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A Two-Stage Group Sampling Plan Based on Truncated Life Tests for Exponentiated Half Logistic Distribution

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Abstract: When the life time of a product follows exponentiated half logistic distribution we can recommend a two-stage group acceptance sampling plan for truncated life tests. The acceptance of the lot can be done at the first or second stage based on the number of failures in each group. In this paper, we obtained the number of groups essential for each of two stages for the underlying lifetime distribution so as to minimize the average sample number under the constraints of satisfying the producer's and consumer's risks simultaneously. Single-stage group sampling plans are also considered as special cases of the stated plan and compared with the proposed plan in terms of the average sample number and the operating characteristics.

Key words: exponentiated half logistic distribution, average sample number, consumer's risk, operating characteristics, producer's risk

ANNOTATIONS

- n_1 – sample size in stage 1,
- n_2 – sample size in stage 2,
- k_1 – number of testers in stage 1,
- k_2 – number of testers in stage 2,
- m – number of items assigned to the tester for testing at time t_0,
- c_1 – number of allowable defective items in the lot from stage 1,
- c_2 – number of allowable defective items in the lot from stage 2,
- P_{a}^1 – probability of accepting the lot in stage 1,
- P_{a}^2 – probability of accepting the lot in stage 2,
- P_{r}^1 – probability of rejecting the lot in stage 1,
- t_q^0 – targeted percentile at test termination time t_0,
- p – probability that a failure arises during termination time t_0,
- p_1 – failure probability corresponding to the consumer's risk,
- p_2 – failure probability corresponding to the producer's risk,
- $L(p)$ – probability that the acceptance of the lot in the proposed two-stage group sampling plan,
- $L(p_1)$ – lot acceptance probability corresponding to consumer's risk,
- $L(p_2)$ – lot acceptance probability corresponding to producer's risk,
- α – producer's risk,
- β – consumer's risk.

I. INTRODUCTION

A commonly used method in quality control is acceptance sampling. A lot will be accepted if the number of failures does not exceed the acceptance number during the test time. The purpose is to conclude about the quality of lot or group of items from a sample. We can accept or reject the entire lot depending on what is found in the sample and the rejected lots can then be scuffled and re-used. Several authors suggested the acceptance sampling plans based on truncated life tests including [1–15].

Usually a tester can test a single product in these plans but in real-world the multiple items may be fitted at a time by the tester. Therefore, a group acceptance sampling plan must be designed and used and this plan is more required than the normal sampling plan in terms of the test time since the previous plan can test more items in a specified test time. In group acceptance sampling plans a simple decision rule is to accept the lot when the number of failures does not exceed the given number from each group. In sudden death testing the group acceptance sampling plans are very useful and these plans are suggested by several authors including [16–22].

In various applications in order to encounter the engineering design purpose the lowest percentile of lifetime is needed. In Refs. [23–26] and lately in [12] the acceptance sampling plans for the lifetime percentiles were developed. Almost all of the above discussed that the average lifetime of an item may not satisfy the constraints of an engineering design. A lot with smaller percentile value than the specified percentile value might be permitted by the acceptance sampling plans. Furthermore, many of the working life distributions are asymmetric. According to [27], the average life might not be sufficient to define the central tendency of the skewed distribution. Moreover, the exponentiated half logistic distribution is not symmetrical distribution; therefore, we need to study the two-stage grouped sampling plans based on percentiles. As far as we know, in the literature the two-stage group sampling plans based on the exponentiated half logistic distribution using percentiles have not been established. Here the objective is to develop the two-stage group sampling plans for the exponentiated half logistic distribution percentiles.

The so far discussed acceptance sampling plans were established for a single sampling plan with respect to the size of a sample. In fact, a double sampling plan works better than a single sampling plan. Hence, Aslam et al. [28] truly said that there is a necessity of evolving a type of a double sampling plan for a life test with groups which are named as a two-stage group sampling plan. The size of the sample will certainly be further reduced by the two-stage group sampling plan. Recently, Aslam et al. [28] studied a time truncated two-stage group sampling plan for Weibull distribution. Aslam et al. [29] developed a two-stage group acceptance sampling plan based on truncated life tests for a general distribution. Rao [30] formulated a two-stage group sampling plan based on truncated life tests for an M-O extended exponential distribution. Rao et al. [31, 32] established a two-stage group sampling plan based on truncated life tests for an exponentiated Frechet distribution. Prasad et al. [33] studied a two stage group sampling plans based on truncated life tests for a Type II generalized log-logistic distribution. Rao et al. [34] developed a time-truncated two-stage group acceptance sampling plan for an odds exponential log-logistic distribution. Sivakumar et al. [35] developed a two-stage group sampling plan based on truncated life tests for an odd generalized exponential log-logistic distribution.

The persistence of this article is to improve a two-stage group sampling plan for the truncated life tests if the lifetime of an item follows the exponentiated half logistic distribution established by Cordeiro et al. [36].

The probability density function (p.d.f.) and cumulative distribution function (c.d.f) of the exponentiated half logistic distribution, respectively, are given by:

$$f\left(t;\,\nu,\,\sigma\right)=\frac{2\nu\left(1-e^{-t/\sigma}\right)^{\nu-1}e^{-t/\sigma}}{\sigma\left(1-e^{-t/\sigma}\right)^{\nu+1}}\ ,\ t\geq 0\,, \tag{1}$$

$$F(t;\ \nu,\ \sigma)=\left[F(t;\,\sigma)\right]^{\,\nu}=\left(\frac{1-e^{-t/\sigma}}{1+e^{-t/\sigma}}\right)^{\nu}\ ,\ t\geq 0\,, \tag{2}$$

where σ is the scale parameter and ν is shape parameter.

The $100q$-th percentile of the exponentiated half logistic distribution is given by

$$t_q=\sigma\eta_q\quad\text{where}\qquad \eta_q=\left[\ln\left(\frac{1+q^{1/v}}{1-q^{1/v}}\right)\right]\,. \tag{3}$$

In this article, for truncated life tests the two-stage group sampling plan will be proposed if the life time of an item follows an exponentiated half logistic distribution. To minimize the mean sample number under the limitations of satisfying the producer's and consumer's risks we need to build the tables for obtaining the number of groups necessary in every step of the proposed plan simultaneously. The proposal of a two-stage plan is given in Sec. 2. The comparison with the single-stage group sampling plan is given in Sec. 3. The procedure is explained with industrial application in Sec. 4 and some inferences are given in Sec. 5.

II. TWO-STAGE GROUP SAMPLING PLAN

Here we establish a two-stage group sampling plans for exponentiated half logistic distribution established. The authors suggested ensuring a two-stage group sampling plan such that the assumption of each group size "m" is prefixed for a tester.

Stage 1. At this stage select a random sample of size n_1 from the lot, then each of k_1 groups (or testers) will be assigned m items so that $n_1 = mk_1$ and set them for testing at time t_0. If the number of failures in each group is c_1 or less then the lot will be accepted. The test will be terminated and the lot will be rejected if the number of failures in any group is greater than c_2 before time t_0. Otherwise go to stage 2.

Stage 2. At this stage select the random sample of size n_2 from the lot, then each of k_2 groups (or testers) will be assigned m items so that $n_2 = mk_2$ and set them for testing at time t_0. If the number of failures in each group is c_1 or less then the lot will be accepted. The test will be terminated and the lot will be rejected if the number of failures in any group is greater than c_1 before time t_0.

The various sampling plans are related to the current group sampling plan. If $m = 1$ the current sampling plan is a case of a double sampling plan. The current two-stage group sampling plan will become a single-stage group sampling plan in case of $c_1 = c_2$. The number of groups from stage 1 and stage 2 are the design parameters of the current sampling plan. The acceptance numbers of c_1 and c_2 can be determined as well, but the average sample number (ASN) is minimizing under the sampling plan in case of $c_1 = 0, c_2 = 1$. The consumers usually prefer the lower acceptance numbers, so that the two-stage sampling plan with $c_1 = 0, c_2 = 1$ is very useful. The probability of accepting the lot from stage 1 in the two-stage group sampling plan is given by

$$P_{\mathrm{a}}^{(1)} = \sum_{i=0}^{c_1} \begin{pmatrix} mk_1 \\ i \end{pmatrix} p^i \, (1-p)^{mk_1 - i} \,, \tag{4}$$

where p is the probability of an item which fails in a group in time t_0. It will be useful to obtain the test termination time t_0 as a number of the stated percentile t_q^0 such that $t_0 = \delta_q^0 t_q^0$ for a constant δ_q^0. Then the probability that a failure arises during the termination time t_0 denoted as $p = F(t_0\,;\,\nu,\ \sigma)$, and is given by

$$p = \left[\frac{1 - e^{-\left(\frac{\eta_q \delta_q^0}{(t_q/t_q^0)}\right)}}{1 + e^{-\left(\frac{\eta_q \delta_q^0}{(t_q/t_q^0)}\right)}} \right]^{\nu} . \tag{5}$$

The probability that the lot will be rejected in stage 1 is given by

$$P_{\mathrm{r}}^{(1)} = 1 - \sum_{i=0}^{c_2} \begin{pmatrix} mk_1 \\ i \end{pmatrix} p^i \, (1-p)^{mk_1 - i} . \tag{6}$$

The probability that the lot will be accepted in stage 2 is given by

$$P_{\mathrm{a}}^{(2)} = \left[1 - \left(P_{\mathrm{a}}^{(1)} + P_{\mathrm{r}}^{(1)}\right)\right] \left[\sum_{i=0}^{c_1} \begin{pmatrix} mk_2 \\ i \end{pmatrix} p^i (1-p)^{mk_2 - i} \right] . \tag{7}$$

Then, the probability that the acceptance of the lot in the recommended two-stage group sampling plan is given by

$$L(p) = P_{\mathrm{a}}^{(1)} + P_{\mathrm{a}}^{(2)} . \tag{8}$$

But, the probability that the lot will be accepted if $c_1 = 0,\ c_2 = 1$, is given by

$$L(p) = (1-p)^{mk_1} + mk_1 p \, (1-p)^{mk_1 - 1} (1-p)^{mk_2} . \tag{9}$$

The two-point approach of obtaining the proposal constraints if the quality level based on the percentile ratio t_q/t_q^0 between the true percentile t_q and the target percentile t_q^0 is to obtain the least number of groups, k_1 and k_2, to satisfy the following two inequalities:

$$L\left(p/(t_q/t_q^0) = \delta_1\right) \le \beta \,, \tag{10}$$

$$L\left(p/(t_q/t_q^0) = \delta_2\right) \ge 1 - \alpha \,, \tag{11}$$

where, δ_1 is the percentile ratio at the consumer's risk and δ_2 is the percentile ratio at the producer's risk. In this study, the ratio δ_1 is set as 1. Let p_1 and p_2 are the failure probabilities of corresponding to consumer's and producer's risks, respectively:

$$p_1 = \left[\frac{1 - e^{-\eta_q \delta_q^0}}{1 + e^{-\eta_q \delta_q^0}} \right]^{\nu} \text{ and } p_2 = \left[\frac{1 - e^{-\left(\frac{\eta_q \delta_q^0}{(t_q/t_q^0)}\right)}}{1 + e^{-\left(\frac{\eta_q \delta_q^0}{(t_q/t_q^0)}\right)}} \right]^{\nu} . \tag{12}$$

Many solutions of the proposal constraints satisfying the Eqs. (10) and (11) may exist; therefore we must select to minimize the ASN for the two-stage group sampling plan. The ASN for the two-stage sampling plan can be determined as

$$\mathrm{ASN} = mk_1 + mk_2 \left(1 - P_{\mathrm{a}}^{(1)} - P_{\mathrm{r}}^{(1)}\right) \,, \tag{13}$$

where $P_{\mathrm{a}}^{(1)}$ and $P_{\mathrm{r}}^{(1)}$ will be estimated at $p = p_2$. So, the proposal constraints for the recommended two-stage group sampling plan can be determined by the result from the following optimization:

$$\text{Minimize } \ \mathrm{ASN}(p_2) = mk_1 + mk_2 \left(1 - P_{\mathrm{a}}^{(1)} - P_{\mathrm{r}}^{(1)}\right), \tag{14a}$$

subject to:

$$L\left(p_1\right) \le \beta, \tag{14b}$$

$$L\left(p_2\right) \ge 1 - \alpha, \tag{14c}$$

$$1 \le k_2 \le k_1 \,, \tag{14d}$$

$$0 \le c_1 < c_2 \,, \tag{14e}$$

$$k_1,\ k_2,\ c_1,\ c_2 \quad \text{are integers.} \tag{14f}$$

If the number of groups in stage 2 is bigger than in stage 1, the constraint (14d) are not necessary.

Tab. 1. The minimum number of groups required in the two-stage sampling plan for EHLD with $\nu = 2$ for 25^{th} percentile. The cells with hyphens (–) indicate that parameters cannot be found to satisfy the conditions

β	t_q/t_q^0	$\delta_q = 0.5,\ m = 5$				$\delta_q = 1.0,\ m = 5$				$\delta_q = 0.5,\ m = 10$				$\delta_q = 1.0,\ m = 10$			
		k_1	k_2	ASN	$L(p_1)$	k_1	k_2	ASN	$L(p_1)$	k_1	k_2	ASN	$L(p_1)$	k_1	k_2	ASN	$L(p_1)$
0.25	4	7	1	35.7	0.9849	2	1	10.8	0.9718	3	1	31.2	0.9855	1	1	11.6	0.9589
	6	7	1	35.3	0.9968	2	1	10.4	0.9938	3	1	30.6	0.9969	1	1	10.8	0.9908
	8	7	1	35.2	0.9990	2	2	10.5	0.9970	3	1	30.3	0.9990	1	1	10.5	0.9970
	10	7	1	35.1	0.9996	2	1	10.1	0.9992	3	1	30.2	0.9996	1	1	10.3	0.9987
0.10	4	10	1	50.9	0.9723	–	–	–	–	5	1	51.9	0.9681	1	1	11.6	0.9589
	6	10	1	50.5	0.9940	3	1	15.6	0.9887	5	1	50.9	0.9930	1	1	10.8	0.9908
	8	10	1	50.3	0.9980	3	1	15.3	0.9962	5	1	50.6	0.9977	1	1	10.5	0.9970
	10	10	1	50.2	0.9992	3	1	15.2	0.9984	5	1	50.4	0.9990	1	1	10.3	0.9987
0.05	4	12	1	61.1	0.9624	–	–	–	–	6	1	62.1	0.9575	–	–	–	–
	6	12	1	60.6	0.9917	3	1	15.6	0.9887	6	1	61.1	0.9906	2	1	21.4	0.9766
	8	12	1	60.3	0.9973	3	1	15.3	0.9962	6	1	60.7	0.9969	2	1	20.9	0.9921
	10	12	1	60.2	0.9989	3	1	15.2	0.9984	6	1	60.4	0.9987	2	1	20.6	0.9966
0.01	4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	6	17	1	85.7	0.9845	4	1	20.7	0.9822	9	1	91.6	0.9812	2	1	21.4	0.9766
	8	17	1	85.5	0.9948	4	1	20.4	0.9940	9	1	91.0	0.9937	2	1	20.9	0.9921
	10	17	1	85.3	0.9978	4	1	20.3	0.9975	9	1	90.6	0.9973	2	1	20.6	0.9966

Tab. 2. The minimum number of groups required in the two-stage sampling plan for EHLD with $\nu = 3$ for 25^{th} percentile. The cells with hyphens (–) indicate that parameters cannot be found to satisfy the conditions

β	t_q/t_q^0	$\delta_q = 0.5,\ m = 5$				$\delta_q = 1.0,\ m = 5$				$\delta_q = 0.5,\ m = 10$				$\delta_q = 1.0,\ m = 10$			
		k_1	k_2	ASN	$L(p_1)$	k_1	k_2	ASN	$L(p_1)$	k_1	k_2	ASN	$L(p_1)$	k_1	k_2	ASN	$L(p_1)$
0.25	4	11	1	55.2	0.9989	2	1	10.2	0.9966	6	1	60.4	0.9986	1	1	10.5	0.9949
	6	11	1	55.0	0.9999	2	1	10.0	0.9997	6	1	60.1	0.9999	1	1	10.1	0.9995
	8	11	1	55.0	1.0000	2	1	10.0	0.9999	6	1	60.1	1.0000	1	1	10.1	0.9999
	10	11	1	55.0	1.0000	2	1	10.0	1.0000	6	1	60.0	1.0000	1	1	10.0	1.0000
0.10	4	17	1	85.3	0.9976	3	1	15.4	0.9937	8	1	80.5	0.9976	1	1	10.5	0.9949
	6	17	1	85.1	0.9998	3	1	15.1	0.9994	8	1	80.1	0.9998	1	1	10.1	0.9995
	8	17	1	85.0	1.0000	3	1	15.1	0.9999	8	1	80.1	1.0000	1	1	10.1	0.9999
	10	17	1	85.0	1.0000	3	1	15.0	1.0000	8	1	80.0	1.0000	1	1	10.0	1.0000
0.05	4	20	1	100.3	0.9968	3	1	15.4	0.9937	10	1	100.7	0.9965	2	1	21.1	0.9868
	6	20	1	100.1	0.9997	3	1	15.1	0.9994	10	1	100.2	0.9997	2	1	20.3	0.9987
	8	20	1	100.0	0.9999	3	1	15.1	0.9999	10	1	100.1	0.9999	2	1	20.1	0.9998
	10	20	1	100.0	1.0000	3	1	15.0	1.0000	10	1	100.1	1.0000	2	1	20.1	0.9999
0.01	4	29	1	145.5	0.9936	4	1	20.5	0.9900	14	1	140.9	0.9936	2	1	21.1	0.9868
	6	29	1	145.2	0.9994	4	1	20.1	0.9990	14	1	140.3	0.9994	2	1	20.3	0.9987
	8	29	1	145.1	0.9999	4	1	20.1	0.9998	14	1	140.1	0.9999	2	1	20.1	0.9998
	10	29	1	145.0	1.0000	4	1	20.0	1.0000	14	1	140.1	1.0000	2	1	20.1	0.9999

Tab. 3. The minimum number of groups required in the two-stage sampling plan for EHLD with $\nu = 1.57$ for 25^{th} percentile. The cells with hyphens (–) indicate that parameters cannot be found to satisfy the conditions

β	t_q/t_q^0	$\delta_q = 0.5\,,\ m = 5$				$\delta_q = 1.0\,,\ m = 5$				$\delta_q = 0.5\,,\ m = 10$				$\delta_q = 1.0\,,\ m = 10$			
		k_1	k_2	ASN	$L(p_1)$	k_1	k_2	ASN	$L(p_1)$	k_1	k_2	ASN	$L(p_1)$	k_1	k_2	ASN	$L(p_1)$
0.25	4	5	1	26.0	0.9612	2	1	10.7	0.9774	–	–	–	–	–	–	–	–
	6	5	1	25.6	0.9881	2	1	10.4	0.9904	3	1	31.4	0.9801	1	1	11.4	0.9670
	8	5	1	25.4	0.9950	2	1	10.3	0.9951	3	1	30.9	0.9915	1	1	10.9	0.9857
	10	5	1	25.2	0.9975	–	–	–	–	3	1	30.7	0.9957	1	1	10.6	0.9926
0.10	4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	6	8	1	40.9	0.974	3	1	15.9	0.9595	4	1	41.7	0.9692	1	1	11.4	0.9670
	8	8	1	40.6	0.9888	3	1	15.6	0.9824	4	1	41.2	0.9867	1	1	10.9	0.9857
	10	8	1	40.4	0.9943	3	1	15.5	0.9909	4	1	40.9	0.9932	1	1	10.6	0.9926
0.05	4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	6	10	1	51.0	0.9623	3	1	15.9	0.9595	5	1	52.1	0.9566	–	–	–	–
	8	10	1	50.7	0.9836	3	1	15.6	0.9824	5	1	51.4	0.981	2	1	21.7	0.9642
	10	10	1	50.5	0.9915	3	1	15.5	0.9909	5	1	51.1	0.9902	2	1	21.2	0.9812
0.01	4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	6	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
	8	14	1	70.9	0.9706	4	1	20.8	0.9726	7	1	71.9	0.9673	2	1	21.7	0.9642
	10	14	1	70.7	0.9847	4	1	20.6	0.9857	7	1	71.4	0.9829	2	1	21.2	0.9812

Thus the plan parameters of the design k_1 and k_2 will be obtained for given α and β, at the consumer's risk the percentile ratio δ_1 and at the producer's risk the percentile ratio δ_2 such that ASN(p_2) should be minimized and for given values of shape parameter ν, test termination ratio δ_q^0, and the number of testers m the inequalities (14b) and (14c) are satisfied simultaneously. The minimum number of groups required for the two-stage group sampling plan according to the values of percentile ratios $\delta_2 = 4, 6, 8, 10$ and $\delta_1 = 1$ when $m = 5$ and $m = 10$ at the four levels of the consumer's risks such as $\beta = 0.25, 0.10, 0.05$ and 0.01 with known parameter $\nu = 2, 3$ and 1.57 are shown in Tabs. 1–3. From the tables we can also observe that the number of groups required decreases as the group size increases from $m = 5$ to $m = 10$ when the other parameters remain the same and also the ASN slightly increases. As the group size increases the sample size (mk_1 or mk_2) decreases, which shows that a larger group will be more economical. We have also found that the number of groups not inclined by the shape parameter.

III. COMPARISONS WITH SINGLE-STAGE GROUP SAMPLING PLANS

We suggest a single-stage group sampling plan (with group size m) as a special case of a recommended two-stage group sampling plan and formulate the table for designing the plan. As discussed earlier, this is the case when $c_1 = c_2 = c$ in the two-stage group sampling plan. The probability that the acceptance of the lot under this plan is given by

$$P_{\mathrm{a}} = \sum_{i=0}^{c} \binom{mk}{i} p^i \, (1-p)^{mk-i} \,, \tag{15}$$

where k is the number of groups needed. Apparently, the ASN is attained by m times k. Similarly as in the two-stage group sampling plan, a table for determining the number of groups and acceptance number required can be prepared according to the value of the specified unreliability at a given consumer's risk.

The minimum number of groups required for the single-stage group sampling plan with $c = 0$ or $c = 1$ when $m = 5$ and $m = 10$ according to the values of percentile ratios $\delta_2 = 4, 6, 8, 10$ and $\delta_1 = 1$ at the four levels of the consumer's risks such as $\beta = 0.25, 0.10, 0.05$ and 0.01 with known parameter $\nu = 2, 3$ and 1.57 are computed to save the space displayed in Tab. 4 for $\nu = 3$ only.

In the sampling plans technique a plan is said to be better than the existing one if its ASN is lower than ASN of the existing plan. Here, when you compare the ASN values of the proposed sampling plan it shows less than the single sampling plan. Also when you see the OC curve of two-stage sampling plan, it is in between the OC curves of single sampling plans at $c = 0$ and at $c = 1$. We observe from Fig. 1 that the planned two-stage group sampling design performs

Tab. 4. The minimum number of groups required in the single-stage sampling plan for EHLD with $\nu = 3.0$ for 25^{th} percentile. The cells with hyphens (–) indicate that parameters cannot be found to satisfy the conditions

β	t_q/t_q^0	$\nu=3.0,\ \delta=0.5,\ m=5$						$\nu=3.0,\ \delta=1.0,\ m=5$					
		single with $c=0$			single with $c=1$			single with $c=0$			single with $c=1$		
		k	ASN	OC	k	ASN	OC	k	ASN	OC	k	ASN	OC
0.25	4	7	35	0.9730	12	60	0.9989	1	5	0.97	2	10	0.9984
	6	7	35	0.9918	12	60	0.9999	1	5	0.9907	2	10	0.9998
	8	7	35	0.9965	12	60	1.0000	1	5	0.996	2	10	1.0000
	10	7	35	0.9982	12	60	1.0000	1	5	0.9980	2	10	1.0000
0.10	4	11	55	0.9575	18	90	0.9976	–	–	–	3	15	0.9962
	6	11	55	0.9872	18	90	0.9998	2	10	0.9816	3	15	0.9996
	8	11	55	0.9946	18	90	1.0000	2	10	0.9921	3	15	0.9999
	10	11	55	0.9972	18	90	1.0000	2	10	0.9960	3	15	1.0000
0.05	4	–	–	–	21	105	0.9968	–	–	–	4	20	0.9933
	6	14	70	0.9837	21	105	0.9997	3	15	0.9725	4	20	0.9994
	8	14	70	0.9931	21	105	0.9999	3	15	0.9882	4	20	0.9999
	10	14	70	0.9964	21	105	1.0000	3	15	0.9939	4	20	1.0000
0.01	4	–	–	–	30	150	0.9936	–	–	–	5	25	0.9897
	6	21	105	0.9756	30	150	0.9994	4	20	0.964	5	25	0.9990
	8	21	105	0.9896	30	150	0.9999	4	20	0.9843	5	25	0.9998
	10	21	105	0.9947	30	150	1.0000	4	20	0.9919	5	25	1.0000
		$\nu=3.0,\ \delta=0.5,\ m=10$						$\nu=3.0,\ \delta=1.0,\ m=10$					
0.25	4	4	40	0.9689	6	60	0.9989	–	–	–	1	10	0.9984
	6	4	40	0.9906	6	60	0.9999	1	10	0.9816	1	10	0.9998
	8	4	40	0.9960	6	60	1.0000	1	10	0.9921	1	10	1.0000
	10	4	40	0.9980	6	60	1.0000	1	10	0.9960	1	10	1.0000
0.10	4	6	60	0.9537	9	90	0.9976	–	–	–	2	20	0.9933
	6	6	60	0.9860	9	90	0.9998	1	10	0.9816	2	20	0.9994
	8	6	60	0.9941	9	90	1.0000	1	10	0.9921	2	20	0.9999
	10	6	60	0.9970	9	90	1.0000	1	10	0.9960	2	20	1.0000
0.05	4	–	–	–	11	110	0.9965	–	–	–	2	20	0.9933
	6	7	70	0.9837	11	110	0.9997	2	20	0.9635	2	20	0.9994
	8	7	70	0.9931	11	110	0.9999	2	20	0.9843	2	20	0.9999
	10	7	70	0.9964	11	110	1.0000	2	20	0.9919	2	20	1.0000
0.01	4	–	–	–	15	150	0.9936	–	–	–	3	30	0.9853
	6	11	110	0.9745	15	150	0.9994	2	20	0.9635	3	30	0.9985
	8	11	110	0.9891	15	150	0.9999	2	20	0.9843	3	30	0.9997
	10	11	110	0.9944	15	150	1.0000	2	20	0.9919	3	30	0.9999

Tab. 5. The minimum number of groups required in the single-stage sampling plan for EHLD with $\nu = 1.57$ for 25^{th} percentile. The cells with hyphens (–) indicate that parameters cannot be found to satisfy the conditions

		$\nu = 1.57\,,\ \ \delta = 0.5\,,\ \ m = 5$						$\nu = 1.57\,,\ \ \delta = 1.0\,,\ \ m = 5$					
β	t_q/t_q^0	single with $c = 0$			single with $c = 1$			single with $c = 0$			single with $c = 1$		
		k	ASN	OC	k	ASN	OC	k	ASN	OC	k	ASN	OC
0.25	4	–	–	–	6	30	0.9604	–	–	–	2	10	0.9632
	6	–	–	–	6	30	0.9878	–	–	–	2	10	0.9888
	8	–	–	–	6	30	0.9949	–	–	–	2	10	0.9953
	10	3	15	0.9631	6	30	0.9974	1	5	0.9635	2	10	0.9976
0.10	4	–	–	–	–	–	–	–	–	–	–	–	–
	6	–	–	–	9	45	0.9738	–	–	–	3	15	0.9752
	8	–	–	–	9	45	0.9887	–	–	–	3	15	0.9894
	10	–	–	–	9	45	0.9942	–	–	–	3	15	0.9946
0.05	4	–	–	–				–	–	–			
	6	–	–	–	11	55	0.9620	–	–	–	4	20	0.9575
	8	–	–	–	11	55	0.9835	–	–	–	4	20	0.9815
	10	–	–	–	11	55	0.9915	–	–	–	4	20	0.9904
0.01	4	–	–	–	–	–	–	–	–	–	–	–	–
	6	–	–	–	–	–	–	–	–	–	–	–	–
	8	–	–	–	5	25	0.9717	–	–	–	5	50	0.9717
	10	–	–	–	5	25	0.9853	–	–	–	5	50	0.9853
		$\nu = 1.57\,,\ \ \delta = 0.5\,,\ \ m = 10$						$\nu = 1.57\,,\ \ \delta = 1.0\,,\ \ m = 10$					
0.25	4	–	–	–	3	30	0.9604	–	–	–	1	10	0.9632
	6	–	–	–	3	30	0.9878	–	–	–	1	10	0.9888
	8	–	–	–	3	30	0.9949	–	–	–	1	10	0.9953
	10	2	20	0.9511	3	30	0.9974	–	–	–	1	10	0.9976
0.10	4	–	–	–	–	–	–	–	–	–	–	–	–
	6	–	–	–	5	50	0.9681	–	–	–	2	20	0.9575
	8	–	–	–	5	50	0.9862	–	–	–	2	20	0.9815
	10	–	–	–	5	50	0.9929	–	–	–	2	20	0.9904
0.05	4	–	–	–				–	–	–			
	6	–	–	–	6	60	0.9556	–	–	–	2	20	0.9575
	8	–	–	–	6	60	0.9805	–	–	–	2	20	0.9815
	10	–	–	–	6	60	0.9899	–	–	–	2	20	0.9904
0.01	4	–	–	–	–	–	–	–	–	–	–	–	–
	6	–	–	–	–	–	–	–	–	–	–	–	–
	8	–	–	–	8	80	0.9668	–	–	–	3	30	0.9604
	10	–	–	–	8	80	0.9826	–	–	–	3	30	0.9791

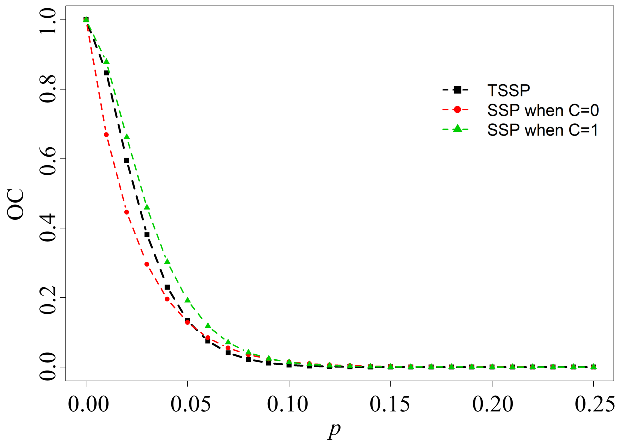

Fig. 1. The OC curves for ASN for different sampling plans

better than the single-stage group sampling plan in terms of the ASN and the OC values (see Tab. 2 with Tab. 4). While comparing the OC curves from Fig.1 at the chosen plan values of $m = 10, \nu = 3, \beta = 0.25, t_q/t_q^0 = 4, \delta = 0.5, t_q = = 0.25$ (25^{th} percentile) the $k_1 = 6$ and $k_2 = 1$ from Tab. 2 represented by the middle curve for ASN for a two-stage sampling plan with $c_1 = 0$ and $c_2 = 1$, and the $k = 4, k = 6$ from Tab. 4 represented by lower and upper curves for the ASN for a single stage sampling plan with $c = 0$ and $c = 1$, respectively. We also observed that the ASN for the two-stage group sampling plan with $c_1 = 0, c_2 = 1$ is much smaller than that of the single-stage group sampling plan with $c = 1$.

IV. INDUSTRIAL APPLICATIONS

At this juncture we use the real data set to demonstrate the methodology of a two-stage group sampling plan for exponentiated half logistic distribution. The real dataset consists of 76 observations of exact times of failures, and this data set was previously used by Abdul-Moniem and Seham [37] and Andrews and Herzberg [38]. For quick reference the data set is reported here, the times of failures (hours): 0.0251, 0.0886, 0.0891, 0.2501, 0.3113, 0.3451, 0.4763, 0.5650, 0.5671, 0.6566, 0.6748, 0.6751, 0.6753, 0.7696, 0.8375, 0.8391, 0.8425, 0.8645, 0.8851, 0.9113, 0.9120, 0.9836, 1.0483, 1.0596, 1.0773, 1.1733, 1.2570, 1.2766, 1.2985, 1.3211, 1.3503, 1.3551, 1.4595, 1.4880, 1.5728, 1.5733, 1.7083, 1.7263, 1.7460, 1.7630, 1.7746, 1.8275, 1.8375, 1.8503, 1.8808, 1.8878, 1.8881, 1.9316, 1.9558, 2.0048, 2.0408, 2.0903, 2.1093, 2.1330, 2.2100, 2.2460, 2.2878, 2.3203, 2.3470, 2.3513, 2.4951, 2.5260, 2.9911, 3.0256, 3.2678, 3.4045, 3.4846, 3.7433, 3.7455, 3.9143, 4.8073, 5.4005, 5.4435, 5.5295, 6.5541, 9.0960. This data set is used to demonstrate the goodness of fit for the exponentiated half logistic distribution and hence methodology of the two-stage group sampling plans. Fig. 2 depicts the estimated density and QQ plot to emphasize the goodness of EHLD for times of failures data set. The maximum likelihood estimates of the parameter of EHLD for the exact times of failures is $\nu = 1.57$, the Kolmogorov-Smirnov test for the maximum distance between the data and the fitted of the EHLD is 0.08903 with p-value is 0.553. Therefore, the two-parameter EHLD provides reasonable fits for lifetimes of items.

Suppose that an experimenter would like to establish the true unknown 25^{th} percentile lifetime for the product for at least 0.1 hours and the experiment will be stopped after 0.2 hours, which leads to $t_q/t_{q_0} = 2$. Assume that times of failures follow EHLD with $\nu = 1.57$, in the laboratory the experimenter has facility to install five items on a group. Suppose that $\beta = 0.10$ and $t_q/t_{q_0} = 2$ with $\alpha = 0.05$ for this experiment from Tab. 3, the two-stage acceptance

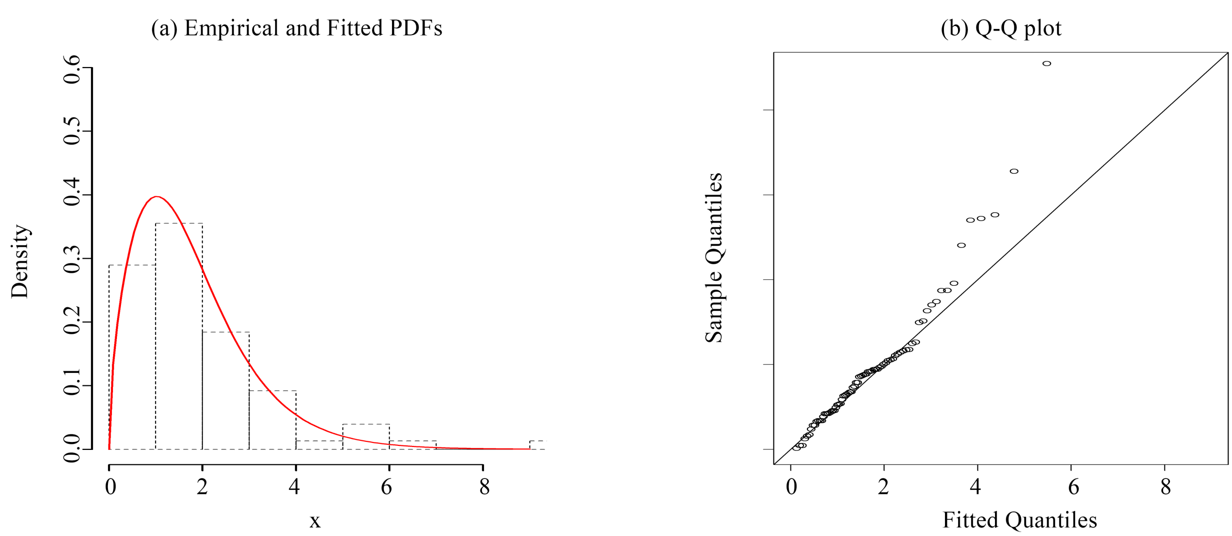

Fig. 2. (a) Estimated density and (b) Q-Q plot for times of failures data

sampling plan parameters at $m = 5$ and $\delta = 0.5$ are $k_1 = 5, k_2 = 1, c_1 = 0$ and $c_2 = 1$. The two-stage acceptance sampling plan is implemented as follows: randomly select 25 items and distribute 5 items into each of 5 testers and accept the product if there is no failure from each tester in 0.05 hours and reject the product if there is more than 1 failure from any tester before 0.05 hours. If one failure is observed from any tester then go to the second stage, then randomly select another 5 items from the lot and distribute 5 items into each one tester. If the total number of failure items from the two-stage testing within 0.05 hours for each stage is less than one then the lot is accepted; otherwise, the lot is rejected. In this example, only one failure before 0.05 hours hence lot is accepted. The probability of acceptance for this plan is 0.9612 and ASN is 26.0.

V. CONCLUSION

In this study, a two-stage group acceptance sampling plan is developed when the lifetime of a product follows exponentiated half logistic distribution percentiles with a known shape parameter. The plan parameters like the number of groups in each stage were determined so as to minimize the ASN subject to satisfy the consumer's and the producer's risks simultaneously under several conditions. Tables for the plan parameters were constructed under various combinations such as the known shape parameter, group sizes, consumer's and the producer's risks, and so on. An industrial example has been presented to illustrate the applications of the proposed two-stage group sampling plan. We made the comparison between the proposed two-stage group acceptance sampling plan and single-stage acceptance sampling plan. We observed from tables that the numbers of groups required decrease as the group size increases from 5 to 10 when other parameters remain the same and also the ASN increases marginally. The sample size also decreases as the group size increases, which indicates that a larger group size may be more economical. Finally, it should be mentioned from tables that the proposed two-stage group acceptance sampling plan performs better in terms of the average sample number and the operating characteristics than a single-stage group acceptance sampling plan.

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