Reflection of Plane Waves at Micropolar Piezothermoelastic Half-space

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Abstract: A problem of reflection at a free surface of micropolar orthotropic piezothermoelastic medium is discussed in the present paper. It is found that there exist five type plane waves in micropolar orthotropic piezothermoelastic medium, namely quasi longitudinal displacement wave (quasi LD wave), quasi thermal wave (quasi T wave), quasi CD-I, quasi CD-II wave and electric potential wave (PE wave). The amplitude ratios corresponding to reflected waves are obtained numerically. The effect of angle of incidence and thermopiezoelectric interactions on the reflected waves are studied for a specific model. Some particular cases of interest are also discussed.

Key words: orthotropic, micropolar, piezothermoelastic, reflection coefficients, angle of incidence

I. INTRODUCTION

The micropolar elasticity theory which takes into consideration the granular character of the medium, describes deformation by a microrotation and a microdisplacement. Eringen first showed that the classical elasticity theory [1] and the coupled stress theory [2] are two special cases of micropolar elasticity. The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effects by Nowacki [3] and Eringen [4]. A comprehensive review on the micropolar thermoelasticity is given by Eringen [5].

In most of the engineering problems, including the response of soils, geological materials and composites, some significant features of the continuum response may not be taken into account by the assumptions of isotropic behavior. The formulation and solution of anisotropic problems is far more difficult and cumbersome than their isotropic counterparts. The number of researchers have paid attention to the elastodynamic response of an anisotropic continuum over the last few years. In particular, transversely isotropic and orthotropic materials, which may not be distinguished from each other in plane strain and plane stress cases, have been more regularly studied.

The static problems of plane micropolar strain of a homogeneous and orthotropic elastic solid, torsion problems of homogeneous and orthotropic cylinders in the linear theory of micropolar elasticity and bending of orthotropic micropolar elastic beams by terminals couple were studied by Iesan [6,7,8]. The finite element method for orthotropic micropolar elasticity was developed by Nakamura et al. [9]. Kumar & Choudhary [10–14] have studied various problems in orthotropic micropolar continua.

Piezoelectric ceramics and composites find applications in many engineering applications e.g. sensors, actuators, intelligent structures, rocket propelled grenades, ultrasonic imaging, when thermal effects are not considered. Piezoelectric ceramics and piezoelectric polymers are pyroelectric media, which are used in small structures and intelligent systems. The thermo-piezoelectric material response entails an interac-
tation of three major fields, namely, mechanical, thermal and electric in the macro-physical world.

The thermopiezoelectric material has one important application to detect the responses of a structure by measurement of the electric charge, sensing, or to reduce excessive responses by applying additional electric forces or thermal forces, actuating. An intelligent structure can be designed by integrating sensing and actuating. The thermopiezoelectric materials are also often used as resonators whose frequencies need to be precisely controlled. It is important to quantify the effect of heat dissipation on the propagation of wave at low and high frequencies, due to the coupling between the thermoelastic and pyroelectric effects.

The theory of thermo-piezoelectricity was first developed by Mindlin [15]. The physical laws for the thermopiezoelectric materials have been explored by Nowacki [16–18]. Chandrasekharaiah [19–20] has generalized Mindlin’s theory of thermo-piezoelectricity to account for the finite speed of propagation of thermal disturbances. Chen [21] derived the general solution for transversely isotropic piezothermoelastic media. Hou et al. [22] constructed Green’s function for a point heat source on the surface of a semi-infinite transversely isotropic pyroelectric media.

Abd-Alla et al. [23–24] investigated reflection and refraction of plane quasi longitudinal waves at an interface of two piezoelectric media under initial stresses. Pang et al. [25] discussed the reflection and refraction of plane waves at the interface between two transversely isotropic piezoelectric and piezomagnetic media. The problems of reflection in piezoelectric media has been studied by such notable researchers as Sharma et al. [26], Kuang and Yuan [27], Abdalla et al. [28], Alshaikh [29–30].


In the present paper, the reflection of plane waves at a free surface of orthotropic micropolar piezothermoelastic medium is studied. A plane quasi wave is incident at a free surface and the amplitude ratios of various reflected waves are depicted. Their variations are shown with angle of incidence.

II. BASIC EQUATIONS

The basic equations of homogeneous orthotropic micropolar piezothermoelastic solid in the absence of body forces, body couples, electric charge density and heat sources are given by

(a) Constitutive relations

\[ t_{kl} = C_{ijkl} \varepsilon_{kl} + A_{ijkl} w_{kl} - g_{ijkl} E_k - \beta_{ij} T, \]  
\[ m_{ji} = D_{ijkl} w_{kl} + A_{ijkl} \varepsilon_{kl} - \epsilon_{ijkl} E_k, \]  
\[ D_i = \epsilon_{ij} E_j + g_{ijkl} \varepsilon_{jk} - \beta_{ij} T, \]  
\[ q_i = -T_0 b_i T + k_{ij} e_j, \]

The deformation and wryness tensor are defined as following:

\[ \varepsilon_{ij} = u_{ij} + \epsilon_{ijk} w_k, \quad w_{ij} = w_{ij}, \]

(b) Balance laws

\[ t_{kl,k} = \rho \ddot{u}_l, \]  
\[ m_{kl,k} + \epsilon_{mn} h_{mn} = \rho J \ddot{w}_l, \]  
\[ D_i = 0, \]  
\[ q_{i,i} = -T_0 S, \]

where \( t_{kl}, \ m_{kl} \) are the stress tensor, couple stress tensor; \( E_i \) is the electric displacement vector, \( E_i \) is the electric field vector, \( q_i \) is the heat flux vector; \( S \) is the entropy; \( T \) is the thermodynamic temperature; \( T_0 \) is the absolute temperature; \( c^* \) is the specific heat at constant strain; \( \rho \) is the bulk mass.
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III. FORMULATION OF THE PROBLEM

By using the transformations, following Slaughter [39] on the set of equations (1) to (9), the equations for micropolar orthotropic piezothermoelastic medium are derived.

We consider a homogeneous centro symmetric, orthotropic micropolar piezothermoelastic medium initially in an undeformed state and at uniform temperature $T_0$, namely medium $M_1$. The origin of the coordinate system is taken on the plane interface and $x_3$ - axis pointing vertically into the medium $M_1$ is taken which is designated as $x_3 \geq 0$. Plane waves are considered such that all the particles on a line parallel to $x_2$ - axis are equally displaced, so that all the partial derivatives with respect to the variable $x_2$ are zero.

Let $\vec{u} = (u_1, 0, u_3), \vec{w} = (0, w_2, 0), \vec{E} = (E_1, 0, E_3), E_i = -\frac{\partial w_i}{\partial x_j}, \phi$ is the electric potential and $\frac{\partial \phi}{\partial x_3} = 0$, so that the field equations and constitutive relations reduce to the following:

$$\begin{align*}
C_{11} \frac{\partial^2 u_1}{\partial x_1^2} + C_{33} \frac{\partial^2 u_1}{\partial x_3^2} + (C_{19} + C_{77}) \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + g_{13} \frac{\partial^2 \phi}{\partial x_1 \partial x_3} + g_{31} \frac{\partial^2 \phi}{\partial x_1^2} - \beta_1 \frac{\partial}{\partial x_1} T &= \rho \frac{\partial^2 u_1}{\partial t^2}, \\
+C_{37} \frac{\partial^2 u_3}{\partial x_1^2} + C_{99} \frac{\partial^2 u_3}{\partial x_3^2} + (C_{33} + C_{91}) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + g_{31} \frac{\partial^2 \phi}{\partial x_1 \partial x_3} + g_{93} \frac{\partial^2 \phi}{\partial x_3^2} - \beta_3 \frac{\partial}{\partial x_3} T &= \rho \frac{\partial^2 u_3}{\partial t^2}, \\
D_{24} \frac{\partial^2 w_2}{\partial x_1^2} + D_{86} \frac{\partial^2 w_2}{\partial x_3^2} + (C_{73} - C_{33}) \frac{\partial u_1}{\partial x_1} + (C_{77} - C_{33}) \frac{\partial u_3}{\partial x_3} + g_{31} \frac{\partial u_1}{\partial x_1} + (g_{31} - g_{71}) \frac{\partial \phi}{\partial x_1} &= \rho J \frac{\partial^2 w_2}{\partial t^2},
\end{align*}$$

where $\beta_1 = C_{11} + C_{19} + C_{13}, \beta_3 = C_{91} + C_{93} + C_{93} - C_{91}, \alpha_3$ are the coefficients of linear thermal expansion. We have used the notations $11 \rightarrow 1, 12 \rightarrow 2, 13 \rightarrow 3, 21 \rightarrow 4, 22 \rightarrow 5, 23 \rightarrow 6, 31 \rightarrow 7, 32 \rightarrow 8, 33 \rightarrow 9$ for the material constants.

The following dimensionless quantities are introduced

$$\begin{align*}
x_1' &= \frac{\omega^* x_1}{c_1}, \\
x_3' &= \frac{\omega^* x_3}{c_1}, \\
u_1' &= \frac{\omega^* u_1}{c_1}, \\
u_3' &= \frac{\omega^* u_3}{c_1}, \\
w_2' &= \frac{C_{11}}{C_{33}} w_2, \\
t_{ij}' &= \frac{1}{C_{11}} t_{ij}, \\
m_{ij}' &= \frac{c_1}{\omega^* D_{24}} m_{ij}, \\
T' &= \frac{T}{T_0}, \\
\phi' &= \frac{\omega^* \phi}{c_1 g_{13}}, \\
D_i' &= \frac{c_1}{\omega^* g_{13}} D_i, \\
\omega^* &= \frac{C_{11}}{\rho}, \\
\omega^* &= \frac{C_{33}}{\rho J},
\end{align*}$$

where $\omega^*$ is the characteristic frequency of the material and $c_1$ is the longitudinal wave velocity of the medium.
By using the dimensionless quantities in equations (11)-(15), we obtain the following equations

\[
\frac{\partial^2 u_1}{\partial x_1^2} + a_1 \frac{\partial^2 u_1}{\partial x_3^2} + a_2 \frac{\partial^2 u_3}{\partial x_1 \partial x_3} + a_3 \frac{\partial w_2}{\partial x_3} + a_4 \frac{\partial^2 \phi}{\partial x_1 \partial x_3} - a_5 \frac{\partial T}{\partial x_1} = a_6 \frac{\partial^2 u_1}{\partial t^2},
\]

(20)

\[
\frac{\partial^2 u_3}{\partial x_1^2} + a_7 \frac{\partial^2 u_3}{\partial x_3^2} + a_8 \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + a_9 \frac{\partial w_2}{\partial x_3} + a_{10} \frac{\partial^2 \phi}{\partial x_1^2} + a_{11} \frac{\partial^2 \phi}{\partial x_3^2} - a_{12} \frac{\partial T}{\partial x_3} = a_{13} \frac{\partial^2 u_3}{\partial t^2},
\]

(21)

\[
\frac{\partial^2 w_2}{\partial x_1^2} + a_{14} \frac{\partial^2 w_2}{\partial x_3^2} + a_{15} \frac{\partial u_1}{\partial x_1} + a_{16} \frac{\partial u_3}{\partial x_1} + a_{17} w_2 + a_{18} \frac{\partial \phi}{\partial x_1} = a_{19} \frac{\partial^2 w_2}{\partial t^2},
\]

(22)

\[
\frac{\partial^2 \phi}{\partial x_1^2} + a_{20} \frac{\partial^2 \phi}{\partial x_3^2} - a_{21} \frac{\partial^2 u_1}{\partial x_1 \partial x_3} - a_{22} \frac{\partial^2 u_3}{\partial x_1^2} - a_{23} \frac{\partial^2 u_3}{\partial x_3^2} = 0,
\]

(23)

\[
\left( \frac{\partial^2 T}{\partial x_1^2} + a_{24} \frac{\partial^2 T}{\partial x_3^2} \right) - \frac{\partial}{\partial t} \left( a_{25} \frac{\partial u_1}{\partial x_1} + a_{26} \frac{\partial u_3}{\partial x_1} \right) = 0
\]

\[
- \frac{\partial}{\partial t} \left( a_{27} \frac{\partial \phi}{\partial x_3} \right) = a_{28} \frac{\partial T}{\partial t}
\]

(24)

IV. PLANE WAVE PROPAGATION

Let us assume the plane wave solution of the form

\[
(u_1, u_3, w_2, \phi, T) = (\bar{u}_1, \bar{u}_3, \bar{w}_2, \bar{\phi}, \bar{T}) e^{i(\omega t - k x_3)},
\]

(26)

where \(\bar{u}_1, \bar{u}_3, \bar{w}_2, \bar{\phi}, \bar{T}\) are functions of \(x_3\) only, \(k\) is the wave number and \(\omega\) is the angular frequency.

Using equation (26) in equations (20)-(24), a system of five homogeneous equations is obtained in terms of \(\omega\) and \(k\) in five unknowns \(\bar{u}_1, \bar{u}_3, \bar{w}_2, \bar{\phi}, \bar{T}\), which for non-trivial solution, using Cramer’s rule yield
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where

\[ A_1 = a_1 a_7 a_{14} a_{20} a_{24} \]

\[ A_2 = a_{24}(h_1 a_1 a_7 + h_2 a_{14} a_{20} + h_4 a_20 - h_6 a_{23}) + h_{19} a_{14} a_{21} + a_1 a_7 a_{14} a_{20} \delta_1 + h_{16} a_{14} \]

\[ A_3 = \delta_1 h_1 a_1 a_7 + h_2 a_{14} a_{20} + h_4 a_20 - a_{23} h_6 \]

\[ A_4 = \delta_1 h_1 a_1 a_7 + \delta_1 h_6 a_{14} a_{20} + h_1 h_2 + a_{20} h_3 - h_4 k^2 \]

\[ A_5 = \delta_1 h_1 a_1 a_7 + \delta_1 h_6 a_{14} a_{20} + h_2 - 2 h_3 h_4 - h_3 h_6 - a_{23} h_8 + h_9 \]

\[ A_6 = \delta_1 h_6 a_{14} a_{20} + h_8 \delta_1 + 2 \delta_1 \delta_6 a_{10} k^2 \]

\[ \frac{d^4}{dx^4} + A_1 \frac{d^6}{dx^3} + A_2 \frac{d^6}{dx^3} + A_3 \frac{d^6}{dx^3} + A_4 \frac{d^4}{dx^3} + A_5 \frac{d^2}{dx^3} + A_6 \] (\(\overline{w}_1, \overline{w}_2, \overline{w}_3, \overline{\varphi}, T\)) = 0, \tag{27} \]

Equation (27) is now in terms of \(\omega\) and \(k\). The roots of the equation (27) gives the velocities of five plane waves in the decreasing order of the velocities, i.e. quasi longitudinal displacement wave (quasi LD wave), quasi thermal wave
(quasi T wave), quasi CD-I, quasi CD-II wave and electric potential wave (PE wave).

V. REFLECTION AND TRANSMISSION

A homogeneous orthotropic micropolar piezothermoelastic half-space is considered. A plane wave making an angle $\theta_0$ with $x_3 -$ axis becomes incident at the free surface. This wave results in five reflected wave modes in medium $M_1$. In medium $M_1$ reflected wave modes are represented by the quasi LD wave, quasi thermal wave, quasi CD-I (transverse) wave, quasi CD-II (micropolar) wave and one other mode corresponding to electric potential wave mode i.e. PE wave mode.

The formal solution for the mechanical displacements, microrotation, electric potential and temperature distribution in medium $M_1$ are

$$u_1(x_3) = (B_{01}e^{-\lambda_{1}x_3} + B_{1}e^{\lambda_{1}x_3} + B_{02}e^{-\lambda_{2}x_3} + B_{2}e^{\lambda_{2}x_3} + B_{03}e^{-\lambda_{3}x_3} + B_{3}e^{\lambda_{3}x_3} + B_{04}e^{-\lambda_{4}x_3} + B_{4}e^{\lambda_{4}x_3} + B_{05}e^{-\lambda_{5}x_3} + B_{5}e^{\lambda_{5}x_3})e^{i(\omega t-kx_1)},$$

$$u_3(x_3) = (m_1B_{01}e^{-\lambda_{1}x_3} + m_1B_{1}e^{\lambda_{1}x_3} + m_2B_{02}e^{-\lambda_{2}x_3} + m_2B_{2}e^{\lambda_{2}x_3} + m_3B_{03}e^{-\lambda_{3}x_3} + m_3B_{3}e^{\lambda_{3}x_3} + m_4B_{04}e^{-\lambda_{4}x_3} + m_4B_{4}e^{\lambda_{4}x_3} + m_5B_{05}e^{-\lambda_{5}x_3} + m_5B_{5}e^{\lambda_{5}x_3})e^{i(\omega t-kx_1)}.$$
Fig. 2. Variations of amplitude ratio $Z_1$ with angle of incidence (QL-wave)

Fig. 3. Variations of amplitude ratio $Z_2$ with angle of incidence (QL-wave)

Fig. 4. Variations of amplitude ratio $Z_3$ with angle of incidence (QL-wave)

Fig. 5. Variations of amplitude ratio $Z_4$ with angle of incidence (QL-wave)
Fig. 6. Variations of amplitude ratio $Z_3$ with angle of incidence (QL-wave)

Fig. 7. Variations of amplitude ratio $Z_4$ with angle of incidence (QT-wave)

Fig. 8. Variations of amplitude ratio $Z_5$ with angle of incidence (QT-wave)

Fig. 9. Variations of amplitude ratio $Z_6$ with angle of incidence (QT-wave)
VI. BOUNDARY CONDITIONS

The appropriate boundary conditions at an interface \( x_3 = 0 \) are given by

\[
t_{33} = 0, \quad t_{31} = 0, \quad m_{32} = 0, \quad \frac{\partial T}{\partial x_3} = 0, \quad D_3 = 0.
\]  
(34)

Using the equations (28)-(32), we find that the boundary conditions are satisfied if and only if:

\[
\sin \theta_0 \frac{v}{\omega} = k,
\]  
(35)

where \( v \) is the velocity of the incident wave at an interface.

Making use of equations (28) to (32) in equation (34) and using equation (35), we obtain a system of five homogeneous equations as:

\[
\sum_{j=1}^{10} a_{ij} B_j = 0; \quad (i = 1, 2, 3, 4, 5),
\]  
(36)

where

\[
a_{1i} = -d_1 \lambda \omega n_i + d_3 \lambda \omega m_i + d_4 \lambda \omega l_i,
\]

\[
a_{1j} = -d_1 \lambda \omega n_i + d_3 \lambda \omega m_i + d_4 \lambda \omega l_i,
\]

\[
a_{2i} = -a_1 \lambda \omega + (d_6 \omega - d_5 \omega) \lambda \omega k, \quad a_{2j} = a_1 \lambda \omega + (d_6 \omega - d_5 \omega) \lambda \omega k,
\]

\[
a_{3i} = -d_7 \lambda \omega n_i, \quad a_{3j} = d_7 \lambda \omega n_i, \quad a_{4i} = -\lambda \omega l_i, \quad a_{4j} = \lambda \omega l_i,
\]

\[
a_{5j} = d_9 \lambda \omega g_i - \lambda k \omega d_{10} + d_{11} \lambda \omega m_1.
\]  
(37)

(a) When quasi LD wave is incident:

\[
B_2 = B_3 = B_4 = B_5 = 0.
\]

Dividing the set of equations throughout by \( B_1 \), we obtain a system of ten non-homogeneous equations in ten unknowns which can be solved by Crammer’s rule and we have

\[
Z_i = \frac{B_i}{B_1} = \frac{\Gamma^1_i}{\Gamma}; \quad i = 1, 2, 3, 4, 5.
\]

(b) When quasi T wave is incident: \( B_1 = B_3 = B_4 = B_5 = 0 \) and

\[
Z_i = \frac{B_i}{B_2} = \frac{\Gamma^2_i}{\Gamma}; \quad i = 1, 2, 3, 4, 5,
\]

where

\[
\Gamma = |a_{ii+5}|_{5 \times 5},
\]

and \( \Gamma^p_i (i = 1, 2, 3, 4, 5) (p = 1, 2, 3, 4, 5) \) can be obtained by replacing, respectively the 1st, 2nd, …, 5th columns of \( \Gamma \) by \( [-a_{1p}, -a_{2p}, a_{3p}, a_{4p}, a_{5p}]^T \).
VII. PARTICULAR CASES

(a) If we neglect the piezoelectric effect in medium $M_1$, we obtain amplitude ratios at the free surface of orthotropic piezothermoelastic solid with changed values of $a_{ij}$ as

$$a_{11} = -d_1 t \nu - d_2 \lambda m_i + d_3 \lambda g_i - d_4 l_i,$$

$$a_{1j} = -d_1 t \nu + d_2 \lambda m_i - d_3 \lambda g_i - d_4 l_i,$$

$$a_{2i} = -a_1 \lambda_i + (d_6 g_i - d_5 m_i) t \nu,$$

$$a_{2j} = a_1 \lambda_i + (d_6 g_i - d_5 m_i) t \nu,$$

$$a_{3i} = -d_7 \lambda_i n_i, a_{3j} = d_7 \lambda_i n_i,$$

$$a_{4i} = -\lambda_i l_i, a_{4j} = \lambda_i l_i.$$

VIII. NUMERICAL RESULTS AND DISCUSSION

The physical data for medium $M_1$ is given by

$$C_{11} = 7.46 \times 10^{10} \text{Nm}^{-2},$$

$$C_{12} = 3.9 \times 10^{10} \text{Nm}^{-2},$$

$$C_{13} = 1.37 \times 10^9 \text{Nm}^{-2},$$

$$C_{22} = 8.39 \times 10^9 \text{Nm}^{-2},$$

$$C_{33} = 0.399 \times 10^9 \text{Nm}^{-2},$$

$$C_{77} = 0.0138 \times 10^9 \text{Nm}^{-2},$$

$$g_{13} = -0.142 \times 10^{-3} \text{cm}^{-2},$$

$$g_{71} = -0.165 \times 10^{-3} \text{cm}^{-2},$$

$$g_{93} = 0.351 \times 10^{-3} \text{cm}^{-2},$$

$$g_{31} = -0.139 \times 10^{-3} \text{cm}^{-2},$$

$$\varepsilon_{11} = 8.29 \times 10^{-11} \text{Nm}^{-2}/K,$$

$$\varepsilon_{33} = 9.07 \times 10^{-11} \text{Nm}^{-2}/K,$$

$$\lambda_3 = 7.6 \times 10^{-6} \text{cm}^{-2}/K,$$

$$\tau = 0.8 \text{s},$$

$$k_1 = 9.5 \text{Wm}^{-1}\text{K}^{-1},$$

$$k_2 = 9.7 \text{Wm}^{-1}\text{K}^{-1},$$

$$\beta_1 = 0.670 \times 10^5 \text{C}^2\text{N}^{-1}\text{m}^{-2},$$

$$\beta_3 = 0.581 \times 10^5 \text{C}^2\text{N}^{-1}\text{m}^{-2},$$

$$\nu = 0.268,$$

$$T_0 = 298 \text{K},$$

$$\rho = 5504 \text{kg} \text{m}^{-3},$$

$$c^* = 2.64 \times 10^2 \text{N m kg}^{-1}\text{s K}^{-1},$$

$$J = 0.02 \times 10^{-11} \text{m}^{-2},$$

$$D_{24} = 0.134 \text{N},$$

$$D_{86} = 0.243 \text{N}.$$

Figs. 2–11 show the variations of amplitude ratios with the angle of incidence for incidence of plane waves at an interface. In Figs. 2–11 MPT corresponds to amplitude ratios in the orthotropic micropolar piezothermoelastic solid, WPE corresponds to amplitude ratios in the orthotropic micropolar thermoelastic solid.

VIII. 1. Incidence of quasi ld wave (ql-wave)

Figs. 2–6 represent the variations of amplitude ratios $|Z_i|$ for MPT and WPE with increase in the angle of incidence. The values for MPT remain more than the values for WPE in the whole range.

VIII. 2. Incidence of quasi t wave (qt-wave)

Figs. 7–11 represent the variations of amplitude ratios $|Z_i|$ for MPT and WPE with increase in the angle of incidence. The values for MPT remain more than the values for WPE in the whole range.

IX. CONCLUSION

The reflection coefficients of various plane quasi waves on incidence of quasi LD wave and quasi T wave at a free surface of orthotropic micropolar piezothermoelastic medium for L-S theory are obtained. It is noticed that when quasi LD wave is incident, the values of amplitude ratios of reflected quasi T, quasi CD-I (Transverse) in the absence of the piezoelectric effect are smaller that reveals the piezoelectric effect.
It is seen that when the quasi T wave is incident, the piezoelectric effect decreases the magnitude of amplitude ratio of the reflected quasi CD-I (transverse) wave and quasi CD-II (micropolar) wave modes.

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