $r_m=a+mh$, $m=0,1,...,N$ and $t_n=nk$; $n=0,1,...,P$. Therefore, $h=\frac{(b-a)}{N}$ is taken as mess width and $k=\frac{t_0}{P}$ is assumed to be the time-step. Here, we assume that t_0 is the final value of time. In the following equations we use the notation u_m^n in place of $u(r_m,t_n)$, $m=0,1,\dots,N$ and $n=0,1,\dots,P$. The finite difference approximations for the partial differential coefficients with respect to the independent variables r and t are obtained as follows (see Ref. [29]):

$$
\begin{aligned}
\frac{\partial y}{\partial r} &= \frac{y_{m+1}^{n} - y_{m-1}^{n}}{2h} + o(h^2), \\
\frac{\partial^2 y}{\partial r^2} &= \frac{y_{m+1}^{n} - 2y_{m}^{n} + y_{m-1}^{n}}{h^2} + o(h^2), \\
\frac{\partial y}{\partial t} &= \frac{y_{m}^{n+1} - y_{m}^{n-1}}{2k} + o(k^2)
\end{aligned}
\tag{20}
$$

In view of equation (20), after detailed manipulations, the equations (13) and (14) are then replaced by the explicit forms of finite difference equations as follows:

$$
\begin{aligned}
u_m^{n+1} \;=\;& 2u_m^n - u_m^{n-1} + \upsilon\left\{1-\beta\left(T_m^n+1\right)\right\}\left[\left(u_{m+1}^n - 2u_m^n + u_{m-1}^n\right) + \frac{h}{2r_m}\left(u_{m+1}^n - u_{m-1}^n\right) - \frac{h^2}{r_m^2}u_m^n\right] \\
&- \frac{\upsilon}{4}\left(T_{m+1}^n - T_{m-1}^n\right)\left[2a_1 h\left\{1-2\beta\left(T_m^n+1\right)\right\} + \beta\left(u_{m+1}^n - u_{m-1}^n\right) + \frac{2h\lambda_1}{r_m}u_m^n\right]
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
T_m^{n+1} \;=\;& 2T_m^n - T_m^{n-1} + a_0\upsilon\left[\left(T_{m+1}^n - 2T_m^n + T_{m-1}^n\right) + \frac{h}{2r_m}\left(T_{m+1}^n - T_{m-1}^n\right)\right] \\
&- \frac{\beta a_0 \upsilon}{4\left\{1-\beta\left(T_m^n+1\right)\right\}}\left(T_{m+1}^n - T_{m-1}^n\right)^2 \\
&- \frac{a_2}{2h}\left[\left(u_{m+1}^{n+1} - 2u_{m+1}^n + u_{m+1}^{n-1}\right) - \left(u_{m-1}^{n+1} - 2u_{m-1}^n + u_{m-1}^{n-1}\right) + \frac{2h}{r_m}\left(u_m^{n+1} - 2u_m^n + u_m^{n-1}\right)\right],
\end{aligned}
\tag{22}
$$

where we have used the notation $\upsilon = \frac{k^2}{h^2}$.

Further, equations (15) and (16) reduce to

$$
\left[\sigma_{rr}\right]_m^n = \left\{1-\beta\left(T_m^n+1\right)\right\}\left[\frac{u_{m+1}^n - u_{m-1}^n}{2h} + \lambda_1\frac{u_m^n}{r_m} - a_1 T_m^n\right]
\tag{23}
$$

$$
\left[\sigma_{\phi\phi}\right]_m^n = \left\{1-\beta\left(T_m^n+1\right)\right\}\left[\lambda_1\frac{u_{m+1}^n - u_{m-1}^n}{2h} + \frac{u_m^n}{r_m} - a_1 T_m^n\right]
\tag{24}
$$

From the initial condition (17) and by using equation (20), we get

$$
\frac{\partial u_m^0}{\partial t} = \frac{u_m^1 - u_m^{-1}}{2k} = 0, \quad \frac{\partial T_m^0}{\partial t} = \frac{T_m^1 - T_m^{-1}}{2k} = 0
\tag{25}
$$

Now, using equation (25) we can eliminate u^{-1} um and T^{-1} from equations (21) and (22) to get the equations satisfied by u_m^n and T_m^n for the first level of t $(i.e., n=0)$ as

$$
\begin{aligned}
u_m^1 \;=\;& u_m^0 + \frac{\upsilon}{2}\left\{1-\beta\left(T_m^0+1\right)\right\}\left[\left(u_{m+1}^0 - 2u_m^0 + u_{m-1}^0\right) + \frac{h}{2r_m}\left(u_{m+1}^0 - u_{m-1}^0\right) - \frac{h^2}{r_m^2}u_m^0\right] \\
&- \frac{\upsilon}{8}\left(T_{m+1}^0 - T_{m-1}^0\right)\left[2a_1 h\left\{1-2\beta\left(T_m^0+1\right)\right\} + \beta\left(u_{m+1}^0 - u_{m-1}^0\right) + \frac{2h\lambda_1}{r_m}u_m^0\right]
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
T_m^1 \;=\;& T_m^0 + \frac{a_0\upsilon}{2}\left[\left(T_{m+1}^0 - 2T_m^0 + T_{m-1}^0\right) + \frac{h}{2r_m}\left(T_{m+1}^0 - T_{m-1}^0\right)\right] \\
&- \frac{\beta a_0\upsilon}{8\left\{1-\beta\left(T_m^0+1\right)\right\}}\left(T_{m+1}^0 - T_{m-1}^0\right)^2 \\
&- \frac{a_2}{2h}\left[\left(u_{m+1}^1 - u_{m+1}^0\right) - \left(u_{m-1}^1 - u_{m-1}^0\right) + \frac{2h}{r_m}\left(u_m^1 - u_m^0\right)\right]
\end{aligned}
\tag{27}
$$

In view of boundary condition (18) and equation (23) , we get for the line $r = a$ as

$$
\frac{u_1^n - u_{-1}^n}{2h} + \lambda_1\frac{u_0^n}{r_0} - a_1 T_0^n = 0 \;\text{ and }\; T_0^n = f(t_n)
\tag{28}
$$

Now, substituting the expression for u_{-1}^n from equation (28) into equation (21) , we get the equation satisfied by u_m^n for $r = a$ (i.e. for the level $m = 0$) as

$$\begin{aligned} u_0^{n+1} &= 2u_0^n - u_0^{n-1} + 2v\left\{1 - \beta\left(T_0^n + 1\right)\right\}\left[\left\{u_1^n - u_0^n + h\left(\lambda_1 \frac{u_0^n}{r_0} - a_1 T_0^n\right)\right\} - \frac{h^2}{2r_0}\left(\lambda_1 \frac{u_0^n}{r_0} - a_1 T_0^n\right) - \frac{h^2}{2r_0^2}u_0^n\right] \\ &- \frac{hv}{2}\left(-3T_0^n + 4T_1^n - T_2^n\right)\left[a_1\left\{1 - 2\beta\left(T_0^n + 1\right)\right\} - \beta\left(\lambda_1 \frac{u_0^n}{r_0} - a_1 T_0^n\right) + \frac{\lambda_1}{r_0}u_0^n\right] \end{aligned} \tag{29}$$

Similarly, by using equation (19), we get for the line $r = b$ as

$$\frac{u_{N+1}^n - u_{N-1}^n}{2h} + \lambda_1 \frac{u_N^n}{r_N} - a_1 T_N^n = 0 \text{ and } T_N^n = 0 \tag{30}$$

Therefore, substituting u_{N+1}^n from equation (30) into equation (21) we obtain the equation for the level $m = N$ as follows:

$$\begin{aligned} u_N^{n+1} &= 2u_N^n - u_N^{n-1} + 2v\left\{1 - \beta\left(T_N^n + 1\right)\right\}\left[\left(-u_N^n + u_{N-1}^n + h\lambda_1 \frac{u_N^n}{r_N}\right) - \frac{h^2\lambda_1}{2r_N^2}u_N^n - \frac{h^2}{2r_N^2}u_N^n\right] \\ &- \frac{hv}{2}\left(3T_N^n - 4T_{N-1}^n + T_{N-2}^n\right)\left[a_1\left\{1 - 2\beta\left(T_N^n + 1\right)\right\} - \beta\lambda_1 \frac{u_N^n}{r_N} + \frac{\lambda_1}{r_N}u_N^n\right] \end{aligned} \tag{31}$$

The equations (21)-(31) therefore constitute the model of finite difference scheme for the present problem to determine the values of the physical field variables u, T , σ_{rr} and $\sigma_{\phi\phi}$ at different points of the solution domain $a \le r \le b$, $0 \le t \le t_0$.

IV. 1. Truncation Error

Now, we expand the finite difference equations (21) and (22) by using Taylor series expansion and subtract from the equations (13) and (14), respectively. Therefore, we find the truncation error associated with finite difference equations (21) and (22) as follows:

$$\begin{aligned} T.E.^u &= k^4\left[\frac{1}{12}\frac{\partial^4 u}{\partial t^4} + \frac{k^2}{360}\frac{\partial^6 u}{\partial t^6} + ...\right] - k^2h^2\left(1 - \beta - \beta T_m^n\right)\left[\left(\frac{1}{12}\frac{\partial^4 u}{\partial r^4} + \frac{h^2}{360}\frac{\partial^6 u}{\partial r^6} + ...\right)\right. \\ &+ \left.\frac{1}{r_m}\left(\frac{1}{6}\frac{\partial^3 u}{\partial r^3} + \frac{h^2}{120}\frac{\partial^5 u}{\partial r^5} + ...\right)\right] \\ &- \beta h^4\left(\frac{1}{6}\frac{\partial^3 T}{\partial r^3} + \frac{h^2}{120}\frac{\partial^5 T}{\partial r^5} + ...\right)\left(\frac{1}{6}\frac{\partial^3 u}{\partial r^3} + \frac{h^2}{120}\frac{\partial^5 u}{\partial r^5} + ...\right) \end{aligned} \tag{32}$$

$$\begin{aligned} T.E.^T &= k^4\left[\frac{1}{12}\frac{\partial^4 T}{\partial t^4} + \frac{k^2}{360}\frac{\partial^6 T}{\partial t^6} + ...\right] - k^2h^2a_0\left[\left(\frac{1}{12}\frac{\partial^4 T}{\partial r^4} + \frac{h^2}{360}\frac{\partial^6 T}{\partial r^6} + ...\right) + \frac{1}{r_m}\left(\frac{1}{6}\frac{\partial^3 T}{\partial r^3} + \frac{h^2}{120}\frac{\partial^5 T}{\partial r^5} + ...\right)\right] \\ &+ \frac{k^2h^2a_0\beta}{\left(1 - \beta - \beta T_m^n\right)}\left[\frac{1}{3}\frac{\partial T}{\partial r}\frac{\partial^3 T}{\partial r^3} + h^2\left\{\frac{1}{36}\left(\frac{\partial^3 T}{\partial r^3}\right)^2 + \frac{1}{60}\frac{\partial T}{\partial r}\frac{\partial^5 T}{\partial r^5}\right\} + ...\right] \\ &+ \frac{k^4 a_2}{r_m}\left[\frac{1}{12}\frac{\partial^4 u}{\partial t^4} + \frac{k^2}{360}\frac{\partial^6 u}{\partial t^6} + ...\right] \end{aligned} \tag{33}$$

The truncated errors given by equations (32) and (33) indicate that $\lim_{(h,k)\to(0,0)} T.E.^u = 0$ and $\lim_{(h,k)\to(0,0)} T.E.^T = 0$. This implies that the difference equations (21) and (22) are consistent. Thus, the finite differences given by(21) and (22) has the accuracy of orders $o(h^4, h^2k^2, k^4)$ and $o(h^2k^2, k^4)$, respectively.

V. NUMERICAL RESULTS AND DISCUSSION

We consider following two types of problems by taking two types of variations in the prescribed temperature distribution $f(t)$ at the inner surface of the cylinder:

Case-I: $f(t) = \mathrm{e}^{-\omega t}$
This implies that the temperature at the inner surface of the cylinder decreases exponentially with time and w is the decaying exponent.

Case-II:
We assume that the temperature at the inner surface of the cylinder varies like the sine function. Hence, we assume $f(t) = \sin(\omega t)$.

For our numerical work, we consider copper material and the physical data for which it is taken as follows [41]:

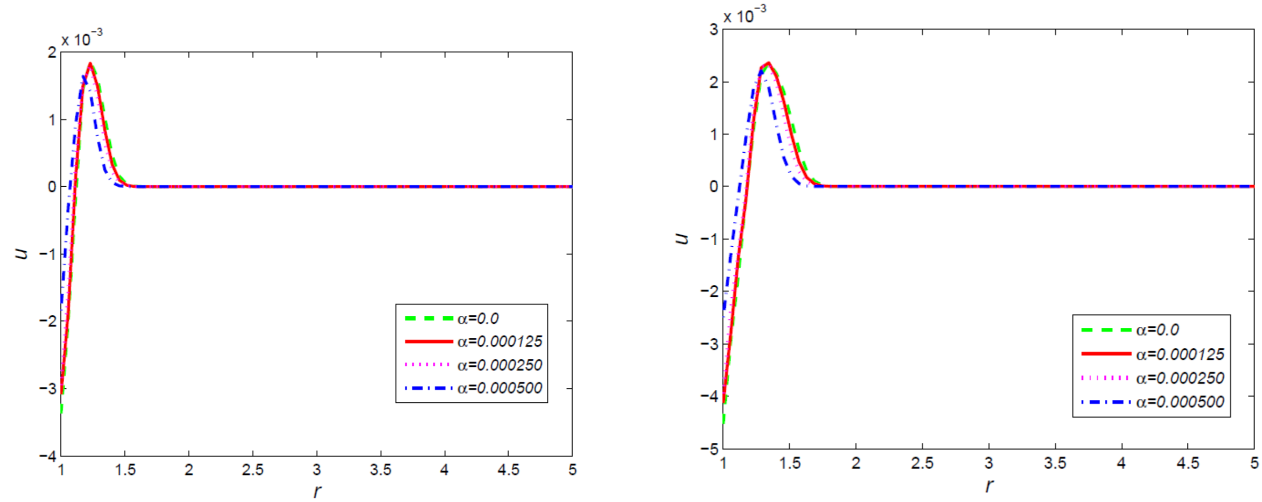
$T_0 = 819\mathrm{K}$, $\lambda_0 = 7.76 \times 10^{10}\mathrm{Nm}^{-2}$, $\mu_0 = 3.86 \times 10^{10}\mathrm{Nm}^{-2}$, $\rho = 8954\ \mathrm{kgm}^{-3}$, $\alpha_t = 1.78 \times 10^{-5}\mathrm{K}^{-1}$, $\eta = 8849.6\ \mathrm{m}^{-2}\mathrm{s}$. We assume $\omega = 0.1$. The inner radius and the outer radius of the cylinder are taken as 1.0 and 5.0, respectively, and we assume $t_0 = 1.0$ and $\upsilon = 0.0156$. Now, by using the equations (21), (22) and (26)-(31), the numerical (discrete) values of dimensionless displacement u and dimensionless temperature T are computed simultaneously for different values of the specified domain. Then, the values of stresses are computed from equations (23) and (24). We get the nature of variations of different field variables like displacement, temperature and stresses inside the medium with the help of computer programming. In order to see the effects of temperature dependency of the material parameters, the computations are done for different values of the parameter α. Clearly, $\alpha = 0$ indicates the case of temperature independent material properties. The results are displayed in different Figures to show the variations of different fields with respect to radial coordinates.

Figs. 1(a,b), 2(a,b), 3(a,b) and 4(a,b) show the variation of displacement u, temperature T, radial stress σ_{rr} and circumferential stress $\sigma_{\phi\phi}$, respectively at two different times $t = 0.4$ and $t = 0.8$ when the inner boundary temperature is varying exponentially (case-I) and Figs. 5(a,b), 6(a,b),7(a,b) and 8(a,b) shows the variations of u, T, σ_{rr} and $\sigma_{\phi\phi}$, respectively at two different times $t = 0.4$ and $t = 0.8$ when the inner boundary temperature is varying as a function of sine (case-II). The nature of variations of various fields observed in different Figures indicate that our system of difference equations (21)-(31) efficiently compute the numerical solutions of the problem and the solutions obtained are in complete agreement with the theoretical boundary conditions of the problem. We observe several important facts evident from the graphical results as mentioned below.

Figs. 1(a,b) and 5(a,b), showing the distributions of displacement, indicate that the nature of variation of displacement inside the annulus is similar in both the cases: whether the inner boundary surface of the annulus is subjected to exponentially varying temperature or to sinusoidal varying temperature. However, there is a significant effect of the temperature dependency on the material parameters. The numerical values of displacement is maximum in the temperature independence case and the values of u decrease with the increase of parameter α. The differences of displacement profiles for the cases of temperature dependent material properties and the case of temperature independent properties increase with the increase of time and the region of influence increases with time. Figs. 1(a,b) and 5(a,b) further indicate that the effect of temperature dependency of material parameters on displacement is more significant in case of exponentially varying temperature applied at the inner surface of the cylinder as compared to the case of sinusoidal temperature distribution applied at the inner boundary. The displacement field achieves zero values in all the cases after some distance from the inner boundary which proves the fact that the theory of thermoelasticity without energy dissipation accounts for finite speeds of elastic as well as thermal disturbances.

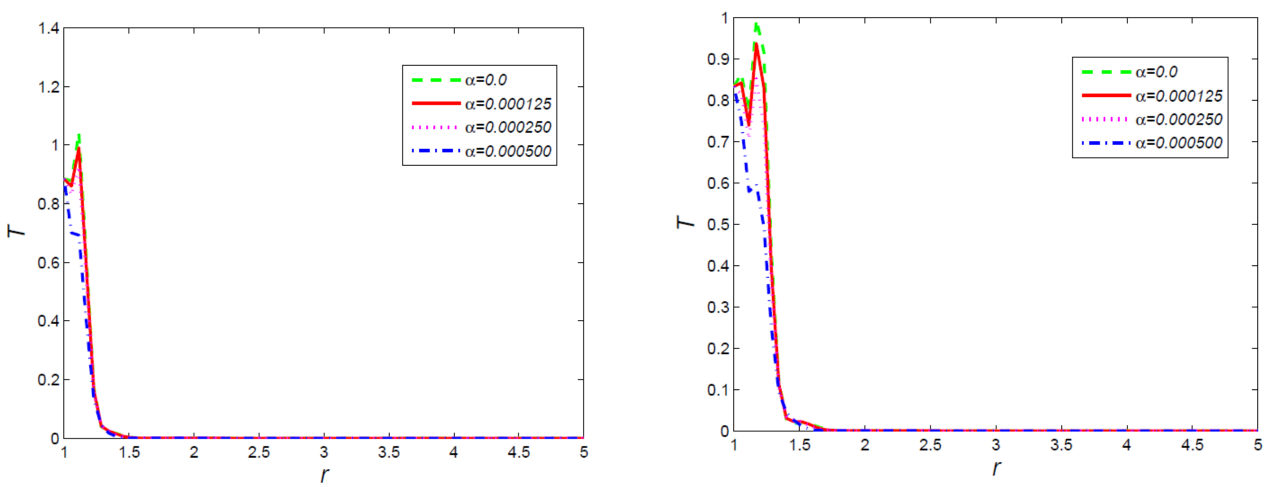
The variation of temperature can be observed from Figs. 2(a,b) and 6(a,b). Figs 2(a, b) show the variation of this field when exponentially decaying temperature is applied at the inner surface of the cylinder and Figs. 6(a,b) depicts the case when the inner boundary is subjected to sinusoidal temperature distribution. The nature of variation of temperature in these two cases are very much different, especially near the inner boundary. The variation is oscillatory in nature near the boundary in the first case as compared to the second case. Temperature shows a larger value in the case when we consider that material parameters are independent of temperature. Furthermore, the effect of temperature dependency of material parameters is very much significant in the first case, while it is almost negligible in the second case of prescribed boundary temperature. The region of influence increases with the increases of time for this field, too.

Figs. 3(a,b) and 7(a,b) show the variation of radial stress and we observe a significant difference in the nature of variation of redial stress in cases of two types of boundary temperatures prescribed at the inner boundary of the annulus. In the second case, the radial stress is fully compressive, while in the first case, it is tensile for some region after some distance from the inner surface and thereafter becoming compressive. Like the temperature field, radial stress also shows the oscillatory nature in the first case. However, the effect of α is very much pronounced in both cases. Numerical values of radial stress is maximum in the case of temperature independent physical parameters and the value of radial stress decreases with the increase of α. Circumferential stress distributions for two different cases can be observed from Figs. 4(a,b) and 8(a,b). Like the case of radial stress, circumferential stress also shows a significantly different trend of variation in two different types of temperature distribution prescribed at the inner surface of the cylinder. The nature is more oscillatory near the inner boundary in the case when exponentially decaying temperature is prescribed. The effect of α is more pronounced in the first case as compared to the second case



(a) (b)

Fig. 1. Variation of displacement, u vs. r at times, $t = 0.4$ and $t = 0.8$, respectively for the Case-I

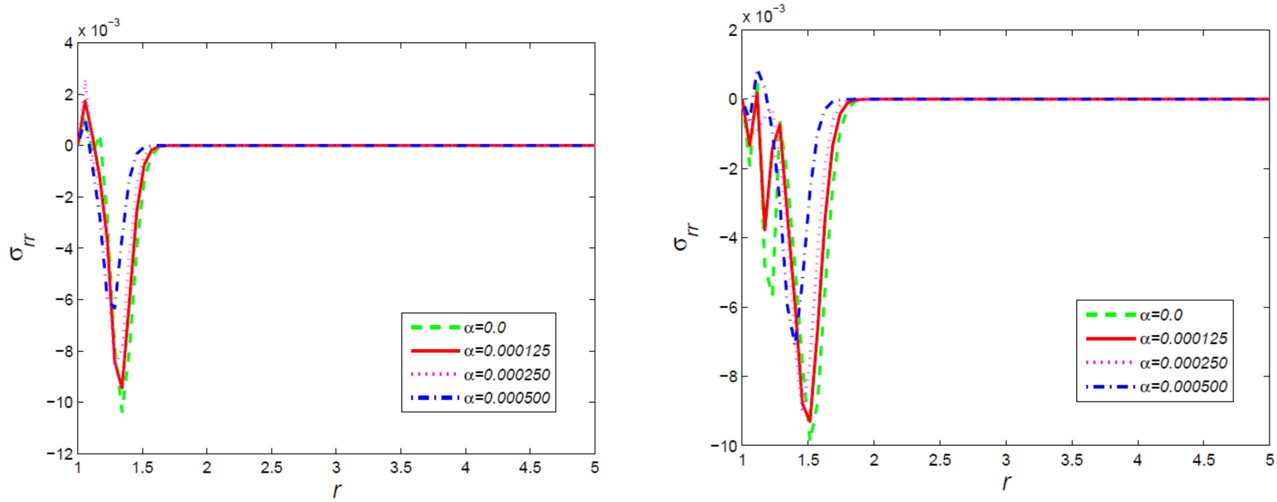


(a) (b)

Fig. 2. Variation of temperature, T vs. r at times, $t = 0.4$ and $t = 0.8$, respectively for the Case-I

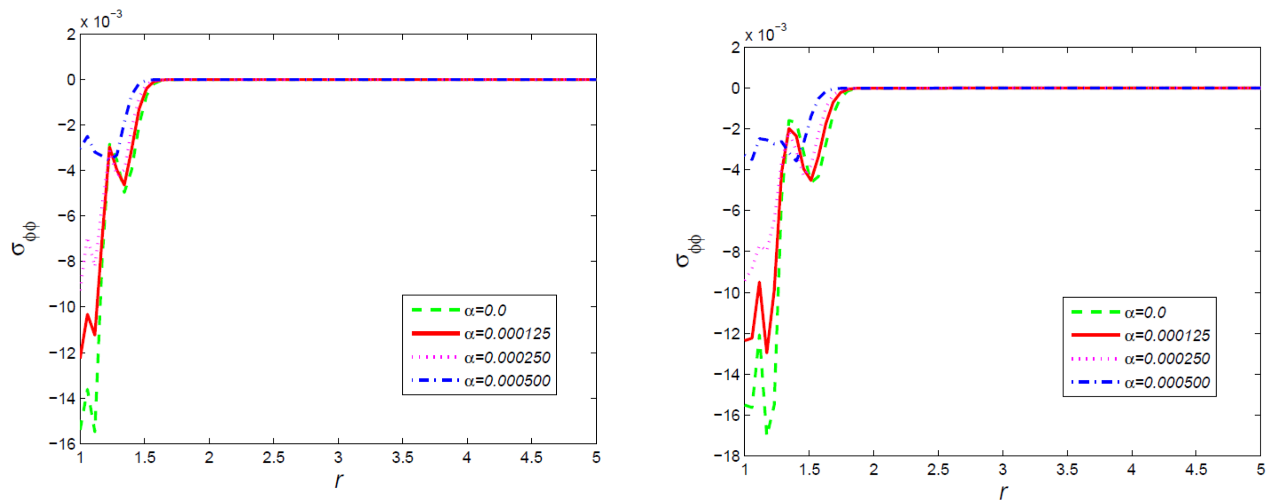
and the absolute value of this stress is maximum in the case of temperature independent physical properties. The difference in the trend of variation increases with the increases of time as well as with α.

It is interesting to make a comparison between the results in case-I of GN-II model and the corresponding results predicted by the LS model as discussed by Mukhopadhyay and Kumar [29] to demonstrate the effects of temperature dependent properties. Fig.1(a) and the corresponding figure for displacement of [29] show that u has a maximum value nearer to the boundary of annulus cylinder in case of both the models while this maximum value is slightly smaller in case of the GN-II model. Furthermore, the displacement decreases as the value of α increases, which agrees with the corresponding figure of the LS model. From Fig.2(a), the temperature distribution shows an oscillatory behaviour in the region nearer to the inner boundary of the annulus in case of GN-II model and there are extreme points. However, the corresponding Figure of the LS model exhibits that temperature distribution shows smooth decrements without any local



(a) (b)

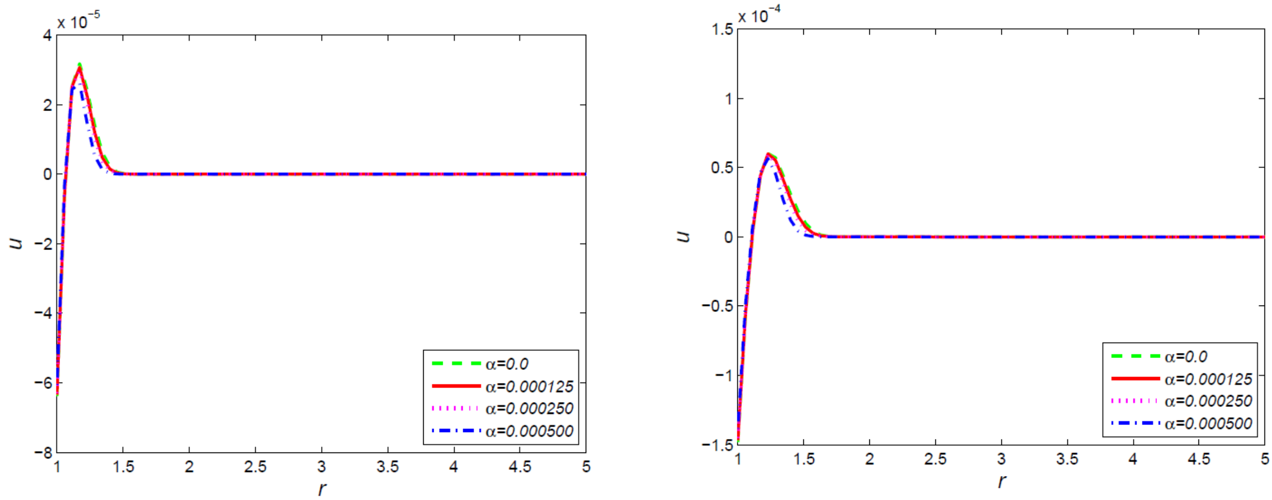
Fig. 3. Variation of radial stress, σ_{rr} vs. r at times, $t = 0.4$ and $t = 0.8$, respectively for the Case-I



(a) (b)

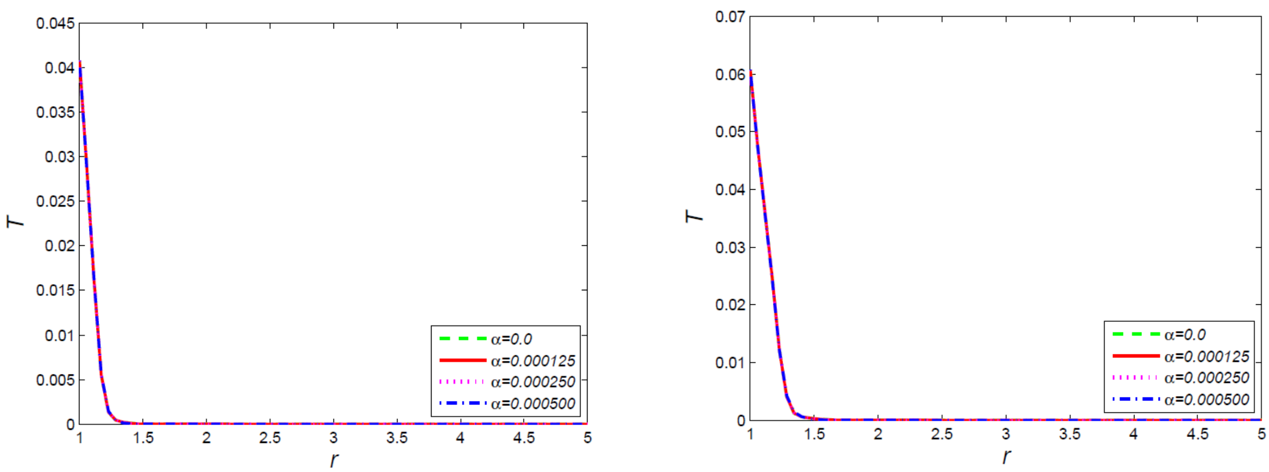
Fig. 4. Variation of transverse stress, $\sigma_{\phi\phi}$ vs. r at times, $t = 0.4$ and $t = 0.8$, respectively for the Case-I

extreme points. The temperature also decreases with the increase of α under both the models. Fig.3(a) shows that in the context of the GN-II model, the radial stress is tensile for some region nearer to the inner surface and thereafter becoming compressive, but in the case of the LS theory, the radial stress, σ_{rr} was shown to be fully compressive (see Ref [29]). Furthermore, it is clear that σ_{rr} is inversely proportional to α for the GN-II model while it is directly proportional to α in case of the LS model, which was found to be a notable difference in two different thermoelasticity theories. In comparison of the Fig.4(a) with the corresponding figure of the LS model, the effect of α is similar for both the models. Furthermore, both the stresses show oscillatory type variation through the radial direction in case of the GN-II model while the LS model shows smooth variation in stresses. The region of influence for each phyical field is observed to be finite under both the theories supporting the fact that thermal wave propagate with finite speed in the context of the LS-model as well as in case of the GN-II model. However, the influence area for all physical quantities is much narrower under the GN-II model (see Figs. 1–4(a)) as compared to the LS model (see Ref. [29]).



(a) (b)

Fig. 5. Variation of displacement, u vs. r at times, $t = 0.4$ and $t = 0.8$, respectively for the Case-II



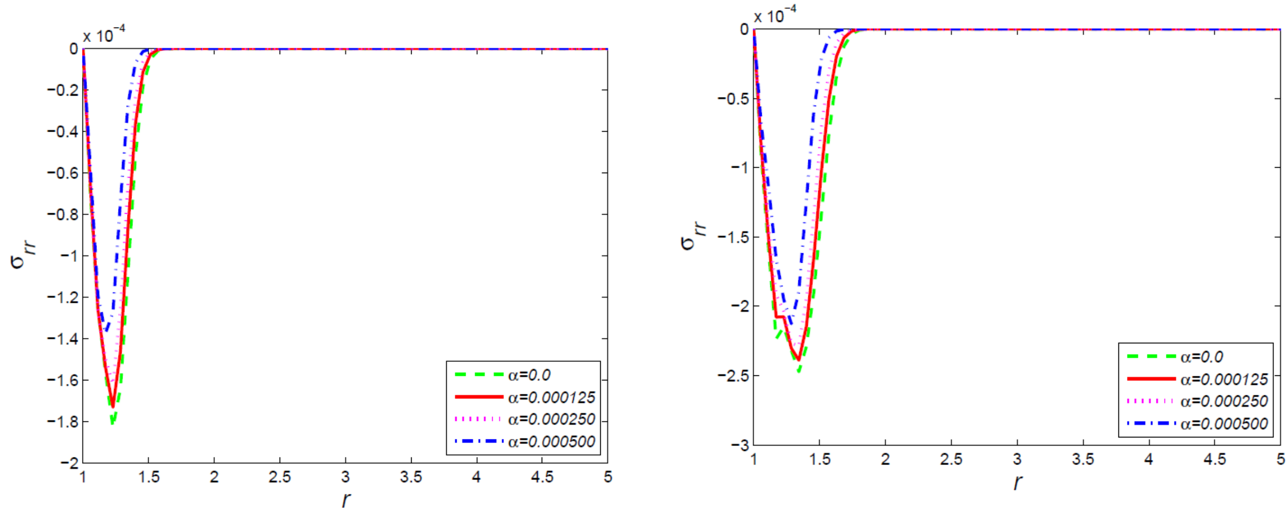
(a) (b)

Fig. 6. Variation of temperature, T vs. r at times, $t = 0.4$ and $t = 0.8$, respectively for the Case-II

VI. CONCLUDING REMARKS

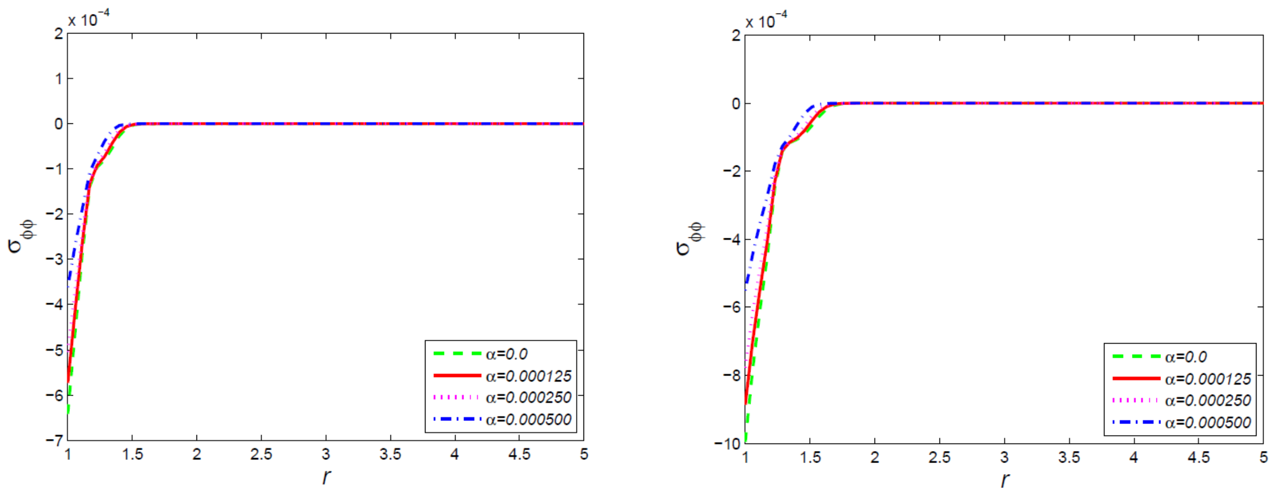
The effects of temperature dependency of material properties on thermo-mechanical responses of an annular cylinder whose inner surface is subjected to time dependent temperature fields has been analyzed by employing the thermoelasticity theory of type GN-II. The governing equations are derived by employing this theory and considering temperature dependent physical properties. Governing equations are obtained as non-linear coupled partial differential equations and we apply the finite difference method for solving the coupled system for the present problem. We showed that the present problem can be efficiently solved by the finite difference method. We obtain the orders of the truncation error for displacement and temperature variable.

Our results highlight the significant effects of temperature dependent material properties on thermoelasticity. We considered two different types of temperature distributions prescribed at the inner boundary surface of the cylinder whereas the outer surface is kept at constant reference temperature. The effects of temperature dependent properties are shown to be of different nature in two cases. However, in both cases it has been observed that under the present context, the difference in the numerical results with temperature



(a) (b)

Fig. 7. Variation of radial stress, σ_{rr} vs. r at times, $t = 0.4$ and $t = 0.8$, respectively for the Case-II



(a) (b)

Fig. 8. Variation of transverse stress, $\sigma_{\phi\phi}$ vs. r at times, $t = 0.4$ and $t = 0.8$, respectively for the Case-II

dependent properties and temperature independent properties are very much pronounced. The region of influence is observed to be finite for all field variables. We note a significant difference in the prediction of results for exponential temperature distribution prescribed at the inner boundary of the cylinder in the present context with the corresponding results under the thermoelasticity theory with one relaxation parameter. The region of influence is much smaller in case of the GN-II model as compared to the same under the LS model. For a more accurate analysis of thermoelastic behavior of structural elements, the temperature dependency needs to be considered. Hence, this study is believed to be useful in characterizing the thermoelastic responses of the structural element with temperature dependent properties under different thermoelastic models.

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