Reflection of Plane Waves at Micropolar Piezothermoelastic Half-space

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Received: 30 December 2016; revised: 15 December 2017; accepted: 18 December 2017; published online: 30 January 2018

Abstract: A problem of reflection at a free surface of micropolar orthotropic piezothermoelastic medium is discussed in the present paper. It is found that there exist five type plane waves in micropolar orthotropic piezothermoelastic medium, namely quasi longitudinal displacement wave (quasi LD wave), quasi thermal wave (quasi T wave), quasi CD-I, quasi CD-II wave and electric potential wave (PE wave). The amplitude ratios corresponding to reflected waves are obtained numerically. The effect of angle of incidence and thermopiezoelectric interactions on the reflected waves are studied for a specific model. Some particular cases of interest are also discussed.

Key words: orthotropic, micropolar, piezothermoelastic, reflection coefficients, angle of incidence

I. INTRODUCTION

The micropolar elasticity theory which takes into consideration the granular character of the medium, describes deformation by a microrotation and a microdisplacement. Eringen first showed that the classical elasticity theory [1] and the coupled stress theory [2] are two special cases of micropolar elasticity. The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effects by Nowacki [3] and Eringen [4]. A comprehensive review on the micropolar theromoelasticity is given by Eringen [5].

In most of the engineering problems, including the response of soils, geological materials and composites, some significant features of the continuum response may not be taken into account by the assumptions of isotropic behavior. The formulation and solution of anisotropic problems is far more difficult and cumbersome than their isotropic counterparts. The number of researchers have paid attention to the elastodynamic response of an anisotropic continuum over the last few years. In particular, transversely isotropic and orthotropic materials, which may not be distinguished from each other in plane strain and plane stress cases, have been more regularly studied.

The static problems of plane micropolar strain of a homogeneous and orthotropic elastic solid, torsion problems of homogeneous and orthotropic cylinders in the linear theory of micropolar elasticity and bending of orthotropic micropolar elastic beams by terminals couple were studied by Iesan [6,7,8]. The finite element method for orthotropic micropolar elasticity was developed by Nakamura et al. [9]. Kumar & Choudhary [10–14] have studied various problems in orthotropic micropolar continua.

Piezoelectric ceramics and composites find applications in many engineering applications e.g. sensors, actuators, intelligent structures, rocket propelled grenades, ultrasonic imaging, when thermal effects are not considered. Piezoelectric ceramics and piezoelectric polymers are pyroelectric media, which are used in small structures and intelligent systems. The thermo-piezoelectric material response entails an interaction of three major fields, namely, mechanical, thermal and electric in the macro-physical world.

The thermopiezoelectric material has one important application to detect the responses of a structure by measurement of the electric charge, sensing, or to reduce excessive responses by applying additional electric forces or thermal forces, actuating. An intelligent structure can be designed by integrating sensing and actuating. The thermopiezoelectric materials are also often used as resonators whose frequencies need to be precisely controlled. It is important to quantify the effect of heat dissipation on the propagation of wave at low and high frequencies, due to the coupling between the thermoelastic and pyroelectric effects.

The theory of thermo-piezoelectricity was first developed by Mindlin [15]. The physical laws for the thermo-piezoelectric materials have been explored by Nowacki [16–18]. Chandrasekharaiah [19–20] has generalized Mindlin's theory of thermo-piezoelectricity to account for the finite speed of propagation of thermal disturbances. Chen [21] derived the general solution for transversely isotropic piezothermoelastic media. Hou et al. [22] constructed Green's function for a point heat source on the surface of a semi-infinite transversely isotropic pyroelectric media.

Abd-Alla et al. [23–24] investigated reflection and refraction of plane quasi longitudinal waves at an interface of two piezoelectric media under initial stresses. Pang et al. [25] discussed the reflection and refraction of plane waves at the interface between two transversely isotropic piezoelectric and piezomagnetic media. The problems of reflection in piezoelectric media has been studied by such notable researchers as Sharma et al. [26], Kuang and Yuan [27], Abdalla et al. [28], Alshaikh [29–30].

Meerschaert and McGough[31] studied attenuated fractional wave equations with anisotropy. Sur and Kanoria [32] investigated fractional heat conduction with finite wave speed in a thermoviscoelastic spherical shell. Abo-Dahab [33] analysed the magnetic effect on three plane waves propagation at an interface between solid-liquid media placed under initial stress in the context of the GL model. Abd-Alla and Abo-Dahab [34] studied the effect of initial stress, rotation and gravity on propagation of surface waves in fibre-reinforced anisotropic solid elastic media.

Vashishth and Sukhija [35] studied reflection and transmission of plane waves from a fluid-piezothermoelastic solid interface. Kumar and Kumar [36] studied the elastodynamic response of thermal laser pulse in a micropolar thermoelastic diffusion medium. Mahmoud [37] presented an analytical solution for the effect of initial stress, rotation, magnetic field and periodic loading in a thermoviscoelastic medium with a spherical cavity. Ezzat, EI-Karamany and EI-Bary [38] discussed a problem of generalized thermoelasticity with memory dependent derivatives involving two temperatures

In the present paper, the reflection of plane waves at a free surface of orthotropic micropolar piezothermoelastic medium is studied. A plane quasi wave is incident at a free surface and the amplitude ratios of various reflected waves are depicted. Their variations are shown with angle of incidence.

II. BASIC EQUATIONS

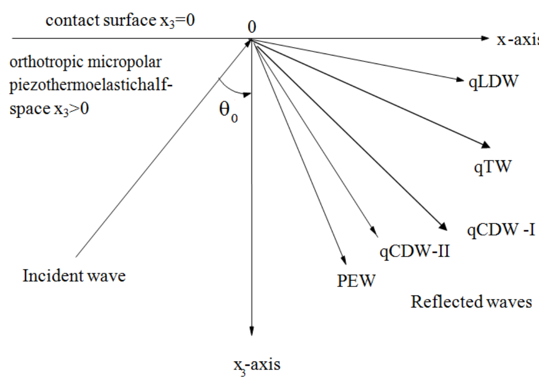

Fig. 1. Geometry of the problem

The basic equations of homogeneous orthotropic micropolar piezothermoelastic solid in the absence of body forces, body couples, electric charge density and heat sources are given by

(a) Constitutive relations

$$t_{kl} = C_{ijkl}\varepsilon_{kl} + A_{ijkl}w_{kl} - g_{ijk}E_k - \beta_{ij}T, \quad (1)$$

$$m_{ji} = D_{ijkl}w_{kl} + A_{ijkl}\varepsilon_{kl} - e_{ijk}E_k, \quad (2)$$

$$D_i = \epsilon_{ij}E_j + g_{ijk}\varepsilon_{jk} - \beta_{ij}T, \quad (3)$$

$$q_i = -T_0 b_i \dot{T} + k_{ij}e_j, \quad (4)$$

The deformation and wryness tensor are defined as following:

$$\varepsilon_{ij} = u_{i,j} + \epsilon_{ijk}w_k, \qquad w_{ij} = w_{i,j}, \quad (5)$$

(b) Balance laws

$$t_{kl,k} = \rho\,\ddot{u}_l, \quad (6)$$

$$m_{kl,k} + \epsilon_{lmn}t_{mn} = \rho J\,\ddot{w}_l, \quad (7)$$

$$D_{i,i} = 0, \quad (8)$$

$$q_{i,i} = -T_0\,\dot{S}, \quad (9)$$

where $t_{kl},\ m_{kl}$ are the stress tensor, couple stress tensor; D_i is the electric displacement vector, E_i is the electric field vector, q_i is the heat flux vector; S is the entropy;T is the thermodynamic temperature; T_0 is the absolute temperature; c^* is the specific heat at constant strain; ρ is the bulk mass density; J is the microinertia; u_l and w_k are the components of displacement vector and microrotation vector, respectively; ε_{ij} are the components of micro-strain tensor, ϵ_{ijk} is the permutation tensor,β_{kl} is the thermal elastic coupling tensor; $C_{ijkl}, G_{ijkl}, D_{ijkl}$ are the characteristic constants of material; g_{ijk} is the electro-elastic coupling moduli where $C_{ijkl},\ D_{ijkl},\ g_{ijk}$ satisfies the symmetric relations

$$C_{ijkl} = C_{klij}, D_{ijkl} = D_{klij}, g_{ijk} = g_{kij}. \tag{10}$$

In a centrosymmetric bodies, all components of A_{ijkl} vanish.

III. FORMULATION OF THE PROBLEM

By using the transformations, following Slaughter [39] on the set of equations (1) to (9), the equations for micropolar orthotropic piezothermoelastic medium are derived.

We consider a homogeneous centrosymmetric, orthotropic micropolar piezothermoelastic medium initially in an undeformed state and at uniform temperature T_0, namely medium M_1. The origin of the coordinate system is taken on the plane interface and x_3- axis pointing vertically into the medium M_1 is taken which is designated as $x_3 \geq 0$. Plane waves are considered such that all the particles on a line parallel to x_2- axis are equally displaced, so that all the partial derivatives with respect to the variable x_2 are zero.

Let $\vec{u} = (u_1,\ 0,\ u_3)$, $\vec{w} = (0,\ w_2,\ 0)$,$\vec{E} = (E_1,\ 0,\ E_3)$, $E_i = -\frac{\partial \phi}{\partial x_i}$, ϕ is the electric potential and $\frac{\partial}{\partial x_2} = 0$, so that the field equations and constitutive relations reduce to the following:

$$\begin{aligned}&C_{11}\frac{\partial^2 u_1}{\partial x_1^2} + C_{73}\frac{\partial^2 u_1}{\partial x_3^2} + (C_{19}+C_{77})\frac{\partial^2 u_3}{\partial x_1 \partial x_3}\\&+(C_{77}-C_{73})\frac{\partial w_2}{\partial x_3} + g_{13}\frac{\partial^2 \phi}{\partial x_1 \partial x_3}\\&+g_{71}\frac{\partial^2 \phi}{\partial x_1 \partial x_3} - \beta_1\frac{\partial}{\partial x_1}T = \rho\frac{\partial^2 u_1}{\partial t^2},\end{aligned} \tag{11}$$

$$\begin{aligned}&C_{37}\frac{\partial^2 u_3}{\partial x_1^2} + C_{99}\frac{\partial^2 u_3}{\partial x_3^2} + (C_{33}+C_{91})\frac{\partial^2 u_1}{\partial x_1 \partial x_3}\\&+(C_{37}-C_{33})\frac{\partial w_2}{\partial x_1} + g_{31}\frac{\partial^2 \phi}{\partial x_1^2}\\&+g_{93}\frac{\partial^2 \phi}{\partial x_3^2} - \beta_3\frac{\partial}{\partial x_3}T = \rho\frac{\partial^2 u_3}{\partial t^2},\end{aligned} \tag{12}$$

$$\begin{aligned}&D_{24}\frac{\partial^2 w_2}{\partial x_1^2} + D_{86}\frac{\partial^2 w_2}{\partial x_3^2} + (C_{73}-C_{33})\frac{\partial u_1}{\partial x_1}\\&+(C_{77}-C_{37})\frac{\partial u_3}{\partial x_1} + (C_{73}-C_{33}-2C_{37})w_2\\&+(g_{31}-g_{71})\frac{\partial \phi}{\partial x_1} = \rho J\frac{\partial^2 w_2}{\partial t^2},\end{aligned} \tag{13}$$

$$\begin{aligned}&-\epsilon_{11}\frac{\partial^2 \phi}{\partial x_1^2} - \epsilon_{33}\frac{\partial^2 \phi}{\partial x_3^2} + (g_{71}+g_{13})\frac{\partial^2 u_1}{\partial x_1 \partial x_3}\\&+g_{31}\frac{\partial^2 u_3}{\partial x_1^2} + g_{93}\frac{\partial^2 u_3}{\partial x_3^2} = 0,\end{aligned} \tag{14}$$

$$\begin{aligned}&k_1\frac{\partial^2 T}{\partial x_1^2} + k_3\frac{\partial^2 T}{\partial x_3^2} - T_0\frac{\partial}{\partial t}(1+\tau_0\frac{\partial}{\partial t})(\beta_1\frac{\partial u_1}{\partial x_1} + \beta_3\frac{\partial u_3}{\partial x_1})\\&-T_0\frac{\partial}{\partial t}(1+\tau_0\frac{\partial}{\partial t})(\lambda_3\frac{\partial \phi}{\partial x_3}) = \rho c^*\frac{\partial T}{\partial t},\end{aligned} \tag{15}$$

$$t_{33} = C_{91}\frac{\partial u_1}{\partial x_1} + C_{93}\frac{\partial u_3}{\partial x_3} - g_{93}\frac{\partial \phi}{\partial x_3} - \beta_3 T, \tag{16}$$

$$t_{31} = C_{73}\frac{\partial u_1}{\partial x_3} + C_{77}\frac{\partial u_3}{\partial x_1} - g_{71}\frac{\partial \phi}{\partial x_1}, \tag{17}$$

$$m_{32} = D_{86}\frac{\partial w_2}{\partial x_3}, \tag{18}$$

where $\beta_1 = C_{11}\alpha_1 + C_{19}\alpha_3,\quad \beta_3 = C_{91}\alpha_1 + C_{99}\alpha_3$,$\alpha_1,\ \alpha_3$ are the coefficients of linear thermal expansion. We have used the notations $11 \to 1,\ 12 \to 2,\ 13 \to 3,\ 21 \to 4,\ 22 \to 5,\ 23 \to 6,\ 31 \to 7,\ 32 \to 8,\ 33 \to 9$ for the material constants.

The following dimensionless quantities are introduced

$$\begin{aligned}x_1^{'} &= \frac{\omega^* x_1}{c_1},\\ x_3^{'} &= \frac{\omega^* x_3}{c_1},\\ u_1^{'} &= \frac{\omega^*}{c_1}u_1,\\ u_3^{'} &= \frac{\omega^*}{c_1}u_3,\\ w_2^{'} &= \frac{C_{11}}{C_{33}}w_2,\\ t_{ij}^{'} &= \frac{1}{C_{11}}t_{ij},\\ m_{ij}^{'} &= \frac{c_1}{\omega^* D_{24}}m_{ij},\\ T^{'} &= \frac{T}{T_0},\\ \phi^{'} &= \frac{\omega^* \epsilon_{11}}{c_1 g_{13}}\phi,\\ D_i^{'} &= \frac{c_1}{\omega^* g_{13}}D_i,\\ t^{'} &= \omega^* t,\quad \tau^{'} = \omega^* \tau,\\ c_1^2 &= \frac{C_{11}}{\rho},\\ \omega^{*2} &= \frac{C_{33}}{\rho J},\end{aligned} \tag{19}$$

where ω^* is the characteristic frequency of the material and c_1 is the longitudinal wave velocity of the medium.

By using the dimensionless quantities in equations (11)-(15), we obtain the following equations

$$\frac{\partial^2 u_1}{\partial x_1^2}+a_1\frac{\partial^2 u_1}{\partial x_3^2}+a_2\frac{\partial^2 u_3}{\partial x_1\partial x_3}+a_3\frac{\partial w_2}{\partial x_3} +a_4\frac{\partial^2\phi}{\partial x_1\partial x_3}-a_5\frac{\partial}{\partial x_1}T=a_6\frac{\partial^2 u_1}{\partial t^2}, \tag{20}$$

$$\frac{\partial^2 u_3}{\partial x_1^2}+a_7\frac{\partial^2 u_3}{\partial x_3^2}+a_8\frac{\partial^2 u_1}{\partial x_1\partial x_3}+a_9\frac{\partial w_2}{\partial x_1} +a_{10}\frac{\partial^2\phi}{\partial x_1^2}+a_{11}\frac{\partial^2\phi}{\partial x_3^2}-a_{12}\frac{\partial}{\partial x_3}T=a_{13}\frac{\partial^2 u_3}{\partial t^2}, \tag{21}$$

$$\frac{\partial^2 w_2}{\partial x_1^2}+a_{14}\frac{\partial^2 w_2}{\partial x_3^2}+a_{15}\frac{\partial u_1}{\partial x_1}+a_{16}\frac{\partial u_3}{\partial x_1}+a_{17}w_2 +a_{18}\frac{\partial\phi}{\partial x_1}=a_{19}\frac{\partial^2 w_2}{\partial t^2} \tag{22}$$

$$\frac{\partial^2\phi}{\partial x_1^2}+a_{20}\frac{\partial^2\phi}{\partial x_3^2}-a_{21}\frac{\partial^2 u_1}{\partial x_1\partial x_3} -a_{22}\frac{\partial^2 u_3}{\partial x_1^2}-a_{23}\frac{\partial^2 u_3}{\partial x_3^2}=0, \tag{23}$$

$$\left(\frac{\partial^2 T}{\partial x_1^2}+a_{24}\frac{\partial^2 T}{\partial x_3^2}\right)-\frac{\partial}{\partial t}\left(a_{25}\frac{\partial u_1}{\partial x_1}+a_{26}\frac{\partial u_3}{\partial x_1}\right) -\frac{\partial}{\partial t}\left(a_{27}\frac{\partial\phi}{\partial x_3}\right)=a_{28}\frac{\partial}{\partial t}T \tag{24}$$

where

$$\begin{aligned} &a_1=\frac{C_{73}}{C_{11}},\ a_2=\frac{C_{19}+C_{77}}{C_{11}},\\ &a_3=\frac{(C_{77}-C_{73})C_{33}}{(C_{11})^2},\\ &a_4=\frac{(g_{13}+g_{71})g_{13}}{C_{11}\epsilon_{11}},\\ &a_5=\frac{\beta_1 T_0}{C_{11}},\ a_6=\frac{\rho c_1^2}{C_{11}},\ a_7=\frac{C_{99}}{C_{37}},\\ &a_8=\frac{C_{33}+C_{91}}{C_{37}},\\ &a_9=\frac{(C_{37}-C_{33})C_{33}}{C_{37}C_{11}},\\ &a_{10}=\frac{g_{31}g_{13}}{C_{37}\epsilon_{11}},\ a_{11}=\frac{g_{93}g_{13}}{C_{37}\epsilon_{11}},\\ &a_{12}=\frac{\beta_3 T_0}{C_{37}},\ a_{13}=\frac{\rho c_1^2}{C_{37}},\ a_{14}=\frac{D_{86}}{D_{24}},\\ &a_{15}=\frac{(C_{73}-C_{33})C_{11}c_1^2}{(C_{33}D_{24})\omega^{*2}},\\ &a_{16}=\frac{(C_{77}-C_{37})C_{11}c_1^2}{(C_{33}D_{24})\omega^{*2}},\\ &a_{17}=\frac{(C_{77}-C_{33}-2C_{37})c_1^2}{D_{24}\omega^{*2}},\\ &a_{18}=\frac{(g_{31}-g_{71})C_{11}g_{13}c_1^2}{C_{33}D_{24}\epsilon_{11}\omega^{*2}},\\ &a_{19}=\frac{\rho J c_1^2}{D_{24}},\ a_{20}=\frac{\epsilon_{33}}{\epsilon_{11}},\\ &a_{21}=\frac{g_{71}+g_{13}}{g_{13}},\\ &a_{22}=\frac{g_{31}}{g_{13}},\ a_{23}=\frac{g_{93}}{g_{13}},\\ &a_{24}=\frac{k_3}{k_1},\ a_{25}=\frac{c_1^2\beta_1}{\omega^* k_1},\\ &a_{26}=\frac{c_1^2\beta_3}{\omega^* k_1},\ a_{27}=\frac{\lambda_3 c_1^2 g_{13}}{\omega^* k_1\epsilon_{11}},\ a_{28}=\frac{\rho c^* c_1^2}{\omega^* k_1} \end{aligned} \tag{25}$$

IV. PLANE WAVE PROPAGATION

Let us assume the plane wave solution of the form

$$(u_1,\, u_3,\, w_2,\, \phi,\, T)=(\overline{u_1},\, \overline{u_3},\, \overline{w_2},\, \overline{\phi},\, \overline{T})e^{\iota(\omega t-kx_1)}, \tag{26}$$

where $\overline{u_1}$, $\overline{u_3}$, $\overline{w_2}$, $\overline{\phi}$, $\overline{T}$ are functions of x_3 only, k is the wave number andω is the angular frequency.

Using equation (26) in equations (20)-(24), a system of five homogeneous equations is obtained in terms of ω and k in five unknowns $\overline{u_1}$, $\overline{u_3}$, $\overline{w_2}$, $\overline{\phi}$, $\overline{T}$, which for non-trivial solution , using Cramer's rule yield

$$\Big(A_1\frac{d^{10}}{dx_3^{10}}+A_2\frac{d^8}{dx_3^8}+A_3\frac{d^6}{dx_3^6}+A_4\frac{d^4}{dx_3^4}+A_5\frac{d^2}{dx_3^2}+A_6\Big)(\overline{u_1},\overline{u_3},\overline{w_2},\overline{\phi},\overline{T})=0, \tag{27}$$

where

$$A_1=a_1a_7a_{14}a_{20}a_{24}$$

$$A_2=a_{24}(h_1a_1a_7+h_2a_{14}a_{20}+h_4a_{20}-h_6a_{23})+h_{19}a_{14}a_{11}+a_1a_7a_{14}a_{20}\delta_{17}+h^*_{18}a_{14},$$

$$A_3=\delta_{17}(h_1a_1a_7+h_2a_{14}a_{20}+h_4a_{20}-a_{23}h_6)+a_{24}(-\delta_{10}k^2a_1a_7+\delta_1\delta_6a_{14}a_{20}+h_1h_2+a_{20}h_3-h_4k^2+h_6\delta_{13}-a_{23}h_7+h_9)+a_{20}h_{14}-a_{23}h_{11}+a_{14}h_{17}+\delta_{10}(a_{11}h_{19}+a_7h_{22})+a_{14}h_{19}\delta_8+a_{11}a_{14}h_{20}+a_7a_{14}h_{23}+\delta_{10}h^*_{18},$$

$$A_4=\delta_{17}(-\delta_{10}k^2a_1a_7+\delta_1\delta_6a_{14}a_{20}+h_1h_2+a_{20}h_3-k^2h_4+\delta_{13}h_6-a_{23}h_7+h_9)+a_{24}(h_{10}-a_{23}h_8-h_3k^2+h_7\delta_{13}+a_{20}h_5-h_2\delta_{10}k^2+h_1\delta_1\delta_6)+h_{11}\delta_{13}+h_{25}-h_{14}k^2+a_{20}h_{15}-a_{23}h_{12}+\delta_{10}(a_7h_{23}+\delta_6h_{22}+h_{17}+a_{11}h_{20}+h_{19}\delta_8)\delta_6a_{14}h_{23}+a_{14}(h_{18}+a_{11}h_{21}+\delta_6h_{23}+\delta_8h_{20}),$$

$$A_5=\delta_{17}(-\delta_{10}k^2h_2+\delta_1\delta_6h_1+a_{20}h_5-k^2h_3+\delta_{13}h_7-a_{23}h_8+h_{10})+a_{24}(-h_5k^2+h_8\delta_{13}-\delta_1\delta_6\delta_{10}k^2)+h_{12}\delta_{13}+h_{26}-h_{15}k^2+a_{20}h_{16}-a_{23}h_{13}+\delta_{10}(a_{11}h_{21}+\delta_8h_{20}+h_{18})+a_{14}\delta_8h_{21}+\delta_6\delta_{10}h_{23}+h_{24}(\delta_6a_{14}+\delta_{10}a_7),$$

$$A_6=\delta_{17}(-h_5k^2+h_8\delta_{13}-\delta_1\delta_6\delta_{10}k^2)+h_{13}\delta_{13}-h_{16}k^2+\delta_{10}(\delta_8h_{21}+\delta_6h_{24})$$

$$h_1=\delta_{10}a_{20}-k^2a_{14},$$
$$h_2=a_1\delta_6+a_7\delta_1-\delta_2\delta_5,$$
$$h_3=(a_1\delta_7-a_3\delta_5)a_{16}\iota k+(\delta_2\delta_7-a_3\delta_6)a_{15},$$
$$h_4=-a_3a_7a_{15},$$
$$h_5=\delta_1\delta_7a_{16}\iota k,$$
$$h_6=\delta_3\delta_{15}a_{14}+a_3a_{11}a_{15}+a_1a_{11},$$
$$h_7=\delta_7\delta_{11}(a_1+\delta_1)+\delta_3(\delta_5\delta_{10}-\delta_7a_{15}-\delta_{11}a_3)+a_3a_{15}\delta_8+a_{11}\delta_1+a_1\delta_8,$$
$$h_8=\delta_1\delta_7\delta_{11}+\delta_1\delta_8,$$
$$h_9=(\delta_{11}a_7+a_{11}a_{16}\iota k)\delta_{12}a_3,$$
$$h_{10}=\delta_{11}\delta_{12}(-\delta_2\delta_7+a_3\delta_6)+a_{16}\delta_{12}(a_3\iota k\delta_8-\iota\delta_3\delta_7k),$$
$$h_{11}=-a_3a_{15}\delta_9\delta_{16},$$
$$h_{12}=\delta_9\delta_{11}\delta_{14}a_3+\delta_4\delta_7\delta_{16}a_{15},$$
$$h_{13}=-\delta_{11}\delta_{14}\delta_4\delta_7,$$
$$h_{14}=a_3a_{15}\delta_9\delta_{15},$$
$$h_{15}=a_3a_{16}\delta_9\delta_{14}\iota k-\delta_4\delta_7\delta_{15}a_{15},$$
$$h_{16}=-\delta_4\delta_7\delta_{14}a_{16}\iota k,$$
$$h_{17}=-\delta_1\delta_9(a_{23}\delta_{16}+\delta_{15}a_{20})-\delta_2\delta_9(\delta_{14}a_{20}-\delta_{12}\delta_{16})+\delta_3\delta_9(\delta_{12}\delta_{15}+\delta_{14}a_{23})+\delta_4\delta_5(a_{20}\delta_{15}+\delta_{16}a_{23}),$$
$$h_{18}=\delta_9(\delta_1\delta_{13}\delta_{16}+\delta_1\delta_{15}k^2-\delta_3\delta_{13}\delta_{14}+\delta_2\delta_{14}k^2)-\delta_4\delta_5(\delta_{15}k^2+\delta_{13}\delta_{16}),$$
$$h_{19}=\delta_2\delta_{12}a_{24},$$
$$h_{20}=\delta_2\delta_{12}\delta_{17}-\delta_4(\delta_{12}\delta_{15}+\delta_{14}a_{23}),$$
$$h_{21}=\delta_4\delta_{13}\delta_{14},$$
$$h_{22}=-\delta_3\delta_{12}a_{24},$$
$$h_{23}=-\delta_3\delta_{12}\delta_{17}+\delta_4(\delta_{12}\delta_{16}-\delta_{14}a_{26}),$$
$$h_{24}=\delta_4\delta_{14}k^2,$$
$$h^*_{18}=-\delta_9a_1(a_{23}\delta_{16}+a_{20}\delta_{15}),$$
$$\delta_1=a_6\omega^2-k^2,\quad \delta_2=-a_2\iota k,\quad \delta_3=-a_4\iota k,$$
$$\delta_4=a_5\iota k(1+\iota\nu\omega),\quad \delta_5=-a_8\iota k,$$
$$\delta_6=a_{13}\omega^2-k^2,\quad \delta_7=-a_9\iota k,$$
$$\delta_8=-a_8k^2,\quad \delta_9=-a_{12},\quad \delta_{10}=a_{19}\omega^2+a_{17}-k^2,$$
$$\delta_{11}=a_{18}\iota k,\quad \delta_{12}=a_{21}\iota k,\quad \delta_1=a_6\omega^2-k^2,$$
$$\delta_2=-a_2\iota k,\quad \delta_3=-a_4\iota k,\quad \delta_4=a_5\iota k(1+\iota\nu\omega),$$
$$\delta_5=-a_8\iota k,\quad \delta_6=a_{13}\omega^2-k^2,\quad \delta_7=-a_9\iota k,$$
$$\delta_8=-a_8k^2,\quad \delta_9=-a_{12},\quad \delta_{10}=a_{19}\omega^2+a_{17}-k^2,$$
$$\delta_{11}=a_{18}\iota k,\quad \delta_{12}=a_{21}\iota k,\quad \delta_{13}=a_{22}k^2,$$
$$\delta_{14}=-a_{25}k\omega,\quad \delta_{15}=-a_{26}\iota\omega,\quad \delta_{16}=-a_{27}\iota\omega,$$
$$\delta_{17}=-a_{28}(\iota\omega-\omega^2\tau)-k^2.$$

Equation (27) is now in terms of $\omega\ and\ k$. The roots of the equation (27) gives the velocities of five plane waves in the decreasing order of the velocities, i.e quasi longitudinal displacement wave (quasi LD wave), quasi thermal wave (quasi T wave), quasi CD-I, quasi CD-II wave and electric potential wave (PE wave).

V. REFLECTION AND TRANSMISSION

A homogeneous orthotropic micropolar piezothermoelastic half-space is considered. A plane wave making an angle θ_0 with x_3- axis becomes incident at the free surface. This wave results in five reflected wave modes in medium M_1. In medium M_1 reflected wave modes are represented by the quasi LD wave, quasi thermal wave, quasi CD-I (transverse) wave, quasi CD-II (micropolar) wave and one other mode corresponding to electric potential wave mode i.e. PE wave mode.

The formal solution for the mechanical displacements, microrotation, electric potential and temperature distribution in medium M_1 are

$$u_1(x_3) = (B_{01}e^{-\lambda_1 x_3} + B_1 e^{\lambda_1 x_3} + B_{02}e^{-\lambda_2 x_3} + B_2 e^{\lambda_2 x_3} + B_{03}e^{-\lambda_3 x_3} + B_3 e^{\lambda_3 x_3} + B_{04}e^{-\lambda_4 x_3} + B_4 e^{\lambda_4 x_3} + B_{05}e^{-\lambda_5 x_3} + B_5 e^{\lambda_5 x_3})e^{i(\omega t - kx_1)}, \tag{28}$$

$$u_3(x_3) = (m_1 B_{01}e^{-\lambda_1 x_3} + m_1 B_1 e^{\lambda_1 x_3} + m_2 B_{02}e^{-\lambda_2 x_3} + m_2 B_2 e^{\lambda_2 x_3} + m_3 B_{03}e^{-\lambda_3 x_3} + m_3 B_3 e^{\lambda_3 x_3} + m_4 B_{04}e^{-\lambda_4 x_3} + m_4 B_4 e^{\lambda_4 x_3} + m_5 B_{05}e^{-\lambda_5 x_3} + m_5 B_5 e^{\lambda_5 x_3})e^{i(\omega t - kx_1)}, \tag{29}$$

$$w_2(x_3) = (n_1 B_{01}e^{-\lambda_1 x_3} + n_1 B_1 e^{\lambda_1 x_3} + n_2 B_{02}e^{-\lambda_2 x_3} + n_2 B_2 e^{\lambda_2 x_3} + n_3 B_{03}e^{-\lambda_3 x_3} + n_3 B_3 e^{\lambda_3 x_3} + n_4 B_{04}e^{-\lambda_4 x_3} + n_4 B_4 e^{\lambda_4 x_3} + n_5 B_{05}e^{-\lambda_5 x_3} + n_5 B_5 e^{\lambda_5 x_3})e^{i(\omega t - kx_1)}, \tag{30}$$

$$\phi(x_3) = (g_1 B_{01}e^{-\lambda_1 x_3} + g_1 B_1 e^{\lambda_1 x_3} + g_2 B_{02}e^{-\lambda_2 x_3} + g_2 B_2 e^{\lambda_2 x_3} + g_3 B_{03}e^{-\lambda_3 x_3} + g_3 B_3 e^{\lambda_3 x_3} + g_4 B_{04}e^{-\lambda_4 x_3} + g_4 B_4 e^{\lambda_4 x_3} + g_5 B_{05}e^{-\lambda_5 x_3} + g_5 B_5 e^{\lambda_5 x_3})e^{i(\omega t - kx_1)}, \tag{31}$$

$$T(x_3) = (l_1 B_{01}e^{-\lambda_1 x_3} + l_1 B_1 e^{\lambda_1 x_3} + l_2 B_{02}e^{-\lambda_2 x_3} + l_2 B_2 e^{\lambda_2 x_3} + l_3 B_{03}e^{-\lambda_3 x_3} + l_3 B_3 e^{\lambda_3 x_3} + l_4 B_{04}e^{-\lambda_4 x_3} + l_4 B_4 e^{\lambda_4 x_3} + l_5 B_{05}e^{-\lambda_5 x_3} + l_5 B_5 e^{\lambda_5 x_3})e^{i(\omega t - kx_1)}, \tag{32}$$

where $\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5$ are the velocities of reflected quasi LD wave, quasi T wave, quasi CD-I wave quasi CD-II wave and PE wave mode respectively in medium M_1 and

$$m_i = \frac{\Delta_1}{\Delta}, n_i = \frac{\Delta_2}{\Delta}, \quad g_i = \frac{\Delta_3}{\Delta}, l_i = \frac{\Delta_4}{\Delta}, \quad i = 1,\ 2,\ 3,\ 4,\ 5 \tag{33}$$

$$\Delta = \begin{vmatrix} a_7\lambda_i^2 + \delta_6 & \delta_7 & a_{11}\lambda_i^2 + \delta_8 & \delta_9\lambda_i \\ -a_{16}\iota k & a_{14}\lambda_i^2 + \delta_{10} & \delta_{11} & 0 \\ -a_{23}\lambda_i^2 + \delta_{13} & 0 & a_{20}\lambda_i^2 - k^2 & 0 \\ \delta_{15}\lambda_i & 0 & \delta_{16}\lambda_i & a_{24}\lambda_i^2 + \delta_{17} \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} \delta_5\lambda_i & \delta_7 & a_{11}\lambda_i^2 + \delta_8 & \delta_9\lambda_i \\ a_{15}\lambda_i & a_{14}\lambda_i^2 + \delta_{10} & \delta_{11} & 0 \\ \delta_{12}\lambda_i & 0 & a_{20}\lambda_i^2 - k^2 & 0 \\ \delta_{14} & 0 & \delta_{16}\lambda_i & a_{24}\lambda_i^2 + \delta_{17} \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} \delta_5\lambda_i & a_7\lambda_i^2 + \delta_6 & a_{11}\lambda_i^2 + \delta_8 & \delta_9\lambda_i \\ a_{15}\lambda_i & -a_{16}\iota k & \delta_{11} & 0 \\ \delta_{12}\lambda_i & -a_{23}\lambda_i^2 + \delta_{13} & a_{20}\lambda_i^2 - k^2 & 0 \\ \delta_{14} & \delta_{15}\lambda_i & \delta_{16}\lambda_i & a_{24}\lambda_i^2 + \delta_{17} \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} \delta_5\lambda_i & a_7\lambda_i^2 + \delta_6 & \delta_7 & \delta_9\lambda_i \\ a_{15}\lambda_i & -a_{16}\iota k & a_{14}\lambda_i^2 + \delta_{10} & 0 \\ \delta_{12}\lambda_i & -a_{23}\lambda_i^2 + \delta_{13} & 0 & 0 \\ \delta_{14} & \delta_{15}\lambda_i & 0 & a_{24}\lambda_i^2 + \delta_{17} \end{vmatrix}$$

$$\Delta_4 = \begin{vmatrix} \delta_5\lambda_i & a_7\lambda_i^2 + \delta_6 & \delta_7 & a_{11}\lambda_i^2 + \delta_8 \\ a_{15}\lambda_i & -a_{16}\iota k & a_{14}\lambda_i^2 + \delta_{10} & \delta_{11} \\ \delta_{12}\lambda_i & -a_{23}\lambda_i^2 + \delta_{13} & 0 & a_{20}\lambda_i^2 - k^2 \\ \delta_{14} & \delta_{15}\lambda_i & 0 & \delta_{16}\lambda_i \end{vmatrix}$$

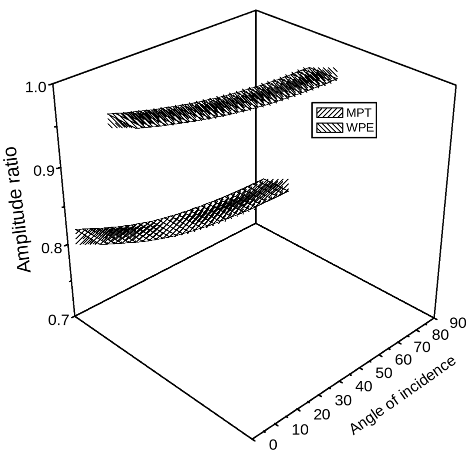

Fig. 2. Variations of amplitude ratio Z_1 with angle of incidence (QL- wave)

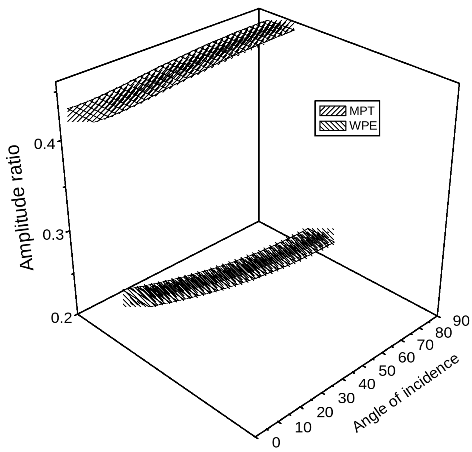

Fig. 3. Variations of amplitude ratio Z_2 with angle of incidence (QL- wave)

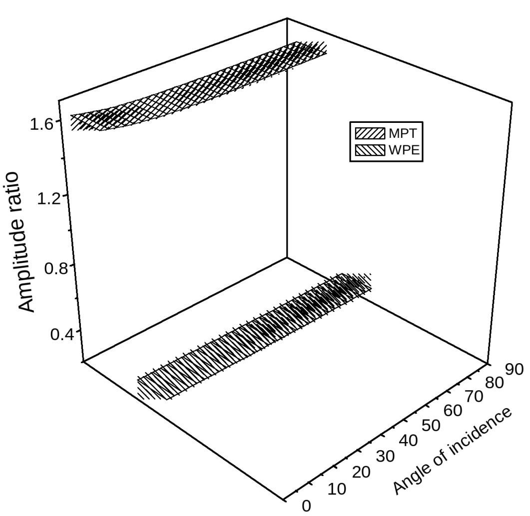

Fig. 4. Variations of amplitude ratio Z_3 with angle of incidence (QL -wave)

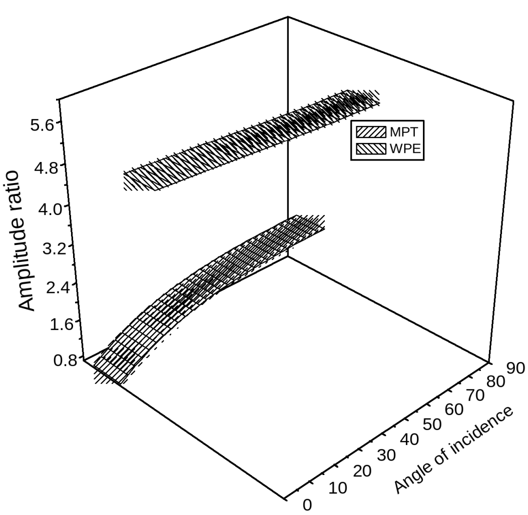

Fig. 5. Variations of amplitude ratio Z_4 with angle of incidence (QL -wave)

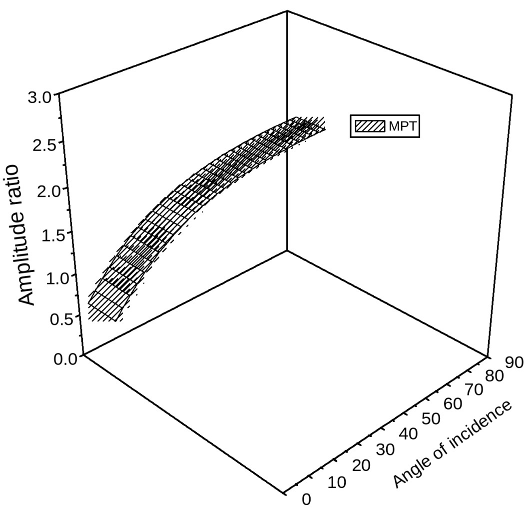

Fig. 6. Variations of amplitude ratio Z_5 with angle of incidence (QL- wave)

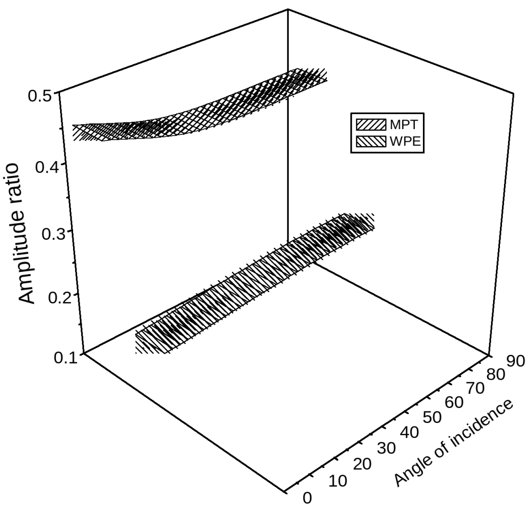

Fig. 7. Variations of amplitude ratio Z_1 with angle of incidence (QT-wave)

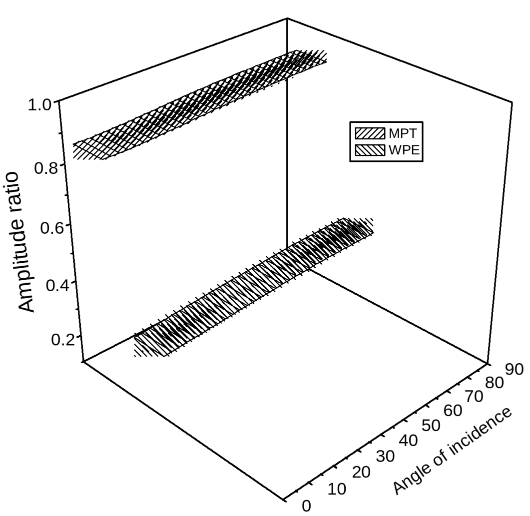

Fig. 8. Variations of amplitude ratio Z_2 with angle of incidence (QT-wave)

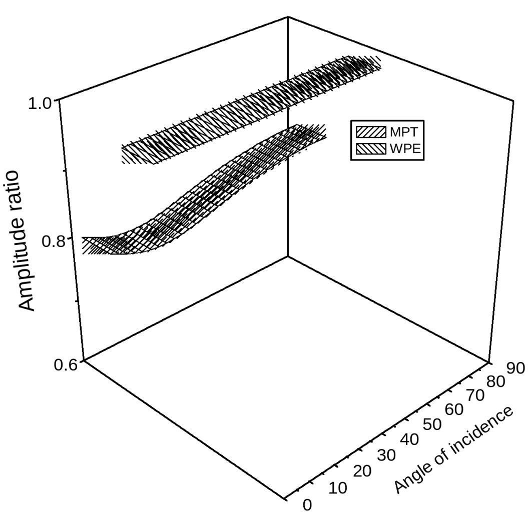

Fig. 9. Variations of amplitude ratio Z_3 with angle of incidence (QT- wave)

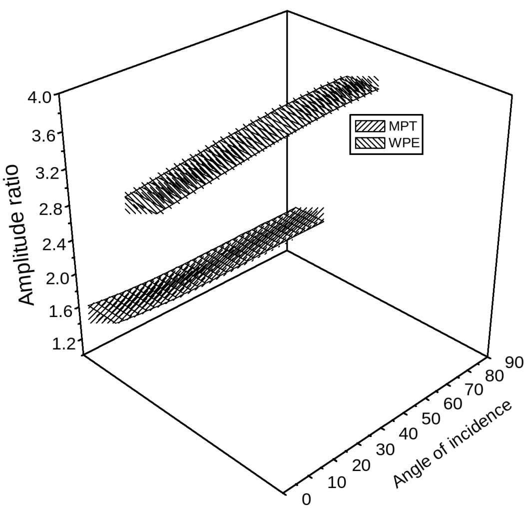

Fig. 10. Variations of amplitude ratio Z_4 with angle of incidence(QT- wave)

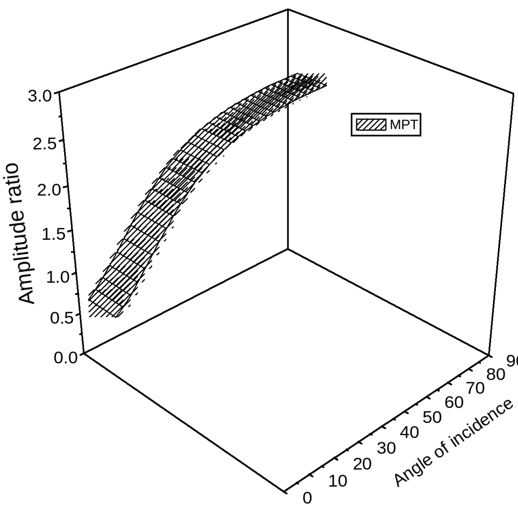

Fig. 11. Variations of amplitude ratio Z_5 with angle of incidence(QT- wave)test

VI. BOUNDARY CONDITIONS

The appropriate boundary conditions at an interface $x_3 = 0$ are given by

$$t_{33} = 0,\ t_{31} = 0,\ m_{32} = 0, \frac{\partial T}{\partial x_3} = 0,\ D_3 = 0. \quad (34)$$

Using the equations (28)-(32), we find that the boundary conditions are satisfied if and only if:

$$\frac{\sin\theta_0}{v} = \frac{k}{\omega}, \quad (35)$$

where v is the velocity of the incident wave at an interface.

Making use of equations (28) to (32) in equation (34) and using equation (35), we obtain a system of five homogeneous equations as:

$$\sum_{j=1}^{10} a_{ij} B_j = 0\ ;\ \ (i = 1,\ 2,\ 3,\ 4,\ 5)\,, \quad (36)$$

where

$$a_{1i} = -d_1 \iota k - d_2\lambda_i m_i + d_3\lambda_i g_i - d_4 l_i,$$

$$a_{1j} = -d_1 \iota k + d_2\lambda_i m_i - d_3\lambda_i g_i - d_4 l_i,$$

$$a_{2i} = -a_1\lambda_i + (d_6 g_i - d_5 m_i)\iota k,\ a_{2j} = \\ = a_1\lambda_i + (d_6 g_i - d_5 m_i)\iota k,$$

$$a_{3i} = -d_7\lambda_i n_i,\ a_{3j} = d_7\lambda_i n_i,\ a_{4i} = -\lambda_i l_i,\ a_{4j} = \lambda_i l_i,$$

$$a_{5j} = d_9\lambda_i g_i - \iota k d_{10} + d_{11}\lambda_i m_1. \quad (37)$$

(a) When quasi LD wave is incident:

$$B_2 = B_3 = B_4 = B_5 = 0.$$

Dividing the set of equations throughout by B_1, we obtain a system of ten non-homogeneous equations in ten unknowns which can be solved by Crammer's rule and we have

$$Z_i = \frac{B_i}{B_1} = \frac{\Gamma_i^1}{\Gamma}\ ;\ i = 1,\ 2,\ 3,\ 4,\ 5.$$

(b) When quasi T wave is incident: $B_1 = B_3 = B_4 = B_5 = 0$ and

$$Z_i = \frac{B_i}{B_2} = \frac{\Gamma_i^2}{\Gamma},\quad ;\ i = 1,\ 2,\ 3,\ 4,\ 5,$$

where

$$\Gamma = |a_{ii+5}|_{5\times 5}\,,$$

and $\Gamma_i^p (i = 1, 2, 3, 4, 5)\,(p = 1,\ 2,\ 3,\ 4,\ 5)$ can be obtained by replacing, respectively the 1st, 2nd,…, 5th columns of Γ by $[-a_{1p},\ -a_{2p},\ a_{3p},\ a_{4p}, a_{5p}]^T$.

VII. PARTICULAR CASES

(a) If we neglect the piezoelectric effect in mediumM_1, we obtain amplitude ratios at the free surface of orthotropic piezothermoelastic solid with changed values of a_{ij} as

$$a_{1i} = -d_1 \iota k - d_2\lambda_i m_i + d_3\lambda_i g_i - d_4 l_i,$$

$$a_{1j} = -d_1 \iota k + d_2\lambda_i m_i - d_3\lambda_i g_i - d_4 l_i,$$

$$a_{2i} = -a_1\lambda_i + (d_6 g_i - d_5 m_i)\iota k,$$

$$a_{2j} = a_1\lambda_i + (d_6 g_i - d_5 m_i)\iota k,$$

$$a_{3i} = -d_7\lambda_i n_i,\ a_{3j} = d_7\lambda_i n_i,$$

$$a_{4i} = -\lambda_i l_i,\ a_{4j} = \lambda_i l_i,$$

VIII. NUMERICAL RESULTS AND DISCUSSION

The physical data for medium M_1 is given by

$C_{11} = 7.46 \times 10^{10}$ Nm^{-2},
$C_{19} = 3.9 \times 10^{10}$ Nm^{-2},
$C_{33} = 1.37 \times 10^{9}$ Nm^{-2},
$C_{99} = 8.39 \times 10^{9}$ Nm^{-2},
$C_{91} = 0.399 \times 10^{9}$ Nm^{-2},
$C_{77} = 0.0138 \times 10^{9}$ Nm^{-2},
$C_{37} = 0.134 \times 10^{9}$ Nm^{-2},
$C_{73} = 1.32 \times 10^{9}$ Nm^{-2},
$g_{13} = -0.142 \times 10^{-3}$ cm^{-2},
$g_{71} = -0.165 \times 10^{-3}$ cm^{-2},
$g_{93} = 0.351 \times 10^{-3}$ cm^{-2},
$g_{31} = -0.139 \times 10^{-3}$ cm^{-2},
$\varepsilon_{11} = 8.29 \times 10^{-11}$ Nm$^{-2}/K$,
$\varepsilon_{33} = 9.07 \times 10^{-11}$ Nm$^{-2}/K$,
$\lambda_3 = 7.6 \times 10^{-6}$ cm$^{-2}/K$,
$\tau = 0.8$ s,
$k_1 = 9.5$ Wm^{-1}K^{-1},
$k_2 = 9.7$ Wm^{-1}K^{-1},
$\beta_1 = 0.670 \times 10^{5}$C^{2}N^{-1}m^{-2},
$\beta_3 = 0.581 \times 10^{5}$C^{2}N^{-1}m^{-2},
$\nu = 0.268$,
$T_0 = 298$ K,
$\rho = 5504$ kg m^{-3},
$c^* = 2.64 \times 10^{2}$ N m kg^{-1}s K^{-1},
$J = 0.02 \times 10^{-11}$ m^{-2},
$D_{24} = 0.134$ N,
$D_{86} = 0.243$ N.

Fig. 2–11 shows the variations of amplitude ratios with the angle of incidence for incidence of plane waves at an interface. In Figs. 2–11 MPT corresponds to amplitude ratios in the orthotropic micropolar piezothermoelastic solid, WPE corresponds to amplitude ratios in the orthotropic micropolar thermoelastic solid.

VIII. 1. Incidence of quasi ld wave (ql-wave)

Figs. 2–6 represent the variations of amplitude ratios ; $Z_i \leq i \leq 5$ with the angle of incidenceθ_0 for incidence of QL-wave. Fig. 2 shows that the values of amplitude ratio $|Z_1|$ for MPT and WPE decrease with increase in the angle of incidence and the values of amplitude ratio for WPE are more than the values for MPT in the whole range. Fig. 3 shows that the values of amplitude ratio $|Z_2|$ for MPT increase in the whole range, except in the initial range where it decreases and remains more than the values for WPE in the whole range. From Fig. 4 it is seen that the values of amplitude ratio $|Z_3|$ for MPT decrease in the range $0^0 \leq \theta_0 < 46^0$ and then increase with the angle of incidence, while the values for WPE decrease in the whole range. The values of the amplitude ratio for MPT are more than the values for WPE in the whole range. Fig. 5 shows that the values of amplitude ratio $|Z_4|$ for WPE are greater than the values for MPT in the whole range. Fig. 7 shows that the values of amplitude ratio $|Z_5|$ for MPT increase as θ_0 increases.

VIII. 2. Incidence of quasi t wave (qt-wave)

Figs. 7–11 represent the variations of amplitude ratios $|Zi|$; $1 \leq i \leq 5$ with the angle of incidence θ_0 for incidence of QT-wave. Fig.7 depicts that the values of amplitude ratio $|Z_1|$ for MPT decrease with the angle of incidence, while the values for WPE increase from normal incidence to grazing incidence. The values for MPT remain more than the value for WPE in the whole range. Fig.8 shows the variation of amplitude ratio $|Z_2|$ with the angle of incidence. The values of amplitude ratio for MPT and WPE increase with the angle of incidence. The values of the amplitude ratio for MPT are more than the values for WPE. Fig.9 shows that the values of amplitude ratio $|Z_3|$ for MPT decrease in the initial range and then increase and the values for WPE get increased with the angle of incidence. The amplitude ratio increases in the absence of the piezoelectric effect. It is seen from Fig. 10 that the amplitude ratio$|Z_4|$ for MPT decrease and WPE get increased in the whole range. The values of amplitude ratio in the absence of the piezoelectric effect are greater than the values in the presence of the piezoelectric effect. Fig. 11 shows that the values of amplitude ratio $|Z_5|$ for MPT increase in the whole range.

IX. CONCLUSION

The reflection coefficients of various plane quasi waves on incidence of quasi LD wave and quasi T wave at a free surface of orthotropic micropolar piezothermoelastic medium for L-S theory are obtained. It is noticed that when quasi LD wave is incident, the values of amplitude ratios of reflected quasi T, quasi CD-I (Transverse) in the absence of the piezoelectric effect are smaller that reveals the piezoelectric effect.

It is seen that when the quasi T wave is incident, the piezoelectric effect decreases the magnitude of amplitude ratio of the reflected quasi CD-I (transverse) wave and quasi CD-II (micropolar) wave modes.

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 DOI:10.12921/cmst.2016.0000069