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Fundamental Solution for the Plane Problem in Magnetothermoelastic Diffusion Media

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Abstract: The aim of the present paper is to study the fundamental solution in orthotropic magneto- thermoelastic diffusion media. With this objective, firstly the two-dimensional general solution in orthotropic magnetothermoelastic diffusion media is derived. On the basis of thegeneral solution, the fundamental solution for a steady point heat source in an infinite and a semi-infinite orthotropic magnetothermoelastic diffusion material is constructed by four newly introduced harmonic functions. The components of displacement, stress, temperature distribution and mass concentration are expressed in terms of elementary functions. From the present investigation, some special cases of interest are also deduced and compared with the previously obtained results. The resulting quantities are computed numerically for infinite and semi-infinite magnetothermoelastic material and presented graphically to depict the magnetic effect.

Key words: fundamental solution, orthotropic, magnetothermoelastic diffusion, semi-infinite, infinite

I. INTRODUCTION

Fundamental solutions or Green's functions play an important role in the solution of numerous problems in the mechanics and physics of solids. They are a basic building block of many further works. For example, fundamental solutions can be used to construct many analytical solutions of practical problems when boundary conditions are imposed. They are essential in the boundary element method as well as the study of cracks, defects and inclusions.

Ding, Chen and Liang [1] derived the general solutions for coupled equations in piezoelectric media. Dunn and Wienecke [2] derived the half space Green's functions for transversely isotropic piezoelectric solid. Pan and Tanon [3] studied the Green's functions for a three-dimensional problem in general anisotropic piezoelectric solids.

When thermal effects are considered, Ding, Guo and Hou [4] obtained a three-dimensional general solution in transversely isotropic piezothermoelastic media. Sharma [5] investigated the fundamental solution for transversely isotropic thermoelastic material in an integral form. Chen et. al. [6] derived the three dimensional general solution for transversely isotropic thermoelastic materials. Hou et. al. [7, 8] investigated the Green's function for two and three-dimensional problems for a steady Point heat source in the interior of a semi-infinite thermoelastic materials. Also, Hou et. al. [9] investigated the two dimensional general solution and fundamental solution for orthotropic thermoelastic materials.

The theory of magnetothermoelasticity is concerned with the interacting effects of the applied magnetic field on the elastic and thermoelastic deformation of a solid body. This theory has drawn the attention of many researchers because of its extensive uses in diverse fields, such as geophysics for understanding the effect of Earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field. Kolaski and Nowacki [10] studied the magnetothermoelastic disturbance in a perfectly conducting elastic half-space in contact with vacuum due to applied thermal disturbance on the plane boundary. Othman and Song [11] investigated the reflection of magnetothermoelastic waves with two relaxation times. Hou et. al. [12] investigated the general solution and fundamental solution for orthotropic magnetothermoelastic materials.

Diffusion is the spontaneous movement of matter (particles) from a region of high concentration to low concentration. Diffusion occurs in response to a concentration gradient expressed as the change in concentration due to the change in position. The example of diffusion is heat transport or movement transport. The net flux of transported quantity (atoms, energy or electrons) is equal to a physical property (diffusivity, thermal conductivity and electric conductivity) multiplied by a gradient (concentration, thermal and electric field gradient). The concept of thermodiffusion is used to describe the process of thermo-mechanical treatment of metals. The study of this phenomenon is of great concern due to its many geophysical and industrial applications. For example, oil companies are interested in the process of thermodiffusion for more efficient extraction of oils from oils deposits. The thermodiffusion process also helps the investigation in the field associated with the advent of semiconductor devices and that advancement of microelectronics. Most of the research associated with the presence of concentration and temperature gradients have been made with metals and alloys.

Nowacki [13-16] developed the theory of thermoelastic diffusion by using the coupled thermoelastic model. Sherief et. al. [17] developed the generalized theory of thermoelastic diffusion with one relaxation time which allows finite speeds of propagation of waves. Kumar and Kansal [18] derived the basic equations for generalized thermoelastic diffusion. When diffusion effects are considered, Kumar and Chawla [19] discussed the Plane wave propagation in the anisotropic three-phase-lag model. Kumar and Chawla [20] investigated the fundamental solution in orthotropic thermoelastic diffusion material. Kumar and Chawla [21] studied the Green's function for a two-dimensional problem in orthotropic thermoelastic diffusion material. However, the important fundamental solution for a two-dimensional problem in magnetothermoelastic diffusion material has not been discussed so far.

The fundamental solution for two-dimensional problem in orthotropic magnetothermoelastic diffusion medium is investigated in this paper. Based on the two-dimensional general solution of orthotropic magnetothermoelastic diffusion media, the fundamental solutions for a steady point heat source acting in an infinite plane and on the surface of a semi-infinite plane are obtained by four newly introduced harmonic functions. From the present investigation, some special cases of interest are also deduced.

II. BASIC EQUATIONS

Following Nayfeh and Nasser [22], Maxwell equations in vector form can be written as

$$\nabla \times \vec{H} = \vec{J} + \dot{\vec{D}}, \tag{1}$$

$$\nabla \times \vec{E} = -\dot{\vec{B}}, \tag{2}$$

$$\nabla \cdot \vec{B} = 0 \tag{3}$$

$$\nabla \cdot \vec{D} = \rho_e \tag{4}$$

$$\vec{B} = \mu_e \vec{H}, \qquad \vec{H} = \vec{H}_0 + \vec{h} \tag{5}$$

$$\vec{D} = \varepsilon \vec{E} \tag{6}$$

and the modified Ohm's law

$$\vec{J} = \sigma \left[\vec{E} + \dot{\vec{u}} \times \vec{B} \right] \tag{7}$$

where ∇ is the del operator, $\vec{H}$ is the magnetic intensity vecor; $\vec{J}$ is the conduction current density, $\vec{D}$ and $\vec{E}$ are, respectively, the electric flux and electric density vecor ($\dot{\vec{D}}$ is commonly termed the electric displacement current), $\vec{B}$ is the magnetic field density; ρ_e is the charge density, the constants μ_e, ε and σ which characterize the medium are known as the magnetic permeability, electric permittivity and electric conductivity, respectively, $\vec{u}$ is the displacement vector.
Using equation (5), equation (7) can be linearized by neglecting the small quantities of the second order giving

$$\vec{J} = \sigma \left[\vec{E} + \mu_e \dot{\vec{u}} \times \vec{H}_0 \right] \tag{8}$$

The above equations (1)-(8) are supplemented by equations of motion and constitutive relations (in case of perfectly electroconducting i.e. $\sigma \to \infty$) in the theory of generalized thermoelastic diffusion, taking into account the Lorentz force (Eringen [23]),
(i) Constitutive relations

$$\sigma_{ij} = c_{ijkm} e_{km} + a_{ij} T + b_{ij} C, \tag{9}$$

(ii) Equations of motion

$$c_{ijkm} e_{km,j} + a_{ij} T_{,j} + b_{ij} C_{,j} + \breve{F}_i = \rho \ddot{u}_i, \tag{10}$$

(iii) Equation of heat conduction

$$\rho C_E \dot{T} + a T_0 \dot{C} - a_{ij} T_0 \dot{e}_{ij} = K_{ij} T_{,ij}, \tag{11}$$

(iv) Equation of mass diffusion

$$-\alpha^*_{ij} b_{km} e_{km,ij} - \alpha^*_{ij} b C_{,ij} + \alpha^*_{ij} a T_{,ij} = -\dot{C}. \tag{12}$$

Here, $F_i = (\vec{J} \times \vec{B})_i$, $c_{ijkm} (= c_{kmij} = c_{jikm} = c_{ijmk})$ are elastic parameters, $a_{ij} (= a_{ji})$, $b_{ij} (= b_{ji})$ are, respectively, the tensor of thermal and diffusion moduli. ρ is the density and C_E is the specific heat at constant strain, a and b are, respectively, coefficient describing the measure of thermoelastic diffusion effects and of diffusion effects, T_0 is the reference temperature assumed to be such that $\left|\frac{T}{T_0}\right| << 1$,$K_{ij}(=K_{ji})$, $\sigma_{ij}(=\sigma_{ji})$ and $e_{ij}=\frac{u_{i,j}+u_{j,i}}{2}$ denote the components of thermal conductivity, stress and strain tensor, respectively, $T(x,y,z,t)$ is the temperature change from the reference temperature T_0 and C is the mass concentration.u_i are components of displacement vector, $\alpha^*_{ij}(=\alpha^*_{ji})$ are diffusion parameters. $\breve{F}_i$ are components of Lorentz force.

In the above equations symbol (",") followed by a suffix denotes differentiation with respect to spatial coordinate and a superposed dot (".") denotes the derivative with respect to time, respectively.

III. FORMULATION OF THE PROBLEM

We consider a homogenous orthotropic magnetothermoelastic diffusion medium. Let us take $Oxyz$ as the frame of reference in Cartesian coordinates, the origin O being any point on the plane boundary.

For a two-dimensional problem, we assume the displacement vector, temperature distribution, mass concentration, applied magnetic field $\vec{H}_0$, induced magnetic field $\vec{h}$, current electric density vector $\vec{E}$ and $\vec{J}$ conduction current density vector are respectively of the form

$$\begin{gathered}\vec{u}=(u,0,w),\qquad T(x,z,t),\qquad C(x,z,t)\\ \vec{H}_0=(0,H_0,0),\qquad \vec{h}=(0,h,0),\\ \vec{E}=(E_1,0,E_3),\quad \vec{J}=(J_1,0,J_3).\end{gathered}\tag{13}$$

It can be easily seen from equation (13) that Lorentz force will have non-vanishing components in $x-z$ direction, that is

$$\breve{F}_1=\mu_0H_0^2\left(\frac{\partial e}{\partial x}-\varepsilon_0\mu_0\frac{\partial^2 u}{\partial t^2}\right),\tag{14a}$$

$$\breve{F}_3=\mu_0H_0^2\left(\frac{\partial e}{\partial z}-\varepsilon_0\mu_0\frac{\partial^2 w}{\partial t^2}\right),\tag{14b}$$

$$e=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}.$$

Moreover, we are discussing steady problem

$$\frac{\partial u}{\partial t}=\frac{\partial w}{\partial t}=\frac{\partial C}{\partial t}=\frac{\partial T}{\partial t}=0.\tag{15}$$

We define the dimensionless quantities as:

$$\begin{gathered}(x',z',u'w')=\frac{\omega_1^*}{v_1}(x,z,u,w),\\ (T',C')=\frac{1}{c_{11}}(a_1T,b_1C),\\ \sigma'_{ij}=\frac{\sigma_{ij}}{a_1T_0},\qquad H'=\frac{a_1v_1}{c_{11}K_1\omega_1^*}H,\end{gathered}\tag{16}$$

where

$$v_1^2=b_1,\qquad \omega_1^*=\frac{ac_{11}}{K_1}.$$

Equations (9) - (12) for orthotropic magnetothermoelastic diffusion material, with the aid of (13)-(16), after suppressing the primes, yields

$$\begin{aligned}&\left(\delta_1\frac{\partial^2}{\partial x^2}+\delta_2\frac{\partial^2}{\partial z^2}\right)u+\left(\delta_3\frac{\partial^2}{\partial x\partial z}\right)w\\ &-\left(\frac{\partial}{\partial x}\right)C-\left(\frac{\partial}{\partial x}\right)T=0,\end{aligned}\tag{17}$$

$$\begin{aligned}&\left(\delta_3\frac{\partial^2}{\partial x\partial z}\right)u+\left(\delta_2\frac{\partial^2}{\partial x^2}+\delta_4\frac{\partial^2}{\partial z^2}\right)w\\ &-\varepsilon_1\left(\frac{\partial}{\partial z}\right)C-\varepsilon_2\left(\frac{\partial}{\partial z}\right)T=0,\end{aligned}\tag{18}$$

$$\left(\frac{\partial^2}{\partial x^2}\right)T+\varepsilon_3\left(\frac{\partial^2}{\partial z^2}\right)T=0,\tag{19}$$

$$\begin{aligned}&\frac{\partial}{\partial x}\left(q_1^*\frac{\partial^2}{\partial x^2}+q_3^*\frac{\partial^2}{\partial z^2}\right)u+\frac{\partial}{\partial z}\left(q_2^*\frac{\partial^2}{\partial x^2}+q_4^*\frac{\partial^2}{\partial z^2}\right)w\\ &-\left(q_5^*\frac{\partial^2}{\partial x^2}+q_6^*\frac{\partial^2}{\partial z^2}\right)C+\left(q_7^*\frac{\partial^2}{\partial x^2}+q_8^*\frac{\partial^2}{\partial z^2}\right)T=0,\end{aligned}\tag{20}$$

where

$$\begin{aligned}&\delta_1=1+\frac{\mu_0H_0^2}{c_{11}},\\ &(\delta_2,\ \delta_3,\ \delta_4)=\frac{1}{c_{11}}(c_{55},\ c_{13}+c_{55}\\ &+\mu_0H_0^2\omega_1^*,\ c_{33}+\mu_0H_0^2\omega_1^*),\\ &\varepsilon_1=\frac{b_3}{b_1},\quad \varepsilon_2=\frac{a_3}{a_1},\quad \varepsilon_3=\frac{K_3}{K_1},\\ &(q_1^*,q_2^*)=\frac{\alpha_1^*\omega_1^*}{c_{11}}\left(b_1,b_3\right),\\ &(q_3^*\ q_4^*)=\frac{\alpha_3^*\omega_1^*}{c_{11}}\left(b_1,b_3\right),\\ &(q_5^*,q_6^*)=\frac{\omega_1^*b}{b_1}\left(\alpha_1^*,\ \alpha_3^*\right),\ (q_7^*,q_8^*)=\frac{a\omega_1^*}{a_1}\left(\alpha_1^*,\ \alpha_3^*\right).\end{aligned}$$

The equations (17)-(20) can be written as

$$D\left\{u,w,C,T\right\}^{tr}=0,\tag{21}$$

where D is the differential operator matrix given by

$$\begin{bmatrix} \delta_1 \frac{\partial^2}{\partial x^2} + \delta_2 \frac{\partial^2}{\partial z^2} & \delta_3 \frac{\partial^2}{\partial x \partial z} & -\frac{\partial}{\partial x} & -\frac{\partial}{\partial x} \\ \delta_3 \frac{\partial^2}{\partial x \partial z} & \delta_2 \frac{\partial^2}{\partial x^2} + \delta_4 \frac{\partial^2}{\partial z^2} & -\varepsilon_1 \frac{\partial}{\partial z} & -\varepsilon_2 \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x}\left(q_1^* \frac{\partial^2}{\partial x^2} + q_3^* \frac{\partial^2}{\partial z^2}\right) & \frac{\partial}{\partial z}\left(q_2^* \frac{\partial^2}{\partial x^2} + q_4^* \frac{\partial^2}{\partial z^2}\right) & -\left(q_5^* \frac{\partial^2}{\partial x^2} + q_6^* \frac{\partial^2}{\partial x^2}\right) & \left(q_7^* \frac{\partial^2}{\partial x^2} + q_8^* \frac{\partial^2}{\partial x^2}\right) \\ 0 & 0 & 0 & \left(\frac{\partial^2}{\partial x^2} + \varepsilon_3 \frac{\partial^2}{\partial z^2}\right) \end{bmatrix}. \tag{22}$$

Equation (21) is a homogeneous set of differential equations in $u,\ w,\ C,\ T$. The general solution by the operator theory as follows

$$u = A_{i1}F,\ w = A_{i2}F,\ C = A_{i3}F,\ T = A_{i4}F, \quad (i = 1, 2, 3, 4) \tag{23}$$

where A_{ij} are algebraic cofactors of the matrix D, of which the determinant is

$$|D| = \left(a^* \frac{\partial^6}{\partial z^6} + b^* \frac{\partial^6}{\partial x^2 \partial z^4} + c^* \frac{\partial^6}{\partial x^4 \partial z^2} + d^* \frac{\partial^6}{\partial x^6}\right) \times \left(\frac{\partial^2}{\partial x^2} + \varepsilon_3 \frac{\partial^2}{\partial z^2}\right), \tag{24}$$

where

$$a^* = \delta_2(\varepsilon_1 q_4^* - \delta_4 q_6^*),$$

$$b^* = \delta_1(\varepsilon_1 q_4^* - \delta_4 q_6^*) - \delta_2(\delta_2 q_6^* + \delta_4 q_5^*) + \delta_2(\varepsilon_1 q_2^* + \delta_3 q_6^*) - q_7^*(\delta_4 + \delta_2 \varepsilon_1) + \delta_3 q_4^*,$$

$$c^* = \delta_1(\varepsilon_1 q_2^* - \delta_4 q_5^*) - \delta_2(\delta_1 q_6^* + \delta_2 q_5^*) + \delta_2(\delta_3 q_5^* - \varepsilon_1 q_1^*) +, \delta_3 q_2^* - \delta_4 q_1^* - \delta_2 q_7^*,$$

$$d = -\delta_2(\delta_1 q_5^* + q_1^*).$$

The function F in equation (23) satisfies the following homogeneous equation

$$|D|\, F = 0. \tag{25}$$

It can be seen that if $i = 1, 2, 3$ are taken in equation (23), three general solutions are obtained in which $T = 0$. These solutions are identical to those without thermal fact and are not discussed here. Therefore if $i = 4$ should be taken in equation (23), the following solution is obtained

$$\begin{aligned} u &= \left(p_1 \frac{\partial^4}{\partial x^4} + q_1 \frac{\partial^4}{\partial z^2 \partial x^2} + r_1 \frac{\partial^4}{\partial z^4}\right) \frac{\partial F}{\partial x}, \\ w &= \left(p_2 \frac{\partial^4}{\partial x^4} + q_2 \frac{\partial^4}{\partial z^2 \partial x^2} + r_2 \frac{\partial^4}{\partial z^4}\right) \frac{\partial F}{\partial z}, \\ C &= \left(p_3 \frac{\partial^6}{\partial z^6} + q_3 \frac{\partial^6}{\partial z^4 \partial x^2} + r_3 \frac{\partial^6}{\partial z^2 \partial x^4} + l_3 \frac{\partial^6}{\partial x^6}\right) F, \\ T &= \left(a^* \frac{\partial^6}{\partial z^6} + b^* \frac{\partial^6}{\partial z^4 \partial x^2} + c^* \frac{\partial^6}{\partial z^2 \partial x^4} + d^* \frac{\partial^6}{\partial x^6}\right) F, \end{aligned} \tag{26}$$

$$p_1 = (q_7^* - q_5^*)\delta_2,\ q_1 = -\delta_2(\varepsilon_1 q_7^* + q_5^* \varepsilon_2) + \delta_2(q_6^* + q_8^*) + \delta_4(q_7^* + q_8^*)q_8^* - \varepsilon_1 q_2^*,$$

$$r_1 = -\delta_2(\varepsilon_1 q_8^* + \varepsilon_2 q_6^*) + \delta_2 q_8^* + \delta_4(q_6^* + q_8^*) + (q_4^* + q_6^*)\delta_4 + \varepsilon_1 q_4^*,$$

$$p_2 = \delta_3(q_5^* + q_7^*) + q_1^*(\varepsilon_2 - \varepsilon_1) - \delta_1(\varepsilon_1 q_7^* + \varepsilon_2 q_5^*),$$

$$r_2 = -\delta_1(\varepsilon_1 q_8^* + \varepsilon_2 q_6^*),$$

$$q_2 = -\delta_1(\varepsilon_1 q_8^* + q_6^* \varepsilon_2) - \delta_2(\varepsilon_1 q_7^* + \varepsilon_2 q_5^*) + \delta_3(q_8^* + q_6^*) + q_7^*(\varepsilon_2 - \varepsilon_1),$$

$$p_3 = (\varepsilon_2 q_4^* + \delta_4 q_8^*)\delta_2,$$

$$q_3 = \delta_1(\delta_4 q_8^* + \varepsilon_2 q_4^*) + \delta_2(\delta_2 q_8^* + \delta_4 q_7^*) + \delta_2(\varepsilon_2 q_2^* - \delta_3 q_8^*) - q_7^*(\delta_2 \varepsilon_2 + \delta_4) + \delta_3 q_4^*,$$

$$r_3 = \delta_1(\delta_2 q_8^* + \delta_4 q_7^*) + \delta_2(\delta_2 q_7^* - \delta_3 q_7^*) + q_2^*(\varepsilon_2 \delta_1 + \delta_3) - \delta_2(q_1^* \varepsilon_2 + q_7^*) - \delta_4 q_1^*,$$

$$l_3 = \delta_2(\delta_1 q_7^* - q_1^*).$$

Equation (25) can be rewritten as

$$\prod_{j=1}^{4} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_j^2}\right) F = 0, \tag{27}$$

where $z_j = s_j z,\ s_4 = \sqrt{\frac{K_1}{K_3}}$ and $s_j (j = 1, 2, 3)$ are three roots (with positive real part) of the following algebraic equation

$$a^* s^6 - b^* s^4 + c^* s^2 - d^* = 0. \tag{28}$$

As known from the generalized Almansi (Proved by Ding et. al. [1]) theorem, the function F can be expressed in terms of four harmonic functions

$$F = F_1 + F_2 + F_3 + F_4 \quad \text{for distinct } s_j\ (j = 1, 2, 3, 4). \tag{29a}$$

$$F = F_1 + F_2 + F_3 + zF_4 \quad \text{for } s_1 \neq s_2 \neq s_3 = s_4. \tag{29b}$$

$$F = F_1 + F_2 + zF_3 + z^2 F_4 \quad \text{for } s_1 \neq s_2 = s_3 = s_4. \tag{29c}$$

$$
\begin{aligned}
F =& F_1 + zF_2 + z^2F_3 + z^3F_4 \\
& \text{for } s_1 = s_2 = s_3 = s_4.
\end{aligned} \tag{29d}
$$

where F_j $(j = 1, 2, 3, 4)$ satisfies the following harmonic equations

$$
\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_j^2}\right) F_j = 0, \quad (j = 1, 2, 3, 4). \tag{30}
$$

The general solution for the case of distinct roots can be derived as follows

$$
\begin{aligned}
u = \sum_{j=1}^{4} p_{1j} \frac{\partial^5 F_j}{\partial x \partial z_j^4} \qquad w = \sum_{j=1}^{4} s_j p_{2j} \frac{\partial^5 F_j}{\partial z_j^5} \\
C = \sum_{j=1}^{4} p_{3j} \frac{\partial^6 F_j}{\partial z_j^6} \qquad T = p_{44} \frac{\partial^6 F_4}{\partial z_4^6}.
\end{aligned} \tag{31}
$$

The general solution for the other three cases can be derived in the similar way. Equation (31) can be further simplified by taking

$$
p_{1j} \frac{\partial^4 F_j}{\partial z_j^4} = \psi_j. \tag{32}
$$

Making use of equation (32) in equation (31), we obtain

$$
\begin{aligned}
u = \sum_{j=1}^{4} \frac{\partial \psi_j}{\partial x}, \qquad w = \sum_{j=1}^{4} s_j P_{1j} \frac{\partial \psi_j}{\partial z_j}, \\
C = \sum_{j=1}^{4} P_{2j} \frac{\partial^2 \psi_j}{\partial z_j^2}, \qquad T = P_{34} \frac{\partial^2 \psi_4}{\partial z_4^2},
\end{aligned} \tag{33}
$$

where

$$
P_{1j} = p_{2j}/p_{1j},\ P_{2j} = p_{3j}/p_{1j},\ P_{34} = p_{44}/p_{14}. \tag{34}
$$

The function ψ_j satisfies the harmonic equations

$$
\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_j^2}\right) \psi_j = 0, \qquad j = 1, 2, 3, 4. \tag{35}
$$

Making use of equations (13), (15) and (16) in equation (9) and after suppressing the primes, with the aid of equation (33), we obtain

$$
\begin{aligned}
\sigma_{xx} = \sum_{j=1}^{4} \big(- f_1 + f_1 s_j^2 P_{1j} \\
- f_1 P_{3j} - f_1 P_{2j} \big) \frac{\partial^2 \psi_j}{\partial z_j^2},
\end{aligned} \tag{36a}
$$

$$
\begin{aligned}
\sigma_{zz} = \sum_{j=1}^{4} \big(- f_2 + h_1 s_j^2 P_{1j} \\
- h_2 P_{3j} - h_3 P_{2j} \big) \frac{\partial^2 \psi_j}{\partial z_j^2},
\end{aligned} \tag{36b}
$$

$$
\sigma_{zx} = \sum_{j=1}^{4} h_4 (1 + P_{1j}) s_j \frac{\partial^2 \psi_j}{\partial x \partial z_j}, \tag{36c}
$$

where

$$
P_{31} = P_{32} = P_{33} = 0, \tag{37a}
$$

and

$$
\begin{aligned}
&(f_1, f_2, h_1, h_2, h_3, h_4) = \\
=& \frac{1}{a_1 T_0} \left(c_{11}, c_{13}, c_{33}, \frac{a_3 c_{11}}{a_1}, \frac{b_3 c_{11}}{b_1}, c_{55} \right).
\end{aligned} \tag{37b}
$$

Substituting the values of σ_{xx}, σ_{zz} and σ_{zx} from equation (36) in equations (10)-(11), with the aid of (13), (15) and (16) gives

$$
\begin{aligned}
f_1 - f_2 s_j^2 P_{1j} + f_1 P_{3j} + f_1 P_{2j} = h_4 (1 + P_{1j}) s_j^2, \\
-f_2 + h_1 s_j^2 P_{1j} - h_2 P_{3j} - h_3 P_{2j} = h_4 (1 + P_{1j}), \\
\left(1 - \varepsilon_3 s_j^2\right) P_{3j} = 0.
\end{aligned} \tag{38}
$$

The general solution (36) with the help of (38) can be simplified as

$$
\begin{aligned}
\sigma_{xx} = - \sum_{j=1}^{4} s_j^2 w_{1j} \frac{\partial^2 \psi_j}{\partial z_j^2}, \qquad \sigma_{zz} = \sum_{j=1}^{4} w_{1j} \frac{\partial^2 \psi_j}{\partial z_j^2}, \\
\sigma_{zx} = \sum_{j=1}^{4} s_j w_{1j} \frac{\partial^2 \psi_j}{\partial x \partial z_j},
\end{aligned} \tag{39}
$$

where

$$
\begin{aligned}
w_{1j} =& \frac{f_1 - f_2 s_j^2 P_{1j} + f_1 P_{3j} + f_1 P_{2j}}{s_j^2} = \\
=& h_4 (1 + \bar{P}_{1j}) = -f_2 + h_1 s_j^2 P_{1j} - h_2 P_{3j} - h_3 P_{3j}.
\end{aligned} \tag{40}
$$

IV. FUNDAMENTAL SOLUTION FOR A POINT HEAT SOURCE IN AN INFINITE ORTHOTROPIC MAGNETOTHERMOELASTIC DIFFUSION MATERIAL

As shown in Fig. 1, we consider an infinite orthotropic magnetothermoelastic diffusion material $z \geq 0$. A point heat source H is applied at the origin and the surface $z = 0$ is free, impermeable and thermally insulated. The complete geometry of the problem is shown in Fig. 1. The general solution given by equations (33) and (39) is derived in this section.

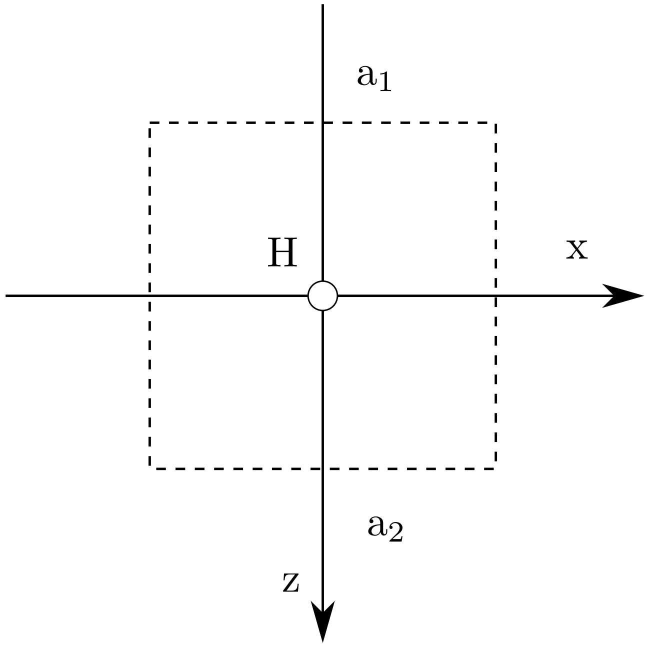

Fig. 1. Geometry of the Problem

The boundary conditions on the surface $z = 0$ are
(i) Mechanical condition

$$\sigma_{zz} = \sigma_{zx} = 0. \tag{41a}$$

(ii) Concentration condition

$$\frac{\partial C}{\partial z} = 0. \tag{41b}$$

(iii) Thermal condition

$$\frac{\partial T}{\partial z} = 0. \tag{41c}$$

Introduce the harmonic functions for magnetothermoelastic diffusion material as

$$\begin{aligned}\psi_j = A_j \Bigg[&\frac{1}{2}(z_j^2 - x^2)\left(\log r_j - \frac{3}{2}\right) \\ &- x z_j \tan^{-1}\frac{x}{z_j}\Bigg], \qquad (j = 1, 2, 3, 4),\end{aligned} \tag{42}$$

where A_j (j=1, 2, 3, 4) are arbitrary constants to be determined and

$$r_j = \sqrt{x^2 + z_j^2}, \qquad (j = 1, 2, 3, 4). \tag{43}$$

Substituting equation (42) in equations (33) and (39), we obtain the components of displacement, mass concentration, temperature change and stress components as follows

$$u = -\sum_{j=1}^{4} A_j \left[x(\log r_j - 1) + z_j \tan^{-1}\frac{x}{z_j} \right], \tag{44a}$$

$$w = \sum_{j=1}^{4} s_j P_{1j} A_j \left[z_j(\log r_j - 1) - x \tan^{-1}\frac{x}{z_j} \right], \tag{44b}$$

$$C = \sum_{j=1}^{4} A_j P_{2j} \log r_j, \tag{44c}$$

$$T = A_4 P_{34} \log r_4, \tag{44d}$$

$$\sigma_{xx} = -\sum_{j=1}^{4} s_j^2 w_{1j} A_j \log r_j, \tag{44e}$$

$$\sigma_{zz} = \sum_{j=1}^{4} w_{1j} A_j \log r_j, \tag{44f}$$

$$\sigma_{zx} = -\sum_{j=1}^{4} s_j w_{1j} A_j \tan^{-1}\frac{x}{z_j}. \tag{44g}$$

Considering the continuity on plane $z = 0$ for w and σ_{zx} yields

$$\sum_{j=1}^{4} s_j P_{1j} A_j = 0, \tag{45}$$

$$\sum_{j=1}^{4} s_j w_{1j} A_j = 0. \tag{46}$$

Substitution the values of w_{1j} from equation (40) in equation (46) gives

$$\sum_{j=1}^{4.} s_j A_j = 0. \tag{47}$$

When the mechanical, concentration and thermal condition for a rectangle of $a_1 \le z \le a_2$ $(0 < a_1 < h < a_2)$ and $-b_1 \le x \le b$ $(b > 0)$ are considered (Fig. 1), three equations can be obtained

$$\int_{-b}^{b} [\sigma_{zz}(x, a_2) - \sigma_{zz}(x, a_1)]\,\mathrm{d}x \tag{48a}$$

$$+ \int_{a_1}^{a_2} [\sigma_{zx}(b, z) - \sigma_{zx}(-b, z)]\,\mathrm{d}z = 0, \tag{48b}$$

$$\int_{-b}^{b} \left[\frac{\partial C}{\partial z}(x, a_2) - \frac{\partial C}{\partial z}(x, a_2) \right] \mathrm{d}x \tag{48c}$$

$$+ \int_{a_1}^{a_2} \left[\frac{\partial C}{\partial x}(b, z) - \frac{\partial C}{\partial x}(-b, z) \right] \mathrm{d}z = 0, \tag{48d}$$

$$-\varepsilon_3 \int_{-b}^{b} \left[\frac{\partial T}{\partial z}(x, a_2) - \frac{\partial T}{\partial z}(x, a_1) \right] \mathrm{d}x \tag{48e}$$

$$- \int_{a_1}^{a_2} \left[\frac{\partial T}{\partial x}(b, z) - \frac{\partial T}{\partial x}(-b, z) \right] \mathrm{d}z = H. \tag{48f}$$

Some useful integrals are given as below

$$\int \tan^{-1}(\frac{x}{z_j})\mathrm{d}z_j = x\log r_j + z_j \tan^{-1}(\frac{x}{z_j}), \tag{49a}$$

$$\int \log r_j \mathrm{d}x = x(\log r_j - 1) + z_j \tan^{-1}(\frac{x}{z_j}), \tag{49b}$$

$$\begin{aligned}\int \frac{\partial T}{\partial z}\mathrm{d}x = s_4 P_{34} A_4 \int \frac{z_4}{r_4^2}\mathrm{d}x = \\ = s_4 P_{34} A_4 \tan^{-1}\frac{x}{z_4},\end{aligned} \tag{49c}$$

$$\begin{aligned}\int \frac{\partial T}{\partial x}\mathrm{d}z = P_{34} A_4 \int \frac{x}{r_4^2}\mathrm{d}z = \\ = -\frac{P_{34}}{s_4} A_4 \tan^{-1}\frac{x}{z_4},\end{aligned} \tag{49d}$$

$$\begin{aligned}\int \frac{\partial C}{\partial z}\mathrm{d}x = \sum_{j=1}^{4} A_j s_j^2 P_{2j} \int \frac{z}{x^2+s_j^2 z^2}\mathrm{d}x = \\ = \sum_{j=1}^{4} A_j s_j P_{2j} \tan^{-1}\frac{x}{s_j z},\end{aligned} \tag{49e}$$

$$\begin{aligned}\int \frac{\partial C}{\partial x}\mathrm{d}z = \sum_{j=1}^{4} A_j P_{2j} \int \frac{x}{x^2+s_j^2 z^2}\mathrm{d}z = \\ = -\sum_{j=1}^{4} \frac{A_j}{s_j} P_{2j} \tan^{-1}\frac{x}{z_j},\end{aligned} \tag{49f}$$

It is noticed that the integrals (49 d, g) are not continuous at $z = 0$, following expression should be used

$$\int_{a_1}^{a_2} \frac{\partial C}{\partial x}\mathrm{d}z = \int_{a_1}^{0^-} \frac{\partial C}{\partial x}\mathrm{d}z + \int_{0^+}^{a_2} \frac{\partial C}{\partial x}\mathrm{d}z, \tag{50a}$$

$$\int_{a_1}^{a_2} \frac{\partial T}{\partial x}\mathrm{d}z = \int_{a_1}^{0^-} \frac{\partial T}{\partial x}\mathrm{d}z + \int_{0^+}^{a_2} \frac{\partial T}{\partial x}\mathrm{d}z. \tag{50b}$$

Substituting equations (44 f, g) in equation (48a) and with the aid of the integrals (49 a, b), we obtain

$$\sum_{j=1}^{4} w_{1j} A_j I_1 = 0, \tag{51}$$

where

$$\begin{aligned} I_1 = &\left[\left(x(\log r_j - 1) + s_j z \tan^{-1}\frac{x}{s_j z}\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b} \\ &- \left[\left(x(\log r_j - 1) + s_j z \tan^{-1}\frac{x}{s_j z}\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=a_2} = 0,\end{aligned} \tag{52}$$

i.e. Equations (48a) and (51) are satisfied automatically. Substituting the value of C from equation (44c) in equation (49b) and using the integrals (49 f, g) and (50 a), we obtain

$$\sum_{j=1}^{4} A_j P_{2j} I_2 = 0, \tag{53}$$

where

$$\begin{aligned} I_2 = &\left[\left(s_j^2 \tan^{-1}\frac{x}{s_j z}\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b} \\ &- \left[\left(\tan^{-1}\frac{x}{s_j z}\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=0^-} \\ &- \left[\left(\tan^{-1}\frac{x}{s_j z}\right)_{x=-b}^{x=b}\right]_{z=0^+}^{z=a_2} = \bar{r}_j\end{aligned} \tag{54}$$

Using equation (54) in equation (53), we obtain

$$\sum_{j=1}^{4} \bar{r}_j A_j P_{2j} = 0. \tag{55}$$

Substituting equation (44d) in equation (48c) and using the integrals (49 c,d) and with the aid of equation (50b) and $s_4 = \sqrt{\frac{K_1}{K_3}}$ on the resulting equation, we obtain

$$A_4 I_3 = \frac{H}{P_{34}\sqrt{K_3/K_1}}, \tag{56}$$

where

$$\begin{aligned} I_2 = &-\left[\left(\tan^{-1}(\frac{x}{s_4 z})\right)_{z=a_1}^{z=a_2}\right]_{x=-b}^{x=b} \\ &+ \left[\left(\tan^{-1}(\frac{x}{s_4 z})\right)_{x=-b}^{x=b}\right]_{z=a_1}^{z=0^-} \\ &+ \left[\left(\tan^{-1}(\frac{x}{s_4 z})\right)_{x=-b}^{x=b}\right]_{z=0^+ a_1}^{z=a_2} = -2\pi.\end{aligned} \tag{57}$$

A_4 can be determined from equation (56) and (57), as follows

$$A_4 = -\frac{H}{2\pi P_{34}\sqrt{K_3/K_1}}. \tag{58}$$

We have determined four constants A_j $(j = 1, 2, 3, 4)$ from four equations including equations (45), (47), (55) and (58) by the method of Cramer rule.

V. FUNDAMENTAL SOLUTION FOR A POINT HEAT SOURCE IN A SEMI-INFINITE ORTHOTROPIC MAGNETOTHERMOELASTIC DIFFUSION MATERIAL

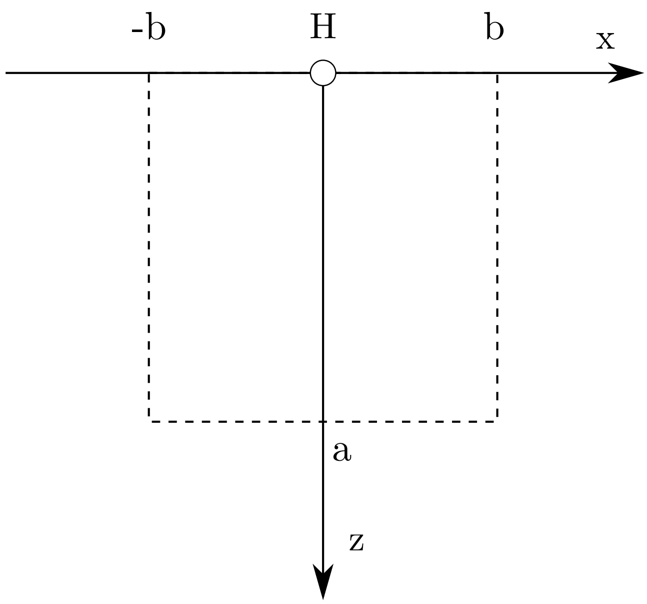

Fig. 2. Geometry of the Problem

As shown in Fig. 2, we consider a semi-infinite orthotropic magnetothermoelastic diffusion material $z \geq 0$. A point heat source H is applied at the origin and the surface $z = 0$ is free, impermeable boundary and thermally insulated. The complete geometry of the problem is shown in Fig. 2. The general solution given by equations (33) and (39) is derived in this section.

The boundary conditions on the surface $z = 0$ are

$$\sigma_{zz} = \sigma_{zx} = 0, \qquad \frac{\partial C}{\partial z} = 0, \qquad \frac{\partial T}{\partial z} = 0. \tag{59}$$

When the mechanical, concentration and thermal condition for a rectangle of $0 \leq z \leq a$ and $-b \leq x \leq b$ $(b > 0)$ are considered (Fig. 2), three equations can be obtained

$$\int_{-b}^{b} \sigma_{zz}(x,a)\mathrm{d}x + \int_{0}^{a} \left[\sigma_{zx}(b,z) - \sigma_{zx}(-b,z)\right] \mathrm{d}z = 0, \tag{60a}$$

$$\int_{-b}^{b} \frac{\partial C}{\partial z}(x,a)\mathrm{d}x + \int_{0}^{a} \left[\frac{\partial C}{\partial x}(b,z) - \frac{\partial C}{\partial x}(-b,z)\right] \mathrm{d}z = 0, \tag{60b}$$

$$\begin{aligned} &-\varepsilon_3 \int_{-b}^{b} \left[\frac{\partial T}{\partial z}(x,a)\right] \mathrm{d}x \\ &- \int_{a_1}^{a_2} \left[\frac{\partial T}{\partial x}(b,z) - \frac{\partial T}{\partial x}(-b,z)\right] \mathrm{d}z = H. \end{aligned} \tag{60c}$$

The harmonic function given by equation (42) and expression for components of displacement, temperature distribution, mass concentration and stress components as given by equations (44a)-(44g) are the same, whereas the constants A_j $(j = 1,2,3,4)$ which are determined by boundary conditions (59) and equilibrium equations (50) are different. By substituting the values of $\sigma_{zz}, \sigma_{zx}, C$ and T from equations (44 c,d,f,g) in equation (59), we obtain

$$\sum_{j=1}^{4} w_{1j} A_j = 0, \tag{61a}$$

$$\sum_{j=1}^{4} s_j w_{1j} A_j = 0, \tag{61b}$$

$$\sum_{j=1}^{4} \frac{z s_j^2 p_{2j}}{x^2 + z_j^2} A_j = 0. \tag{61c}$$

Equation (61 c) shows that $\frac{\partial C}{\partial z}$ is satisfied automatically at the surface $z = 0$ and similarly $\frac{\partial T}{\partial z}$ is also satisfied automatically at the surface $z = 0$. Making use of the values of σ_{zz}, σ_{zx} from equations (44 f, g) in equation (60 a) and using the integrals (49 a,b), we obtain

$$\sum_{j=1}^{4} w_{1j} A_j I_{4,} = 0, \tag{62}$$

where

$$\begin{aligned} I_3 = &\left[x\left(\log\sqrt{x^2 + s_j^2 a^2} - 1\right) + s_j a \tan^{-1}\frac{x}{s_j a}\right]_{x=-b}^{x=b} \\ &-2\left[z_j \tan^{-1}\frac{b}{s_j z} + b\log\sqrt{b^2 + s_j^2 z^2}\right]_{z=0}^{z=a} = \\ &=2b(\log b - 1). \end{aligned} \tag{63}$$

By virtue of the equation (63), the equation (62) degenerates to equation (61a) i.e. equations (60a) and (62) are satisfied automatically.

$$A_4 I_5 = \frac{H}{P_{34}\sqrt{K_3/K_1}}, \tag{64}$$

$$\begin{aligned} I_5 = &-\left[\tan^{-1}(\frac{x}{s_4 a})\right]_{x=-b}^{x=b} \\ &+\left[\tan^{-1}(\frac{b}{s_4 z})\right]_{z=0_1}^{z=a} = -\pi. \end{aligned} \tag{65}$$

A_4 can be determined from equation (64) and (65), as follows

$$A_4 = -\frac{H}{\pi P_{34}\sqrt{K_3/K_1}}. \quad (66)$$

Substituting the value of C from equation (44c) in equation (60 b) and using the integrals (49 f, g), we obtain

$$\sum_{j=1}^{4} r_j A_j P_{2j} = 0, \quad (67)$$

where

$$r_j = \left[s_j^2 \tan^{-1}\left(\frac{x}{s_j a}\right)\right]_{x=-b}^{x=b} - \left[\tan^{-1}\left(\frac{b}{s_j z}\right) - \tan^{-1}\left(\frac{-b}{s_j z}\right)\right]_{z=0}^{z=a}.$$

We have determined four constants A_j $(j = 1, 2, 3, 4)$ from four equations, including equations (61 a), (61 b), (66) and (67) by the method of Cramer's rule.

VI. SPECIAL CASES

(I) In the absence of magnetic effect, equations (44a)-(44f) reduces to

$$u = -\sum_{j=1}^{4} A_j \left[x(\log r_j - 1) + z_j \tan^{-1}\frac{x}{z_j}\right], \quad (68a)$$

$$w = \sum_{j=1}^{4} s_j P_{1j} A_j \left[z_j(\log r_j - 1) - x \tan^{-1}\frac{x}{z_j}\right], \quad (68b)$$

$$C = \sum_{j=1}^{4} A_j P_{2j} \log r_j, \quad (68c)$$

$$T = A_4 P_{34} \log r_4, \quad (68d)$$

$$\sigma_{xx} = -\sum_{j=1}^{4} s_j^2 w_{1j} A_j \log r_j, \quad (68e)$$

$$\sigma_{zz} = \sum_{j=1}^{4} w_{1j} A_j \log r_j, \quad (68f)$$

$$\sigma_{zx} = -\sum_{j=1}^{4} s_j w_{1j} A_j \tan^{-1}\frac{x}{z_j}. \quad (68g)$$

In this case s_j $(j = 1, 2, 3)$ are the roots of the (28) in the absence of magnetic effect. Which are similar to the results as those obtained by Kumar and Chawla [24].

(II) In the absence of magnetic and diffusion effect, equations (44a)-(44f) reduces to

$$u = -\sum_{j=1}^{3} A_j \left[x(\log r_j - 1) + z_j \tan^{-1}\frac{x}{z_j}\right], \quad (69a)$$

$$w = \sum_{j=1}^{3} s_j P_{1j} A_j \left[z_j(\log r_j - 1) - x \tan^{-1}\frac{x}{z_j}\right], \quad (69b)$$

$$T = A_4 P_{34} \log r_4, \quad (69c)$$

$$\sigma_{xx} = -\sum_{j=1}^{3} s_j^2 w_{1j} A_j \log r_j, \quad (69d)$$

$$\sigma_{zz} = \sum_{j=1}^{3} w_{1j} A_j \log r_j, \quad (69e)$$

$$\sigma_{zx} = -\sum_{j=1}^{3} s_j w_{1j} A_j \tan^{-1}\frac{x}{z_j}. \quad (69f)$$

In this case s_j $(j = 1, 2)$ are the roots of the (28) in the absence of magnetic and diffusion effect. The above results are similar as obtained by Hou et. al. [13].

VII. NUMERICAL RESULTS AND DISSCUSSION

For the purpose of numerical computation, we take the following values of the relevant parameters as

$$c_{11} = 18.78 \times 10^{10}\ \mathrm{kg\,m^{-1}\,s^{-2}},$$

$$c_{13} = 8.0 \times 10^{10}\ \mathrm{kg\,m^{-1}\,s^{-2}},$$

$$c_{33} = 10.2 \times 10^{10}\ \mathrm{kg\,m^{-1}\,s^{-2}},$$

$$c_{55} = 10.06 \times 10^{10}\ \mathrm{kg\,m^{-1}\,s^{-2}},$$

$$T_0 = 0.293 \times 10^3\ \mathrm{K},$$

$$\alpha_1 = 1.96 \times 10^{-5}\ \mathrm{K^{-1}},$$

$$\alpha_3 = 1.4 \times 10^{-5}\ \mathrm{K^{-1}},$$

$$\alpha_{1c} = 1.1 \times 10^{-4}\ \mathrm{m^3\,kg^{-1}},$$

$$\alpha_{3c} = 0.43 \times 10^{-4}\ \mathrm{m^3\,kg^{-1}},$$

$$K_1 = 0.12 \times 10^3\ \mathrm{W\,m^{-1}\,K^{-1}},$$

$$K_3 = 0.33 \times 10^3\ \mathrm{W\,m^{-1}\,K^{-1}},$$

$$a = 1.4 \times 10^4\ \mathrm{m^2\,s^{-2}\,K^{-1}},$$

$$b = 9 \times 10^5\ \mathrm{kg^{-1}\,m^5\ s^{-2}},$$

$$\alpha_1^* = 0.58 \times 10^{-8}\ \mathrm{m^{-3}\,s\,kg},$$

$$\alpha_3^* = 0.52 \times 10^{-8}\ \mathrm{m^{-3}\,s\,kg},$$

$$H_0 = 0.38,$$

$$\mu_0 = 1.$$

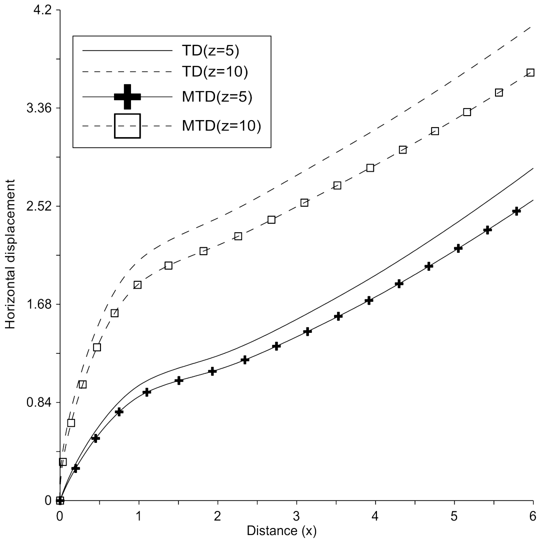

Fig. 3. Variation of horizontal displacement (u) w.r.t. x (for infinite plane)

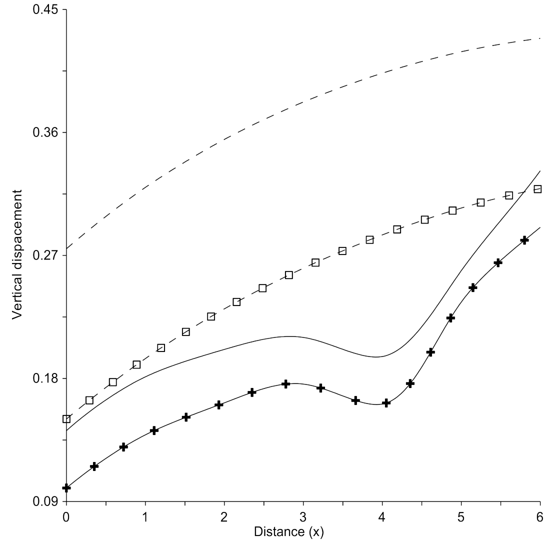

Fig. 4. Variation of vertical dispacement (w) w.r.t. x (for infinite plane)

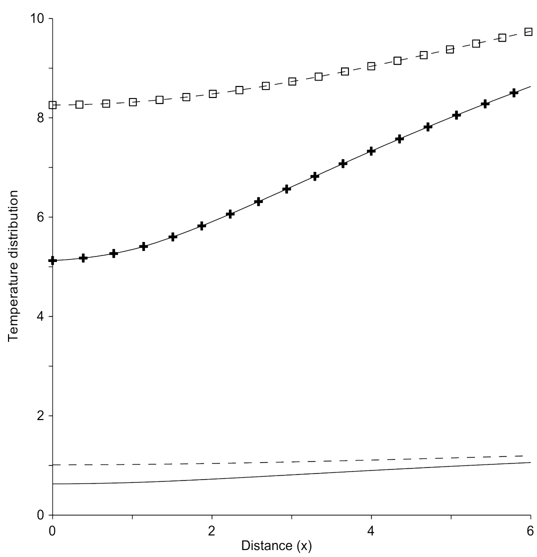

Fig. 5. Variation of temperature distribution (T) w.r.t. x (for infinite plane)

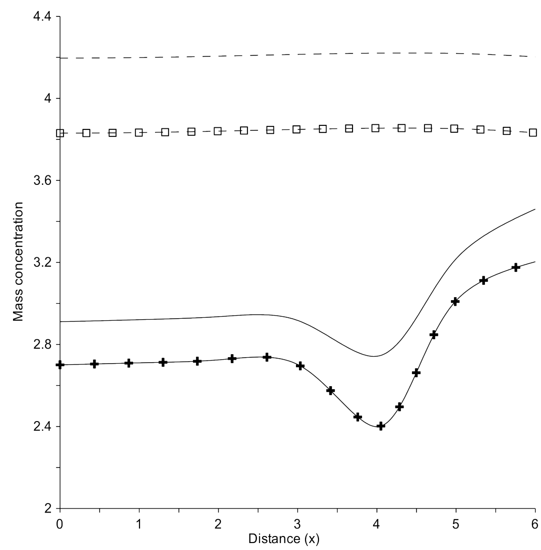

Fig. 6. Variation of mass concentration w.r.t. x (for infinite plane)

Figs. 3-6 depict the variation of components of displacement, temperature distribution and mass concentration for infinite magnetothermoelastic diffusion material and Figs. 7-10 exhibit the variation of components of displacement, temperature distribution and mass concentration for semi-infinite magnetothermoelastic diffusion material. The solid lines and dotted lines correspond to thermoelastic diffusion material (TD) and center symbol on these lines correspond to magnetothermoelastic diffusion material (MTD).

Fig. 3 depicts the variation of horizontal displacement u with x. It is noticed that the values of u increase for smaller values of x, but for higher values of x, the values of u increase monotonically. It is evident that the values of u in case of TD ($z = 5$) remain more (in comparison with MTD ($z = 5$)) and similar behavior is observed for the cases of TD ($z = 5$) and MTD ($z = 10$). Fig. 4 depicts the variation of vertical displacement (w) with x. It indicates that the values of w increase for the cases of TD and MTD ($z = 10$), although for the cases of TD, MTD ($z = 5$), the values of w remain oscillatory for smaller values of x, but for higher values of x, the values of w increase monotonically. It is observed that the values of w in case of TD ($z = 10$) remain more in comparison with others.

Fig. 5 exhibits the variation of temperature distribution (T) with x. It is noticed that the values of T for both cases TD and MTD increase for all values of x. It is evident that the values of T in case of MTD remain more in comparison with TD. Fig. 6 depicts the variation of mass concentration (C) with x, and it indicates that for the cases of TD and MTD ($z = 10$), the values of C decrease slightly, although for the case of TD and MTD ($z = 5$) the values of C oscillates for smaller values of x, but for higher values of x, the values of C increase. It is evident that the values of C for TD ($z = 10$) remain more in comparison with others.

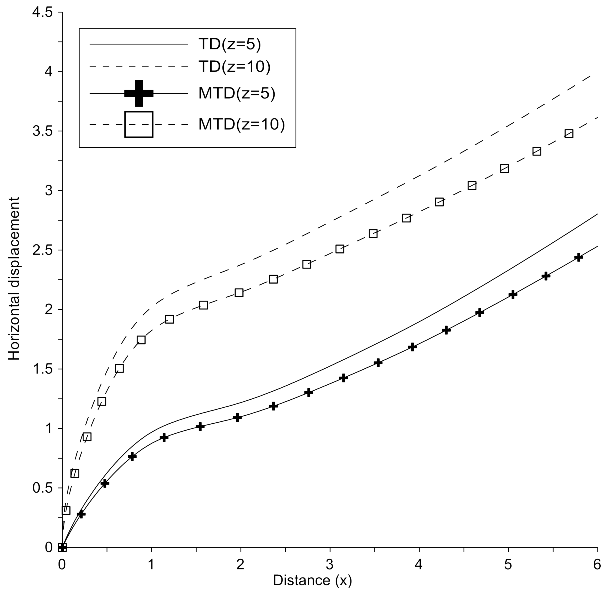

Fig. 7. Variation of horizontal displacement w.r.t. x (for semi-infinite plane)

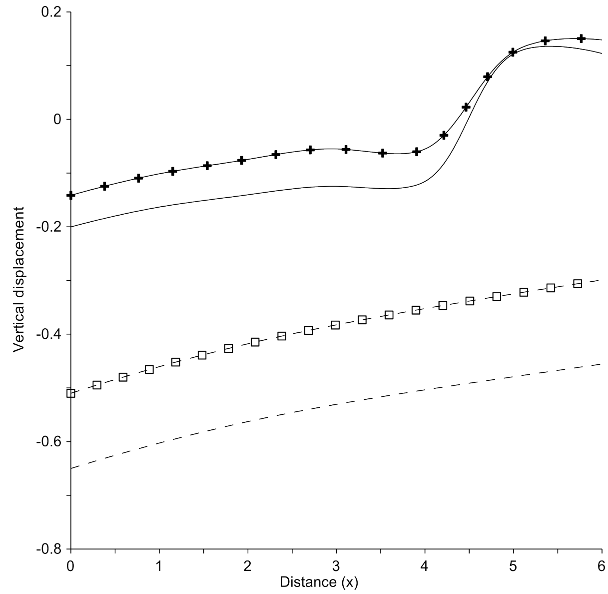

Fig. 8. Varition of Vertical displacement (w) w.r.t. x (for semi-infinite plane)

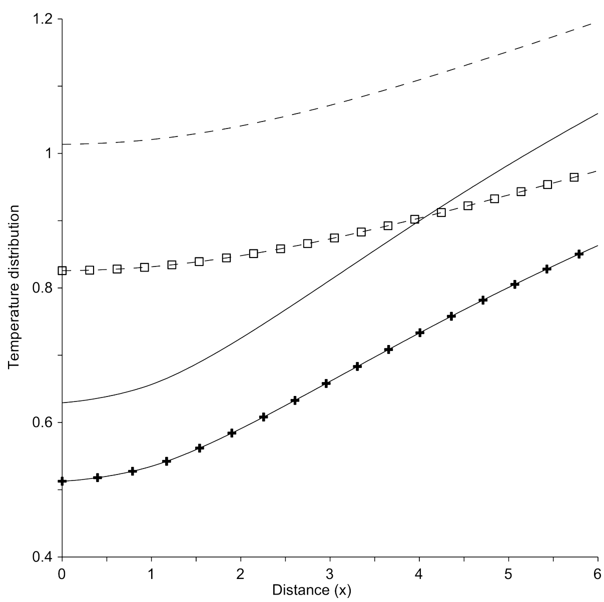

Fig. 9. Variation of temperature distribution (T) w.r.t. x (for semi-infinite plane)

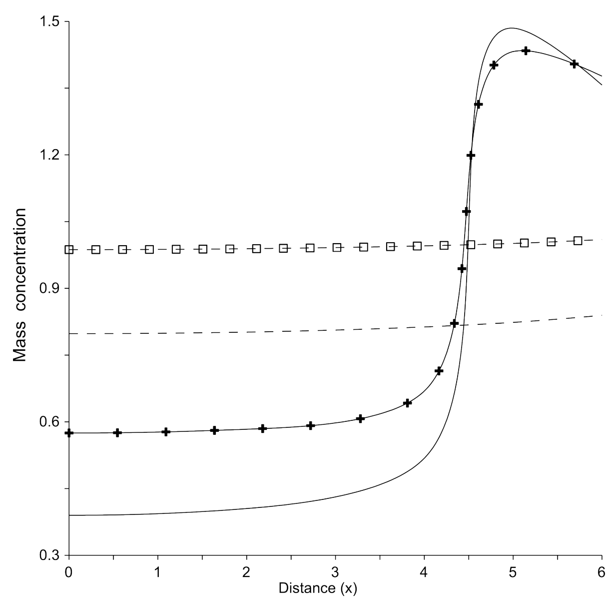

Fig. 10. Variation of mass concentration (C) w.r.t. x (for semi-infinite plane)

Fig. 7 exhibits the variation of horizontal displacement (u) with x. It is evident that the behavior and variation of u for the semi-infinite magnetothermoelastic diffusion material is similar as for the infinite magnetothermoelastic diffusion material, although the magnitude values of u in case of semi-infinite magnetothermoelastic diffusion material are different from the infinite magnetothermoelastic diffusion material. Fig. 8 depicts the variation of vertical displacement

(w) with x, and it shows that the values of w, for the cases of cases TD($z = 5$), MTD ($z = 5$) increase for all values of x, whereas for the cases of TD and MTD ($z = 10$), the values of w increase for smaller values of x, but for higher values of x, the values of w remain oscillatory. It is evident that the values of w in case of MTD ($z = 5$) and MTD ($z = 10$) remain more (in comparison with TD ($z = 5$) and TD ($z = 10$)).

Fig. 9 depicts the variation of temperature distribution (T) with x, and it indicates that, for both cases TD and MTD, the values of T increase. It is noticed that the values of T in case of TD ($z = 10$) remain more in comparison with TD ($z = 5$) and MTD ($z = 5$,$z = 10$). Fig. 10 depicts the variation of mass concentration (C) with x, and it shows that the values of C slightly increase for the case of TD, MTD ($z = 10$) whereas for the case of TD, MTD ($z = 5$), the values of C remain oscillatory.

VIII. CONCLUDING REMARKS

The fundamental solution for two-dimensional in orthotropic magnetothermoelastic diffusion material has been derived. With this objective, the two-dimensional general solution in magnetothermoelastic diffusion material is derived firstly for the case of distinct roots. By virtue of the two-dimensional general solution of orthotropic magnetothermoelastic diffusion material, the fundamental solution for a steady point heat source acting in an infinite plane and on the surface of a semi-infinite plane are obtained by four newly introduced harmonic functions $\psi_j(j = 1, 2, 3, 4)$. The general expression for components of displacement, stress, mass concentration and temperature change are expressed in terms of elementary functions. Since all the components are expressed in terms of elementary functions, it is convenient to use them. From the present investigation, some special cases of interest are also deduced. The components of displacement, mass concentration and temperature distribution are computed numerically and depicted graphically to depict the magnetic effect.

From the numerical results we conclude that, due to the magnetic effect, the values of horizontal displacement (u), vertical displacement (w) and mass concentration (C) become smaller, although for the values to T become higher due to the magnetic effect. In case of the semi-infinite plane it is noticed that, due to themagnetic effect, the values of horizontal displacement (u) and temperature change (T) become smaller, whereas the values of mass concentration (C) become higher due to the magnetic effect. Appreciable magnetic effect is observed on components of displacement, temperature distribution and mass concentration for the infinite magnetothermoelastic diffusion material and semi-infinite magnetothermoelastic diffusion material.

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