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Propagation of Harmonic Plane Waves in a Rotating Elastic Medium under Two-Temperature Thermoelasticity with Relaxation Parameter

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Abstract: The present work investigates the propagation of harmonic plane waves in an isotropic and homogeneous elastic medium that is rotating with uniform angular velocity by employing the two-temperature generalized thermoelasticity, recently introduced by Youssef (IMA Journal of Applied Mathematics, 71, 383-390, 2006). Dispersion relation solutions for longitudinal as well as transverse plane waves are obtained analytically. Asymptotic expressions of several important characterizations of the wave fields, such as phase velocity, specific loss, penetration depth, amplitude coefficient factor and phase shift of thermodynamic temperature are obtained for high frequency as well as low frequency values. A critical value of the two-temperature parameter for the low frequency case is obtained. Using Mathematica, numerical values of the wave fields at intermediate values of frequency and for various values of the two-temperature parameter are computed. A detailed analysis of the effects of rotation on the plane wave is presented on the basis of analytical and numerical results. An in-depth comparative analysis of our results with the corresponding results of the special cases of absence of rotation of the body and with the case of generalized thermoelasticity is also presented. The most significant points are highlighted.

Key words: two-temperature thermoelasticity, generalized thermoelasticity, rotating elastic medium, phase velocity, specific loss, penetration depth.

I. INTRODUCTION

A lot of attention has been paid in recent years to the theory of thermoelasticity with two-temperatures. It must be recalled that this theory was first formulated by Chen and Gurtin [1] and Chen et al. [2] and it proposes that the heat conduction on a deformable body depends upon two different temperatures: the conductive temperature and the thermodynamic temperature, the difference between these two-temperatures being proportional to the heat supply. In the absence of heat supply the two-temperatures are equal for the time-independent situation. However, for time-dependent cases, these two temperatures are in general different, regardless of the heat supply [2]. Prior to this, Gurtin and Willium [3] pointed out that "there are no a priori grounds for assuming that the second law of thermodynamics for continuous bodies involves only a single temperature and that it is more logical to assume a second law in which the entropy contribution due to heat-conduction is governed by one temperature, and that of the heat supply by another". They assumed the Clausius-Duhem inequality (second law of thermodynamics) in the form

$$\frac{d}{dt}\int_B s dv \geq -\int_{\partial B} \frac{q.n}{\varphi} dA + \int_B \frac{r}{\theta} dv,$$

where q is the heat flux vector, r is the heat supplied per unit volume from an external source, s is the entropy per unit volume, φ is the conductive temperature and θ is the thermodynamic temperature. The key element that makes this theory different from the classical theory of thermoelasticity is the material parameter α (> 0) and in the limiting case when $\alpha \to 0, \varphi \to \theta,$ so that this theory reduces to the classical theory. The uniqueness and reciprocity theorems for the two-temperature thermoelasticity theory [2] in case of a homogeneous and isotropic solid are given by Iesan [4]. Subsequently, several investigations (see Warren and Chen [5], Warren [6], Amos [7], Chakrabarti [8], Ting [9], Colton and Wimp [10] and the references therein) have been pursued by employing the linearized version of this theory. This two-temperature thermoelasticity theory, also known as 2TT is being revisited once again during the last few years. The existence, structural stability, convergence and spatial behavior in 2TT have been discussed in detail by Quintanilla [11]. The propagation of harmonic plane waves in the same theory is discussed by Puri and Jordan [12]. Youssef [13] has introduced the modification of 2TT in the form of two generalized thermoelasticity theories, namely Lord-Shulman theory [14] and the Green-Lindsay theory [15], by introducing thermal relaxation parameters into the governing equations. Magana and Quintanilla [16] studied the uniqueness and growth of solutions for the equations under Youssef's theory [13]. Several research works [17-27] have been carried out very recently on the basis of this theory and indicated some significant features of the theory.

In the present work we propose to investigate the propagation of harmonic plane waves in an infinite rotating elastic medium under the theory proposed by Youssef [13]. It is worth mentioning that the propagation of harmonic plane waves in elastic medium have been the subject of interest for several years due to its great applications in engineering science. Chadwick and Sneddon [28] and Chadwick [29] studied the propagation of plane waves in classical thermoelasticity. The propagation of plane waves in the context of generalized thermoelasticity with one relaxation time introduced by Lord and Shulman [14] is discussed by Nayfeh and Nemat-Nasser [30] and later on by Puri [31]. The propagation and stability of harmonically time-dependent thermoelastic plane waves in temperature-rate-dependent thermoelasticity theory developed by Green and Lindsay [15] is reported by Agarwal [32]. Investigation on plane waves in the context of the thermoelasticity theory without energy dissipation (Green–Naghdi [33]) is discussed by Chandrasekharaiah [34]. In a recent work, Puri and Jordan [35] have investigated the propagation of plane waves in the context of the GN-III thermoelasticity theory [36]. Wave propagation in an infinite rotating elastic solid medium was investigated by Schenberg and Censor [37] and later on by several other researchers like Puri [38], Chandrasekharaiah and Srikantiah [39], Roychoudhuri [40], Roychoudhuri and Bandyopadhyay [41], Chandrasekharaiah [42, 43], Othman [44], Auriault [45], Sharma and Othman [46].

In the present work, we consider a homogeneous and isotropic rotating elastic medium and employ the linear theory of two-temperature generalized thermoelasticity. After obtaining the dispersion relation solutions of both the longitudinal and transverse plane waves, we find the asymptotic expansions of several qualitative characterizations of the wave fields, such as phase velocity, specific loss, penetration depth, amplitude coefficient factor and phase shift of the thermodynamic temperature for the high and low frequency values. It should be mentioned here that in earlier studies concerning plane waves only the behavior of the phase velocity, specific loss, penetration depth, etc., are discussed on the basis of the asymptotic expressions for the high and low frequency values. However, being motivated by the work reported by Puri and Jordan [12] we also make an attempt to observe the behavior of the above-mentioned quantities for intermediate values of frequency with the help of computational work. For this, the numerical values of wave characterizations for intermediate values of frequency and for various values of rotational angle are computed, and the analytical results are examined. The results are shown in several graphs. A detailed analysis of the results highlighting the effects of rotation on various wave fields is presented. The basic differences in the behavior of wave characterizations under generalized thermoelasticity reported by Lord and Shulman [14] and the two-temperature generalized thermoelasticity introduced by Youssef [13] are highlighted in a detailed way, which have not been investigated till date.

II. BASIC GOVERNING EQUATIONS AND PROBLEM FORMULATION

We consider a linear homogeneous isotropic thermally conducting elastic medium that is rotating uniformly with the angular velocity $\mathbf{\Omega} = \Omega_0\, \mathbf{p},$ where $\mathbf{p}$ is the unit vector that represents the direction of the axis of rotation. The displacement equation of motion in the rotating frame of reference involves two additional terms: the centripetal acceleration $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})$ due to the time-varying motion only and the coriolis acceleration $2\mathbf{\Omega} \times \dot{\mathbf{u}},$ where $\mathbf{u}$ is the displacement vector. The equations governing the displacement and thermal fields in the absence of body forces and heat sources under the two-temperature generalized thermoelasticity theory (Youssef [13]) with usual indicial notations are therefore taken as follows:

Stress-strain temperature relations:

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \gamma\theta\delta_{ij}. \tag{1}$$

Strain-displacement relations:

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right). \tag{2}$$

Heat conduction equation without heat source:

$$K\varphi_{,ii} = \rho c_E\left(\frac{\partial\theta}{\partial t} + \tau_1\frac{\partial^2\theta}{\partial t^2}\right) + \gamma\varphi_0\left(\frac{\partial}{\partial t} + \tau_1\frac{\partial^2}{\partial t^2}\right)e_{kk}. \tag{3}$$

The stress equation of motion in a rotating medium without body force:

$$\begin{aligned}(\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) + \mu\nabla^2\mathbf{u} - \gamma\nabla\theta &= \\ &= \rho\left[\ddot{\mathbf{u}} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u}) + 2\mathbf{\Omega}\times\dot{\mathbf{u}}\right].\end{aligned} \tag{4}$$

The thermodynamic temperature, θ is related to the conductive temperature, φ as

$$\varphi - \theta = \alpha\nabla^2\varphi. \tag{5}$$

In the above set of Eqs. (1-5), u is the component of displacement vector, σ_{ij} and e_{ij} are the components of stress tensor, and strain tensor, respectively. θ and φ, respectively, are the thermodynamic temperature and conductive temperature measured from a constant reference temperature φ_0. λ, μ are Lame's elastic constants, ρ is the mass density, K is thermal conductivity of the material, τ_1 is the thermal relaxation parameter and c_E is the specific heat at constant strain. $\gamma = (3\lambda+2\mu)\alpha_t$, where α_t is the coefficient of linear thermal expansion. The comma notation is used to represent the partial derivatives with respect to the space variables, the over-headed dots denote partial derivative with respect to time variable, t and the bold faced notations are used to denote the vector quantities. $\alpha > 0$, (a scalar), is the two-temperature parameter.

Now, using Eq. (5), we write the equation of motion (4) in the form

$$\begin{aligned}(\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) + \mu\nabla^2\mathbf{u} - \gamma\nabla\left(\varphi - \alpha\nabla^2\varphi\right) &= \\ &= \rho\left[\ddot{\boldsymbol{u}} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u}) + 2\mathbf{\Omega}\times\dot{\mathbf{u}}\right].\end{aligned} \tag{6}$$

From Eqs. (3) and (5), we get

$$\begin{aligned}&\left[K + \alpha\rho c_E\left(\frac{\partial}{\partial t} + \tau_1\frac{\partial^2}{\partial t^2}\right)\right]\nabla^2\varphi = \\ &= \rho c_E\left(\dot{\varphi} + \tau_1\ddot{\varphi}\right) + \gamma\varphi_0\left(\frac{\partial}{\partial t} + \tau_1\frac{\partial^2}{\partial t^2}\right)\nabla\cdot\mathbf{u}\end{aligned} \tag{7}$$

Now, we use the following dimensionless quantities and notations:

$$x' = c_0\eta x,\ \tau_1' = c_0^2\eta\tau_1,\ \ t' = c_0^2\eta t,\ \varphi' = \frac{\varphi}{\varphi_0},\ u' = c_0\eta u,$$

$$c_0{}^2 = \frac{(\lambda+2\mu)}{\rho},\ \eta = \frac{\rho c_E}{K},\ \Omega = \frac{\Omega}{c_0^2\eta},\ a_1 = \frac{\gamma\varphi_0}{(\lambda+2\mu)},$$

$$a_2 = \frac{\gamma}{K\eta},\ \alpha' = c_0^2\eta^2\alpha,\ \ \mu_1 = \frac{\mu}{\lambda+2\mu},\ \lambda_1 = 1-\mu_1.$$

Equations (6) and (7) then transform to the dimensionless forms (after dropping the primes for convenience) as

$$\begin{aligned}\lambda_1\nabla(\nabla\cdot\mathbf{u}) + \mu_1\nabla^2\mathbf{u} - a_1\nabla\left(\varphi - \alpha\nabla^2\varphi\right) &= \\ &= \ddot{\mathbf{u}} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{u}) + 2\mathbf{\Omega}\times\dot{\mathbf{u}},\end{aligned} \tag{8}$$

$$\begin{aligned}&\left[1 + \alpha\left(\frac{\partial}{\partial t} + \tau_1\frac{\partial^2}{\partial t^2}\right)\right]\nabla^2\varphi = \\ &= \left(\frac{\partial}{\partial t} + \tau_1\frac{\partial^2}{\partial t^2}\right)\varphi + a_2\left(\frac{\partial}{\partial t} + \tau_1\frac{\partial^2}{\partial t^2}\right)\nabla\cdot\mathbf{u}.\end{aligned} \tag{9}$$

III. PLANE HARMONIC WAVES SOLUTIONS AND DISPERSION RELATIONS

In order to study the propagation of plane harmonic waves, the solutions of Eqs. (8) and (9) are assumed in the form

$$(\mathbf{u}, \varphi) = (\mathbf{a}, b)\exp\left[i\left(\omega t - \eta\mathbf{n}\cdot x\right)\right], \tag{10}$$

where $\mathbf{a},\ b$ are arbitrary constants not both zero, ω is the dimensionless angular frequency, η is the wave number of the wave and $\mathbf{n}$ is the unit vector along the direction of propagation of the wave. ω is assumed to be a positive real and $\mathbf{a}, b, \eta$ are allowed to be complex. In order to ensure the bounded amplitudes, $\mathrm{Im}[\eta] \le 0$ must hold.

Substituting (10) into the Eqs. (8) and (9), we get

$$\begin{aligned}&\left(\omega^2 + \Omega_0{}^2 - \mu_1\eta^2\right)\mathbf{a} - \lambda_1\eta^2(\mathbf{a}\cdot\mathbf{n})\mathbf{n} + \\ &-(\mathbf{\Omega}\cdot\mathbf{a})\mathbf{\Omega} - 2i\omega(\mathbf{\Omega}\times\mathbf{a}) = -ia_1 b\eta\left(1+\alpha\eta^2\right)\mathbf{n},\end{aligned} \tag{11}$$

$$\begin{aligned}&\left[-\eta^2 + \left(1+\alpha\eta^2\right)\left(-i\omega + \tau_1\omega^2\right)\right]b = \\ &= a_2(\mathbf{a}\cdot\mathbf{n})\eta\omega(1+i\tau_1\omega).\end{aligned} \tag{12}$$

Here, $\Omega_0 = |\Omega|$ is the magnitude of $\mathbf{\Omega}$. We note that if $\mathbf{a} = 0$, then Eq. (11) yields $b = 0$, but $\mathbf{a}$ and b both cannot vanish together for the waves of the desired type to occur. Therefore, we take $\mathbf{a}$ to be a non-zero vector. Now we will analyze purely shear waves and purely dilatational waves on the basis of Eqs. (11) and (12).

Case 1: Shear waves

For purely shear waves, we have $\mathbf{a}\cdot\mathbf{n}=0$, and Eqs. (11) and (12) become

$$\begin{aligned}\left(\omega^2+\Omega_0{}^2-\mu_1\eta^2\right)\mathbf{a}-(\mathbf{\Omega}\cdot\mathbf{a})\mathbf{\Omega}-2i\omega(\mathbf{\Omega}\times\mathbf{a})\\ =-ia_1b\eta\left(1+\alpha\eta^2\right)\mathbf{n},\end{aligned} \tag{13}$$

$$\left[\left(1+\alpha\eta^2\right)\left(-i\omega+\tau_1\omega^2\right)-\eta^2\right]b=0. \tag{14}$$

From Eq. (14) it is evident that the thermal field is uncoupled with purely shear waves. Taking a scalar product with $\mathbf{a}$ of Eq. (13), we obtain the following secular equation for purely elastic shear waves in the presence of the rotation of the body:

$$\omega^2\Gamma-\mu_1\eta^2=0, \tag{15}$$

where $\Gamma=1+q^2\sin^2\phi$ with $q^2=\dfrac{\Omega_0{}^2}{\omega^2}$, (16)

and ϕ is the angle between the directions of $\mathbf{\Omega}$ and $\mathbf{u}$. In the absence of the rotation of the body, i.e., when $\Omega_0=0$, the Eq. (15) reduces to that which holds in the case of a non-rotating body. We further find that when the axis of the rotation of the body is aligned with the direction of the displacement vector, i.e., when $\phi=0$ or π, Eq. (15) becomes identical to the corresponding equation of the non-rotating body and thus no effect of rotation will be observed in this case.

The positive root of Eq. (15) is given by

$$\eta_s=\omega\sqrt{\frac{\Gamma}{\mu_1}}. \tag{17}$$

Therefore the phase velocity of shear wave is given by

$$V_S=\frac{\omega}{\mathrm{Re}[\eta_s]}=\sqrt{\frac{\mu_1}{\Gamma}}. \tag{18}$$

Equation (17) clearly indicates that due to rotation of the body, the velocity of shear wave reduces by factor $\sqrt{\Gamma}$.

Case 2: Dilatational waves

For purely dilatational waves $\mathbf{u}$ and $\mathbf{n}$ have the same directions, so that $\mathbf{a}\cdot\mathbf{n}=a$ where $a=|\mathbf{a}|$ In this case, Eqs. (11) and (12), on taking the scalar product with $\mathbf{n}$, become

$$\left(\omega^2\Gamma-\eta^2\right)a+ia_1\eta\left(1+\alpha\eta^2\right)b=0, \tag{19}$$

$$\begin{aligned}a_2\eta\omega(1+i\tau_1\omega)a+\\ -\left[-\eta^2+\left(1+\alpha\eta^2\right)\left(-i\omega+\tau_1\omega^2\right)\right]b=0,\end{aligned} \tag{20}$$

where Γ is given by Eq. (16).

Equations (19) and (20) clearly imply that the thermal field is coupled with the elastic dilatational wave.

For a non-trivial solution, the determinant of the coefficient matrix in the above system of Eqs. (19) and (20) must be zero, i.e.

$$\begin{vmatrix}\omega^2\Gamma-\eta^2 & ia_1\eta\left(1+\alpha\eta^2\right)\\ a_2\eta\omega(1+i\tau_1\omega) & -\left(1+\alpha\eta^2\right)\left(-i\omega+\tau_1\omega^2\right)+\eta^2\end{vmatrix}=0. \tag{21}$$

Therefore, we have a bi-quadratic dispersion relation

$$\begin{aligned}\eta^4\left(1-\omega^2\tau_1\alpha h+i\omega\alpha h\right)+\\ -\eta^2\left[\left(\omega^2-\omega^4\tau_1\alpha\right)\Gamma+\omega^2\tau_1h-i\left\{\omega h-\omega^3\alpha\Gamma\right\}\right]+\\ +\left(\omega^4\tau_1-i\omega^3\right)\Gamma=0.\end{aligned} \tag{22}$$

On setting $Z=\eta/\sqrt{\omega}$ and simplifying, Eq. (22) reduces to

$$\begin{aligned}Z^4\left[1-\omega^2\tau_1\alpha h+i\omega\alpha h\right]+\\ -Z^2\left[\omega\Gamma\left(1-\omega^2\tau_1\alpha\right)+\omega\tau_1h-i\left(h-\omega^2\alpha\Gamma\right)\right]+\\ +\omega\Gamma\left(\omega\tau_1-i\right)=0.\end{aligned} \tag{23}$$

Now, multiplying the above equation throughout by $\left[\left(1-\omega^2\tau_1\alpha h\right)+i\omega\alpha h\right]$, we arrive at the simplified form of the following dispersion relation equation for dilatational waves in a rotating body in the context of the two-temperature generalized thermoelasticity theory:

$$\begin{aligned}Z^4\left[1+\omega^2A_1+\omega^4\left(\tau_1\alpha h\right)^2\right]-Z^2\left[P-iQ\right]+\\ +\Gamma\left[\left(\tau_1-\alpha h\right)\omega^2-\tau_1^2\alpha h\omega^4-i\omega\right]=0,\end{aligned} \tag{24}$$

where we have used the notations

$$P=A_2\omega+A_3\omega^3+\left(\tau_1\alpha\right)^2\Gamma h\omega^5,$$

$$Q=h+\alpha\varepsilon\Gamma\omega^2,\ \varepsilon=a_1a_2,\ h=1+\varepsilon,$$

$$A_1=\left(\alpha h\right)^2-2\alpha\tau_1h,\ \ A_2=\Gamma+\tau_1h-\alpha h^2,$$

$$A_3=-\Gamma\alpha\tau_1\left(1+h\right)-\alpha\left(\tau_1h\right)^2+\alpha^2h\Gamma.$$

IV. EXPRESSIONS FOR ATTENUATION COEFFICIENT AND WAVE NUMBER

The roots of Eq. (22) can be obtained as $\pm\eta_1$ and $\pm\eta_2$, where

$$\frac{\left(\eta_{1,2}\right)^2}{\omega}=\left(Z_{1,2}\right)^2=\frac{P-iQ\pm\sqrt{D(\omega)}}{2\left[1+\omega^2A_1+\omega^4\left(\tau_1\alpha h\right)^2\right]} \tag{25}$$

with

$$\mathrm{Re}\left[D(\omega)\right]=-h^2+\left(A_2^2-2h\alpha\varepsilon\Gamma-4(\tau_1-\alpha h)\Gamma\right)\omega^2+$$
$$+\left(2A_2A_3-(\alpha\varepsilon\Gamma)^2+4\Gamma\tau_1^2\alpha h-4A_1\Gamma(\tau_1-\alpha h)\right)\omega^4+$$
$$+\left\{A_3^2+2A_2(\tau_1\alpha)^2h+4A_1\tau_1^2\alpha h\Gamma+4(\tau_1\alpha h)^2(\tau_1-\alpha h)\Gamma\right\}\omega^6+$$
$$+\omega^8\left\{2A_3(\tau_1\alpha)^2\Gamma h+2\Gamma\tau_1^4(\alpha h)^3\right\}+(\tau_1\alpha)^4(\Gamma h)^2\omega^{10}$$

and

$$\mathrm{Im}[D(\omega)]=(-2A_2h+4\Gamma)\omega+$$
$$+(-2A_2\alpha\varepsilon\Gamma-2A_3h+4A_1\Gamma)\omega^3+$$
$$+\left(-2A_3\alpha\varepsilon\Gamma+2\Gamma(\tau_1\alpha h)^2\right)\omega^5-2\tau_1^2\alpha^3\varepsilon h\Gamma\omega^7.$$

Now, out of the four roots of η as given by Eq. (24) or (25), we are interested in only those two roots which have negative imaginary parts, as only these two roots yield the negative values of the decay coefficient, Im[η] for the desired waves. These two roots can be obtained from Eq. (25) and by employing the theorem of complex analysis (see Ref. [47]). These two roots correspond to two different modes of the dilatational wave. One of these is predominately elastic and the other is predominately thermal in nature. Let the value of η associated with the former one be denoted by η_1 and the other one by η_2. It should be mentioned here that in the absence of rotation (i.e., when $\Omega_0=0$), the dispersion relation (23) reduces to the corresponding relation as reported by Kumar and Mukhopadhyay [24]. Furthermore, when $\phi=0$ or π, this equation becomes identical with the dispersion relation obtained by Kumar and Mukhopadhyay [24]. If we assume $\tau_1=0$ and $\Omega_0=0$, then the reduced dispersion relation corresponds to the relation under the two-temperature thermoelasticity theory without any relaxation parameter and it is obtained by Puri and Jordan [12].

V. THERMODYNAMIC TEMPERATURE: MODULUS AND ARGUMENT

Using Eqs. (10) and (5), we can write the thermodynamic temperature as

$$\begin{aligned}\theta(x,t)&=(1+\alpha\eta^2)b\exp\left[i(\omega t-\eta\mathbf{n}\cdot x)\right]=\\&=(1+\alpha\eta^2)\varphi(x,t)=M\exp(i\psi)\varphi(x,t),\end{aligned}\tag{26}$$

where $M=\left|1+\alpha\eta^2\right|$, $\psi=\mathrm{Arg}\left(1+\alpha\eta^2\right)$.

Here, $|.|$ and $\mathrm{Arg}[.]$ denote the modulus and the principal value of the argument, respectively, of a complex quantity.

We assume

$$M_{E,T}=\left|1+\alpha\eta_{1,2}^2\right| \text{ and } \psi_{E,T}=\mathrm{Arg}\left(1+\alpha\eta_{1,2}^2\right).\tag{27}$$

Therefore M_E and ψ_E, respectively, can be termed as the amplitude coefficient factor and the phase-shift of the elastic mode dilatational wave associated with the thermodynamic temperature. M_T and ψ_T, respectively, are the amplitude coefficient factor and the phase-shift associated with the thermal mode dilatational wave for the thermodynamic temperature.

VI. ANALYTICAL RESULTS

In this section, two different cases which correspond to the waves of small frequency and waves of high frequency will be considered to analyze two different dilatational waves as mentioned above. In order to analyze the effects of rotation on both waves, we consider the important quantities, like phase velocity, specific loss, and penetration depth which are of importance in the study of harmonic plane waves. The phase velocity, specific loss, and penetration depth for elastic mode and thermal mode dilatational wave are given by

$$V_{E,T}=V_{1,2}=\frac{\omega}{\mathrm{Re}\left[\eta_{1,2}\right]},$$

$$\left(\frac{\Delta W}{W}\right)_{E,T}=\left(\frac{\Delta W}{W}\right)_{1,2}=4\pi\left|\frac{\mathrm{Im}\left[\eta_{1,2}\right]}{\mathrm{Re}\left[\eta_{1,2}\right]}\right|$$

and $\delta_{E,T}=\left|\dfrac{1}{\mathrm{Im}\left[\eta_{1,2}\right]}\right|$, respectively [12]. (28-30)

In the above formulae, V_E, $(\Delta W/W)_E$ and δ_E denote the phase velocity, specific loss, and penetration depth, respectively associated with the elastic mode dilatational wave and V_T, $(\Delta W/W)_T$ and δ_T denote the phase velocity, specific loss, and penetration depth, respectively associated with the thermal mode dilatational wave.

1. High frequency asymptotics

We consider $\omega>>1$. Therefore expanding the expressions for $\eta_{1,2}$ from Eq. (25) in powers of ω^{-1} and neglecting the higher powers for smallness, we obtain, after a long and straight forward algebraic manipulations, the asymptotic expressions for η_1, η_2 and the quantities as defined by Eqs. (27)-(30) as follows:

(a) For the elastic mode dilatational wave

$$\eta_1 \approx \omega\sqrt{\frac{\Gamma}{h}}\left[1-\frac{\varepsilon}{2\tau_1\alpha h\omega^2}-\frac{i\varepsilon}{2\tau_1^2\alpha h\omega^3}\left(1-\frac{\tau_1\varepsilon}{2\alpha h}\right)\right],$$

$$V_E \approx \sqrt{\frac{h}{\Gamma}}\left[1+\frac{\varepsilon}{2\tau_1\alpha h\omega^2}\right],$$

$$\left(\frac{\Delta W}{W}\right)_E \approx \frac{2\pi\varepsilon}{\omega^3\tau_1^2\alpha h}\left(1+\frac{\varepsilon}{2\tau_1\alpha h\omega^2}\right)\left(1-\frac{\tau_1\varepsilon}{2\alpha h}\right),$$

$$\delta_E \approx \frac{2\omega^2\tau_1^2\alpha h^{3/2}}{\varepsilon\sqrt{\Gamma}}\left(1+\frac{\tau_1\varepsilon}{2\alpha h}\right), \qquad (31\text{-}36)$$

$$M_E \approx \frac{\alpha\Gamma\omega^2}{h}\left[1+\frac{1}{\omega^2}\left(\frac{h}{\alpha\Gamma}-\frac{\varepsilon}{\tau_1\alpha h}\right)\right], \qquad \psi_E \to 0,$$

$$(\omega\to\infty).$$

(b) For the thermal mode dilatational wave:

$$\eta_2 \approx \frac{1}{\sqrt{\alpha}}\left[\frac{1}{\omega^3}\left[\left(\frac{1-\varepsilon}{\tau_1^2\alpha h\Gamma}\right)^2+\left(\frac{2}{\tau_1^2}-\frac{\varepsilon+3}{\tau_1\alpha h}\right)^3\right]^{1/2}+\right.$$
$$\left.-i\left[1-\frac{1}{2\omega^2}\left(\frac{2}{\tau_1^2}-\frac{\varepsilon+3}{\tau_1\alpha h}\right)\right]\right],$$

$$V_T \approx \omega^4\sqrt{\alpha}\left[\left(\frac{(1-\varepsilon)}{\tau_1^2\alpha h\Gamma}\right)^2+\left(\frac{2}{\tau_1^2}-\frac{(\varepsilon+3)}{\tau_1\alpha h}\right)^3\right]^{-1/2},$$

$$\left(\frac{\Delta W}{W}\right)_T \approx 4\pi\omega^3\left[1-\frac{1}{2\omega^2}\left(\frac{2}{\tau_1^2}-\frac{\varepsilon+3}{\tau_1\alpha h}\right)\right]\times$$
$$\times\left[\left(\frac{1-\varepsilon}{\tau_1^2\alpha h\Gamma}\right)^2+\left(\frac{2}{\tau_1^2}-\frac{\varepsilon+3}{\tau_1\alpha h}\right)^3\right]^{-1/2},$$

$$\delta_T \approx \sqrt{\alpha}\left[1+\frac{1}{2\tau_1\alpha\omega^2}\left(\frac{2\alpha}{\tau_1}-\frac{\varepsilon+3}{h}\right)\right],$$

$$M_T \approx \frac{1}{\omega^2\tau_1}\left(\frac{2}{\tau_1}-\frac{\varepsilon+3}{\alpha h}\right), \qquad \psi_T \to -\pi, \qquad (37\text{-}42)$$

$$(\omega\to\infty).$$

2. Low-frequency asymptotics

We consider $\omega \ll 1$. Then expanding the expressions for $\eta_{1,2}$ from Eq. (25) in powers of ω and neglecting the higher powers for smallness, we obtain, after a long but straight forward algebraic manipulations, the asymptotic expansions for $\eta_{1,2}$ and for different wave characterizations of elastic mode and thermal mode dilatational wave as defined by Eqs. (27)-(30) for the case of low frequency values as follows:

(a) For the elastic mode dilatational wave:

$$\eta_1 \approx \omega\sqrt{\frac{\Gamma}{h}}\left[1+\frac{\omega^2\Delta}{2}-i\frac{\omega(1+h)\Gamma}{2h^2}\right],$$

$$V_E \approx \sqrt{\frac{h}{\Gamma}}\left[1-\frac{\omega^2\Delta}{2}\right], \quad \left(\frac{\Delta W}{W}\right)_E \approx \frac{2\pi\,\Gamma(1+h)\,\omega^2}{h^2},$$

$$\delta_E \approx \frac{2h^{5/2}}{\omega^2(1+h)\Gamma^{3/2}}, \quad M_E \approx 1+\frac{\alpha\Gamma}{h}\omega^2, \ \psi_E \to 0, \qquad (43\text{-}48)$$

$$(\omega\to 0)$$

where we used the notation

$$\Delta=\frac{\Gamma}{2h^4}\left[-3\Gamma+2h\left\{\Gamma(1+6\Gamma)+2h\left[2\tau_1(3\Gamma-1)+9\Gamma\right]+\right.\right.$$
$$\left.\left.+h^2\left[6\Gamma(9\tau_1+\alpha-3\Gamma)-\alpha\varepsilon\right]+3\Gamma h^3\left(19\tau_1^2+15(\alpha h)^2\right)\right\}\right].$$

(b) For the thermal mode dilatational wave:

$$\eta_2 \approx \begin{cases}\sqrt{\frac{\omega h}{2}}\left[1+\omega\frac{\left(\Gamma\varepsilon+(\tau_1-\alpha h)h^2\right)}{2h^2}-i\left\{1-\omega\frac{\left(\Gamma\varepsilon+(\tau_1-\alpha h)h^2\right)}{2h^2}\right\}\right], & \alpha\neq\alpha^*\\ \sqrt{\frac{\omega h}{2}}\left[1+\omega^2\theta_1-i\left\{1+\omega^2\theta_1\right\}\right], & \alpha=\alpha^*\end{cases}$$

$$V_T \approx \begin{cases}\sqrt{\frac{2\omega}{h}}\left[1-\omega\frac{\left(\Gamma\varepsilon+(\tau_1-\alpha h)h^2\right)}{2h^2}\right], & \alpha\neq\alpha^*\\ \sqrt{\frac{2\omega}{h}}\left[1-\omega^2\theta_1\right], & \alpha=\alpha^*\end{cases}$$

$$\left(\frac{\Delta W}{W}\right)_T \approx \begin{cases}4\pi\left[1-\omega\frac{\left(\Gamma\varepsilon+h(\tau_1-\alpha h)\right)}{h^2}\right], & \alpha\neq\alpha^*\\ 4\pi, & \alpha=\alpha^*\end{cases}$$

$$\delta_T \approx \begin{cases}\sqrt{\frac{2}{\omega h}}\left[1+\frac{\omega\left(\Gamma\varepsilon+(\tau_1-\alpha h)h^2\right)}{2h^2}\right], & \alpha\neq\alpha^*\\ \sqrt{\frac{2}{\omega h}}\left[1-\omega\theta_1\right], & \alpha=\alpha^*\end{cases},$$

$$M_T \approx \begin{cases}1+\frac{\omega^2\alpha}{h}\left[\Gamma\varepsilon+\tau_1 h^2\right], & \alpha\neq\alpha^*\\ 1+\frac{(\alpha h)^2\omega^2}{2}, & \alpha=\alpha^*\end{cases},$$

$$\psi_T \approx \begin{cases} \tan^{-1}(-\omega\alpha h), & \alpha \neq \alpha^* \\ \tan^{-1}(-\omega\alpha h), & \alpha = \alpha^* \end{cases}, \qquad (49\text{-}54)$$

$(\omega \to 0)$

where

$$\theta_1 = \left\{ \frac{-\Gamma^2\varepsilon - \Gamma h^2 \alpha\left(-2\tau_1 + \alpha h + \alpha h^2\right) + \alpha h^5 (2\tau_1 - \alpha h)}{2h^4} \right\}$$

and $\alpha^* = (\Gamma\varepsilon + \tau_1 h^2)/h^3$ is obtained as the critical value of the two-temperature parameter α. Clearly, the critical value depends on the thermoelastic coupling constant as well as the magnitude of rotation.

VII. NUMERICAL RESULTS

Now, in order to examine the behavior of phase velocity, specific loss, penetration depth, amplitude coefficient factor and phase shift of the thermodynamic temperature due to rotation for intermediate values of frequency and to illustrate the asymptotic results obtained in the previous section, we carry out computational work by using the computational tool, Mathematica. We assume $\varepsilon = 0.0168$ (Puri and Jordan [12]) and we consider the case when the frequency of motion is equal to the rotational frequency of the body, i.e., we take $q = 1$. Using the formulae given by (27)-(30), we compute the values of the quantities directly from Eq. (25) for various values of frequency, ω and the rotational angle, ϕ (*i.e.*, for $\phi = 0, \pi/4, \pi/2$). For the purpose of examining the influence of the critical value of the two-temperature parameter on the wave fields, we compute the values of the wave characterizations for various values of two-temperature parameter, namely $\alpha = 0$, $\alpha = \alpha^*$ and $\alpha = 0.071301$. Clearly, the case when $\alpha = 0$ corresponds to the case under the generalized thermoelasticity theory (Lord-Shulman theory) and the analytical results for this case are discussed by Puri [38] and Chandrasekharaiah and Srikantiah [39]. The cases when $\alpha \neq 0$ correspond to the case under two-temperature generalized thermoelasticity. We have plotted the results in different figures. In all the figures, the solid lines, the dashed line and the dotted lines represent the variations for $\phi = 0$, $\phi = \pi/4$ and $\phi = \pi/2$, respectively. The thick black lines, thin black lines and the gray lines show the nature of variations for the cases when $\alpha = 0, \alpha = \alpha^*$ and $\alpha = 0.071301$, respectively.

Phase Velocity

Figures 1(a-f) display the variation of phase velocity of the elastic mode and thermal mode dilatational waves with respect to ω for different values of two-temperature parameter and for different values of ϕ. Figs. 1(a-c) reveal that the variation of phase velocity of the elastic mode wave (V_E) is affected by the variation of α. This implies that V_E shows a different behavior under generalized thermoelasticity (Lord-Shulman model) as compared to two-temperature generalized thermoelasticity (Youssef-model). Although under both theories V_E shows constant limiting values when $\omega \to 0$ as well as when $\omega \to \infty$ and the limiting values being the same when $\omega \to 0$, but they are different when $\omega \to \infty$. Furthermore, this field shows two stationary values (one minimum value followed by one maximum) under the Youssef-model, whereas under the Lord-Shulman model no such stationary value is observed. In the absence of rotation, a similar behavior was observed by Puri and Jordan [12] under the two-temperature thermoelasticity theory and by Kumar and Mukhopadhyay [24] under the two-temperature generalized thermoelasticity theory (Youssef-model). The effect of rotation on this wave field is very much pronounced in all cases. The phase velocity decreases with the increase of angle of rotation ϕ. For both the high and low frequency values the limiting values of V_E is $\sqrt{h/\Gamma}$ which is clearly influenced by rotation. This is in complete agreement with our theoretical results (see Eqs. (32, 44)).

The effect of rotation on the thermal mode dilatational wave is depicted in Figs. 1(d-f) which indicate that the effect of rotation on phase velocity of thermal mode wave is significant for the case when $\alpha = \alpha^*$ as compared to the other two cases. Moreover, the effect of rotation is more prominent for lower frequency values (see Figs. 1(c, d)). It is clear from these figures that this wave field is an increasing function of ω under the Youssef-model (i.e., for $\alpha = 0.071301$ and $\alpha = \alpha^*$), whereas it reaches to a constant limiting value as $\omega \to \infty$ under the Lord- Shulman model (see Fig. 1(f)). This fact is in agreement with the analytical prediction given by (38).

Specific Loss

Figures 2(a, b) display the variation of specific loss of the elastic mode wave. A significant difference between the specific loss profiles under the Youssef-model and those under the Lord- Shulman model is observed from these two figures. In the former case, three stationary values, two maxima separated by one minimum, are met. However, in the later case, only two stationary values, one maximum and one minimum, are seen for all values of rotational term. A similar behavior is reported for the case of a non-rotating body under the two-temperature thermoelasticity theory as well as under the Youssef-model as described by Puri and Jordan [12] and by Kumar and Mukhopadhyay

[24], respectively. For low frequency values, the effect of rotation is pronounced for all three values of α, but for high frequency values the effect is negligible in the case when $\alpha = 0.071301$. Specific loss decreases with the increase of angle ϕ. In the case when $\alpha = \alpha^{*}$ a similar trend of variation of specific loss with ϕ is observed for high

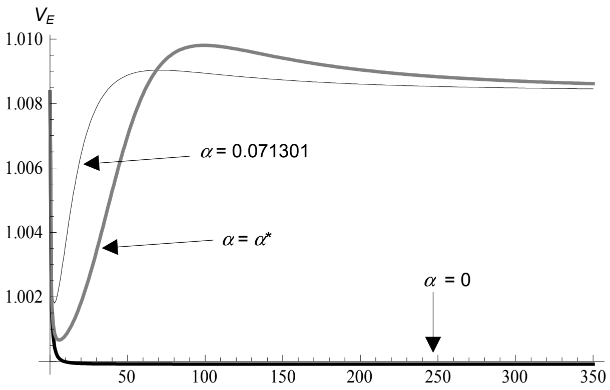

Fig. 1(a). V_E vs. ω for $\phi = 0$

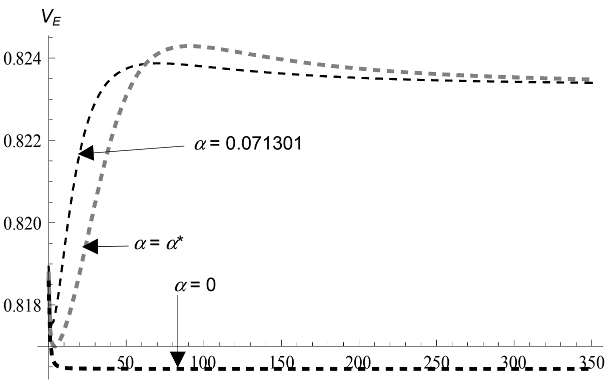

Fig. 1(b). V_E vs. ω for $\phi = \pi/4$

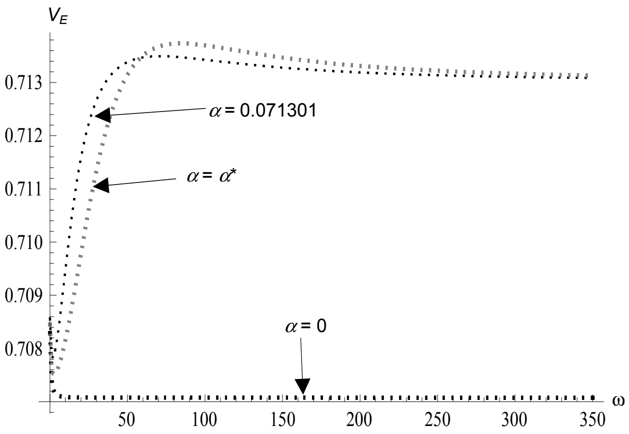

Fig. 1(c). V_E vs. ω for $\phi = \pi/2$

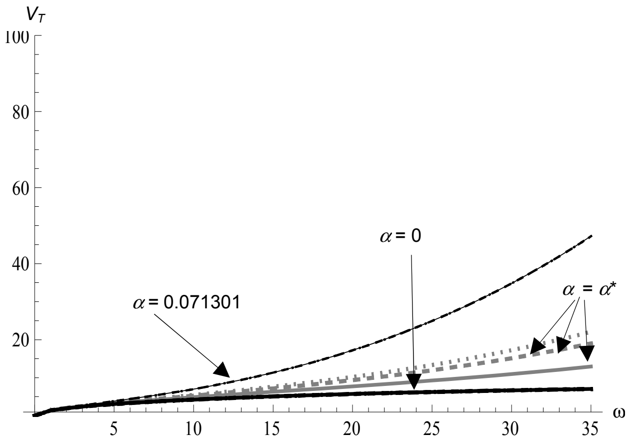

Fig. 1(d). V_E vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

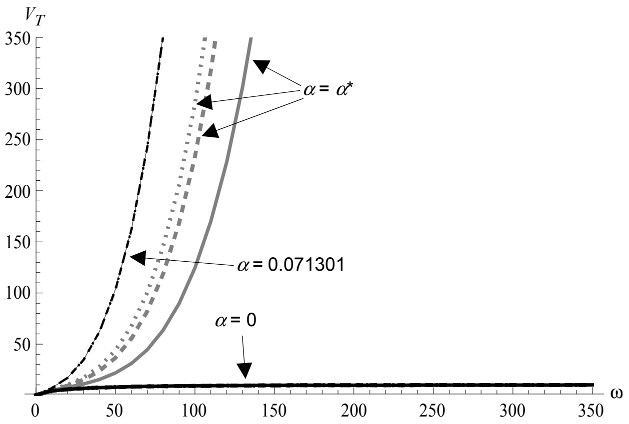

Fig. 1(e). V_T vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

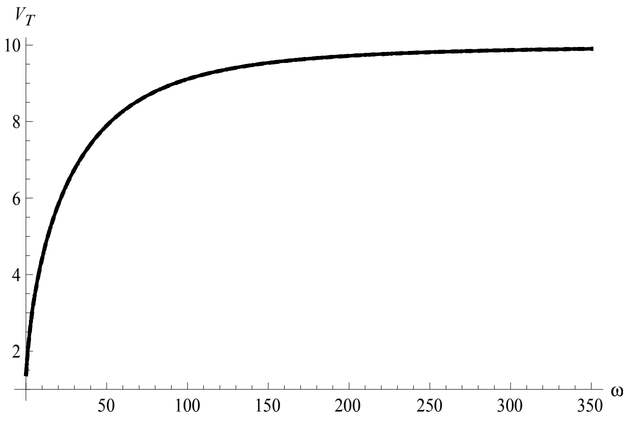

Fig. 1(f). V_T vs. ω for $\alpha = 0$. Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$ (all lines merged together)

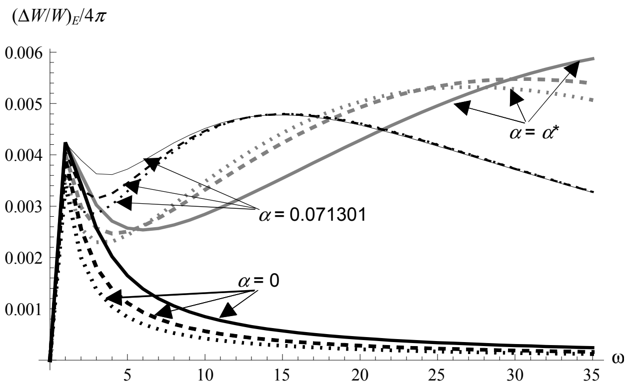

Fig. 2(a). $(\Delta W/W)_E$ vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

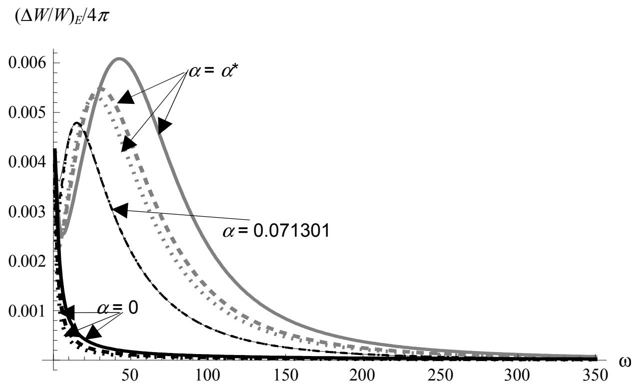

Fig. 2(b). $(\Delta W/W)_E$ vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

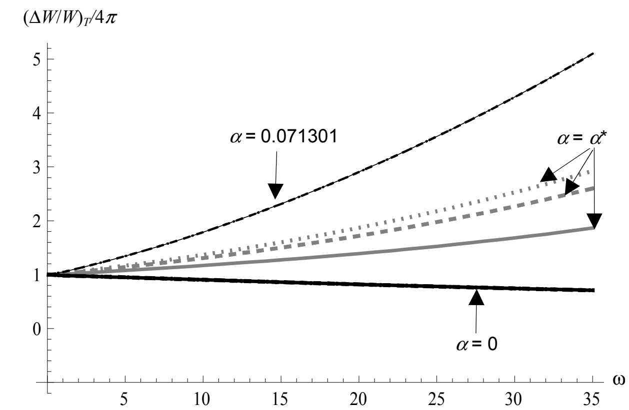

Fig. 2(c). $(\Delta W/W)_T$ vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

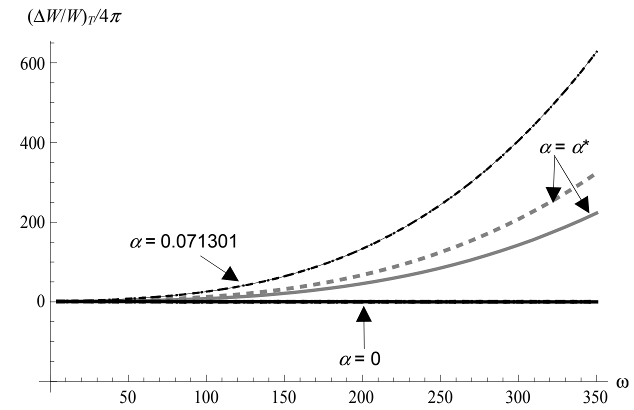

Fig. 2(d). $(\Delta W/W)_T$ vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

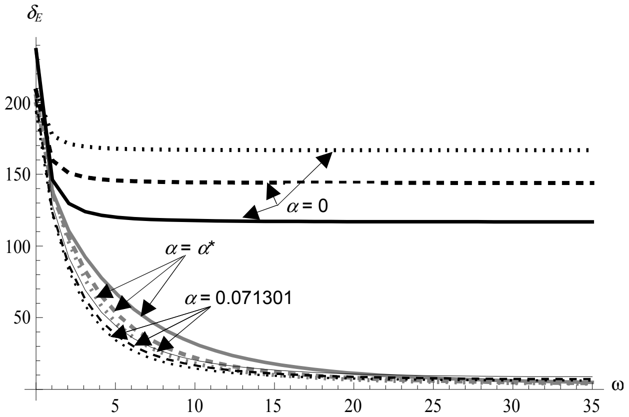

Fig. 3(a). δ_E vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

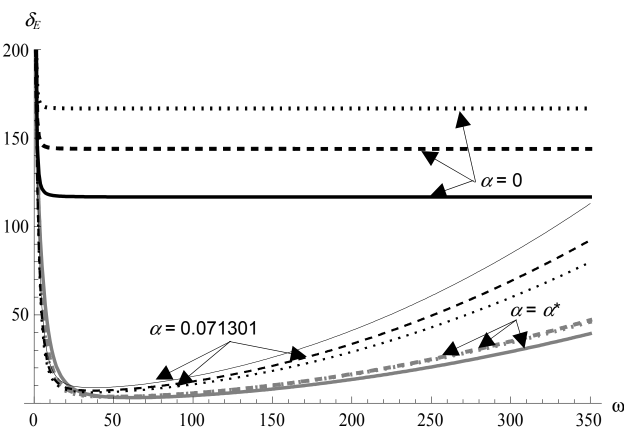

Fig. 3(b). δ_E vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

and low frequency ranges. However, the trend is reversed for intermediate values of frequency. Finally, this wave field approaches to zero value but with different paces in all the cases, although in case when $\alpha = 0$, the specific loss

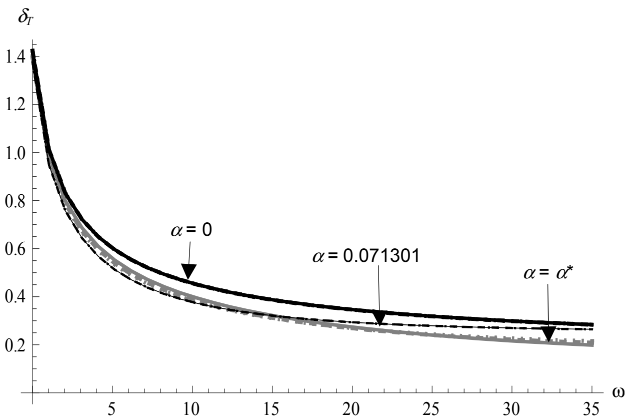

Fig. 3(c). δ_T vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

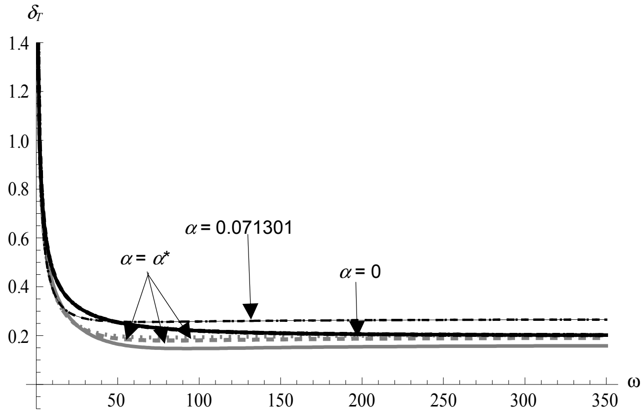

Fig. 3(d). δ_T vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

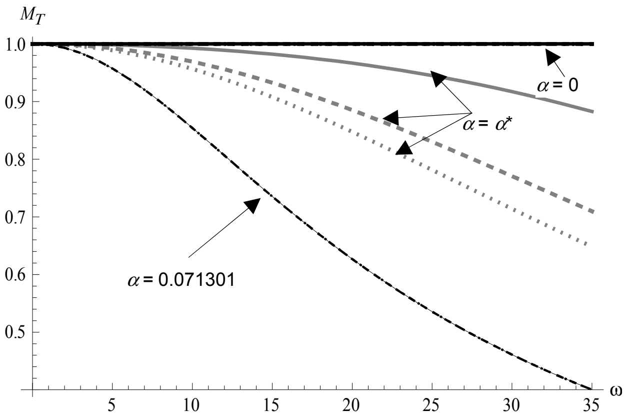

Fig. 4(a). M_T vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

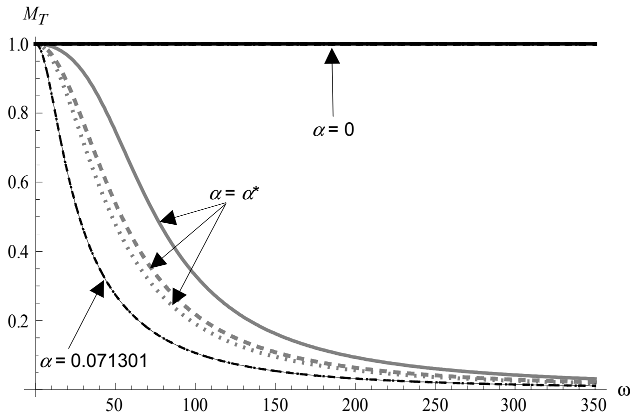

Fig. 4(b). M_T vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

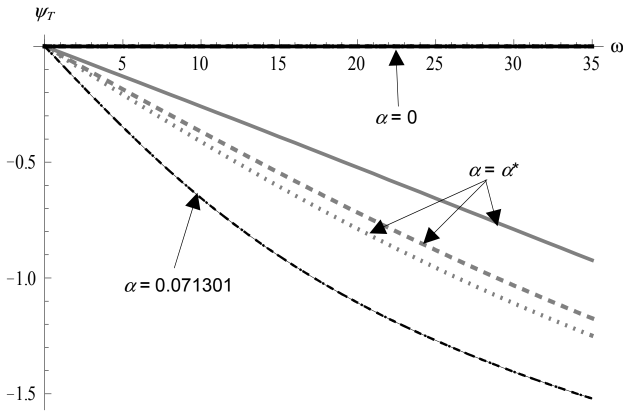

Fig. 4(c). Ψ_T vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

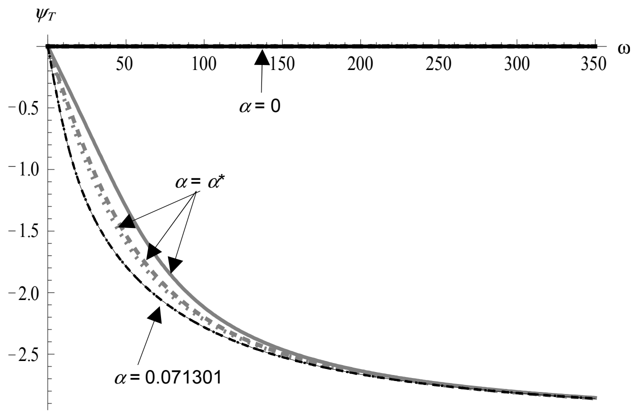

Fig. 4(d). Ψ_T vs. ω: Solid line for $\phi = 0$; dashed line for $\phi = \pi/4$; dotted line for $\phi = \pi/2$

rapidly approaches to zero as compared to the other two cases. The nature of specific of the thermal mode wave can be observed in Fig. 2(c,d) which indicates that the effect of rotation on specific loss of the thermal mode dilatational wave is significant for $\alpha = \alpha^*$ but it is negligible in other two cases. It is clear from the theoretical as well as numerical results (see Eq. (39) and Fig. 2(c,d)) that this field is an increasing function of ω under two-temperature generalized thermoelasticity. However, it shows a constant limiting value under the Lord-Shulman theory (see Puri [38] and Chadrasekharaiah and Srikantiah [39]).

Penetration Depth

The variation of penetration depth profile of elastic mode wave is displayed in Figs. 3(a, b). From these figures we note that the effect of rotation on this wave field is very much pronounced for all three values of α. A significant difference between the results predicted by the Lord-Shulman model and the Youssef-model is observed for this profile, too. It approaches to infinity as $\omega \to \infty$ under two-temperature thermoelasticity. However, in the context of the Lord-Shulman theory it decreases rapidly and approaches to its constant limiting value which is dependent on the rotational term. Moreover, the penetration depth increases with the increase of rotational angle under the Lord-Shulman theory, whereas under the Youssef theory (for α = 0.071301), the opposite is true. This is in complete agreement with our theoretical result given by Eq. (34) and the result as reported by Puri [38] and Chadrasekharaiah and Srikantiah [39].

Figures 3(c,d) shows that the penetration depth, like other two wave fields, related to the thermal mode dilatational wave is affected due to rotation only in the case when $\alpha = \alpha^*$. Under both theories, this field decreases rapidly and finally reaches its limiting values as $\omega \to \infty$ (see Eq. (40)).

Amplitude coefficient factor and phase shift

Variations of the amplitude coefficient factor and phase shift profiles (M_T, ψ_T) of the thermal mode dilatational wave associated with thermodynamic temperature with ω are depicted in Figs. 4(a-c) which reveal a prominent effect of rotation on both profiles only for the case when $\alpha = \alpha^*$ and the effect is more significant for low frequency values. The value decreases as the angle of rotation decreases. Furthermore, Figs. 4(a-c) indicate that $|M_T b| < |b|$, which implies that the thermodynamic temperature exhibits a lesser magnitude as compared to conductive temperature and it has a phase shift $\psi_T < 0$ where $\lim_{\omega \to \infty} \psi_T = -\pi$ (see Eq. (42) and Fig. 4(d)).

VIII. CONCLUSIONS

Harmonic plane waves propagating in a rotating elastic medium under two-temperature thermoelasticity with the thermal relaxation parameter are investigated. The transverse wave is observed to be unaffected due to thermal field where as the dilatational wave is coupled with the thermal field. Analytical expressions for high and low frequency asymptotics for different wave characterizations for longitudinal elastic (predominated) and thermal waves (predominated) are found out. The numerical values of these wave fields for intermediate values of frequency and for various values of rotational term are computed. Detailed analysis of the results highlighting the effects of rotation on the waves propagating inside the medium is presented on different graphs. An analysis concerning the differences in the results predicted by the two-temperature generalized thermoelasticity theory (Youssef-theory) as compared to the generalized thermoelasticity theory (Lord-Shulman theory) is also presented. We summarize the following important facts:

1. All waves are dispersive in nature.
2. There exists a critical value of two-temperature parameter α given by $\alpha^* = (\Gamma\varepsilon + \tau_1 h^2)/h^3$ and this critical value clearly depends on the thermoelastic coupling constant, thermal relaxation parameter as well as the two-temperature parameter and it is influenced by rotation.
3. The wave characterizations show qualitatively different behavior under the Youssef-theory as compared to the Lord-Shulman theory.
4. Effects of rotation on shear wave and elastic mode dilatational wave is very much prominent for all three values of α.
5. Phase velocity decreases and the specific loss increases with the increase of the angle between the axis of rotation and the displacement vector.
6. The penetration depth related to the elastic mode dilatational wave approaches to infinity as $\omega \to \infty$ and it shows a minimum value under the Youssef-theory. However, it decreases rapidly and approaches to its constant limiting value under the Lord-Shulman theory. The effect of rotation is pronounced in all cases. Due to the increase of rotational angle, the penetration depth increases under the Lord-Shulman theory, where as opposite is true under the Youssef-theory.
7. The wave fields corresponding to the thermal mode longitudinal wave is not effected significantly due to rotation for the cases when $\alpha \neq \alpha^*$. However, all the wave fields are affected significantly due to rotation for the case when $\alpha = \alpha^*$. The effect is more prominent for lower values of frequency.
8. The thermodynamic temperature θ exhibits a lesser magnitude as compared to the conductive temperature and it experiences a negative phase shift. Both the

amplitude coefficient factor and the phase shift related to the thermal mode wave is affected due to rotation only for critical value of the two-temperature parameter.

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References

[1] P.J. Chen, M.E. Gurtin, *On a theory of heat conduction involving two-temperatures*. J. Appl. Math. Phys. (ZAMP) 19, 614-627, (1968).
[2] P.J. Chen, M.E. Gurtin, W.O. Williams, *On the thermodynamics of non-simple elastic materials with two temperatures*. J. Appl. Math. Phys. (ZAMP) 20, 107-112 (1969).
[3] M.E. Gurtin, W.O. Williams, *On the Clausius-Duhem inequality*. Z. angew. Math Phys. 17, 626-633 (1966).
[4] D. Iesan, *On the thermodynamics of non-simple elastic materials with two temperatures*. J. Appl. Math. Phys. (ZAMP) 21, 583-591 (1970).
[5] W.E. Warren, P.J. Chen, *Wave propagation in the two temperature theory of thermoelasticity*. Acta Mech. 16, 21-33 (1973).
[6] W.E. Warren, *Thermoelastic wave propagation from cylindrical and spherical cavities in the two-temperature theory*. J. Appl. Phys. 43, 3595-3597 (1972).
[7] D.E. Amos, *On a half-space solution of a modified heat equation*. Quart. Appl. Math. 27, 359-369 (1969).
[8] S. Chakrabarti, *Thermoelastic waves in non-simple media*. Pure Appl. Geophys. 109, 1682-1692 (1973).
[9] T.W. Ting, *A cooling process according to two-temperature theory of heat conduction*. J. Math. Anal. Appl. 45, 23-31 (1974).
[10] D. Colton, J. Wimp, *Asymptotic behavior of the fundamental solution to the equation of heat conduction in two-temperature*. J. Math. Anal. Appl. 69, 411-418 (1979).
[11] R. Quintanilla, *On existence, structural stability, convergence and spatial behavior in thermoelasticity with two temperatures*. Acta Mech. 168, 61-73 (2004).
[12] P. Puri, P.M. Jordan, *On the propagation of harmonic plane waves under the two- temperature theory*. Int. J. Engg. Sci. 44, 1113-1126 (2006).
[13] H.M. Youssef, *Theory of two-temperature-generalized thermoelasticity*. IMA J. Appl. Math. 71, 383-390 (2006).
[14] H.W. Lord, Y. Shulman, *A Generalized dynamical theory of thermoelasticity*. J. Mech. Phys. Solids 15, 299-309 (1967).
[15] A.E. Green, K.A. Lindsay, *Thermoelasticity*. J. Elasticity 2, 1-7 (1972).
[16] A. Magana, R. Quintanilla, *Uniqueness and growth of solutions in two temperature generalized thermoelastic theories*. Math Mech. Solids 14, 622-634 (2009).
[17] H.M. Youssef, E.A. Al-Lehaibi, *State-space approach of two-temperature generalized thermoelasticity of one dimensional problem*. Int. J. Solids Struct. 44, 1550-1562 (2007).
[18] H.M. Youssef, *Problem of generalized thermoelastic infinite cylindrical cavity subjected to a ramp-type heating and loading*. Arch. Appl. Mech. 75, 553-565 (2006).
[19] H.M. Youssef, *Theory of two-temperature-generalized thermoelasticity*. IMA J. Appl. Math. 71, 383-390 (2006).
[20] H.M. Youssef, *Two-temperature generalized thermoelastic infinite medium with cylindrical cavity subjected to moving heat source*. Arch. Appl. Mech. 80, 1213-1224 (2010).
[21] R. Kumar, R. Prasad, S. Mukhopadhyay, *Variational and reciprocal principles in two- temperature generalized thermoelasticity*. J. Therm. Stresses 33, 161-171 (2010).
[22] S. Mukhopadhyay, R. Kumar, *Thermoelastic interaction on two-temperature generalized thermoelasticity in an infinite medium with a cylindrical cavity*. J. Therm. Stresses 32, 341-360 (2009).
[23] R. Quintanilla, P.M. Jordan, *A note on the two temperature theory with dual phase lag delay: some exact solutions*. Mech. Res. Comm. 36, 796-803 (2009).
[24] R. Kumar, S. Mukhopadhyay, *Effects of thermal relaxation time on plane wave propagation under two-temperature thermoelasticity*. Int. J. Engg. Sci. 48, 128-139 (2010).
[25] R. Kumar, R. Prasad, S. Mukhopadhyay, *Some theorems on two-temperature generalized thermoelasticity*. Arch. Appl. Mech. 81, 1031-1040 (2011).
[26] M. Ezzat, F. Hamza, E. Awad, *Electro-magneto-thermoelastic plane waves in micropolar solid involving two temperatures*. Acta Mech. Solida Sinica. 23, 200-212 (2010).
[27] S. Mukhopadhyay, R. Prasad, R. Kumar, *On the theory of two-temperature generalized thermoelasticity with dual phase lags*. J. Therm. Stresses 34, 352-365 (2011).
[28] P. Chadwick, I.N. Sneddon, *Plane waves in an elastic solid conducting heat*. J. Mech. Phys Solids 6, 223-230 (1958).
[29] P. Chadwick, *Thermoelasticity: the dynamic theory*. in: R. Hill, I.N. Sneddon (Eds.), Progress in Solid Mechanics, North-Holland, Amsterdam, I, 263-328 (1960).
[30] A. Nayfeh, S. Nemat-Nasser, *Thermoelastic waves in solids with thermal relaxation*. Acta Mech. 12, 53-69 (1971).
[31] P. Puri, *Plane waves in generalized thermoelasticity*. Int. J. Eng. Sci. 11, 735-744 (1973).
[32] V.K. Agarwal, *On plane waves in generalized thermoelasticity*. Acta Mech. 31, 185-198 (1979).
[33] A.E. Green, P.M. Naghdi, *Thermoelasticity without energy dissipation*. J. Elast. 31, 189-208 (1993).
[34] D.S. Chandrasekharaiah, *Thermoelastic plane waves without energy dissipation*. Mech. Res. Commun. 23, 549-555 (1996).
[35] P. Puri, P.M. Jordan, *On the propagation of plane waves in type-III thermoelastic media*. Proc. Royal Soc. A 460, 3203--3221 (2004).
[36] A.E. Green, M. Naghdi, *On undamped heat waves in an elastic solid*. J. Therm. Stresses 15, 253-264 (1992).
[37] M. Schoenberg, D. Censor, *Elastic waves in rotating media*. Quart. Appl. Math. 31, 115-125 (1973).
[38] P. Puri, *Plane thermoelastic waves in rotating media*. Bull. Acad. Polon. Sci. Ser. Sci. Tech. 24, 137-144 (1976).
[39] D.S. Chandrasekharaiah, K.R. Srikantiah, *Thermoelastic plane waves in a rotating solid*. Acta Mech. 50, 211-219 (1984).
[40] S.K. Roychoudhari, *Effect of rotation and relaxation times on plane waves in generalized thermoelasticity*. J. Elasticity 21, 59-68 (1985).

[41] S.K. Roychoudhari, N. Bandyopadhyay, *Thermoelastic wave propagation in rotating elastic medium without energy dissipation*. Int. J. Math. and Math. Sci. 1, 99-107 (2005).
[42] D.S. Chandrasekharaiah, *Thermoelastic plane waves without energy dissipation in a rotating body*. Mech. Res. Comm. 24, 551-560 (1997).
[43] D.S. Chandrasekharaiah, *Plane waves in a rotating elastic solid with voids*. Int. J. Engg. Sci. 25, 591-596 (1987).
[44] Mohamed I.A. Othman, *Effect of rotation on plane waves in generalized thermo-elasticity with two relaxation times*. Int. J. Solids and Structures, 41, 2939-2956 (2004).
[45] J.L. Auriault, *Body wave propagation in rotating elastic media*. Mech. Res. Comm. 31, 21-27 (2004).
[46] J.N. Sharma, Mohamed I.A. Othman, *Effect of rotation on generalized thermo-viscoelastic Rayleigh–Lamb waves*. Int. J. Solids and Structures 44, 4243-4255 (2007).
[47] S. Punnusamy, *Foundation of complex analysis.* Narosa Publishing House (2001).

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