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Optically Detected Electrophonon Resonance Effect in Quantum Wires

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Abstract: Electrophonon resonance (EPR) effects are investigated in cylindrical quantum wires (CQWR) in the presence of a laser field using a projection technique. We obtain the analytical expressions and graphs of absorption power. From graphs of the absorption power we obtain line-widths as a profile of the curves. The dependence of line-widths on different factors is considered. The conditions for EPR and optically detected electrophonon resonance (ODEPR) are also discussed.

Key words: electrophonon resonance, quantum wire, absorption power

I. INTRODUCTION

The study of optical conductivity in low-dimensional electron system has been considered as a special study by both theoretical and experimental condensed matter physicists in recent years. Numerous studies of two dimensional electron-gas systems have been undertaken [1-3]. Unfortunately, the study of this phenomena in 1D electron gas is still limited. The behavior of electrons in these systems is affected by their interaction with phonons. Thus, the most popular way to look into the structure of the systems is to investigate the scattering mechanisms. Among the many approaches to this problem, we are interested in the quantum-statistical operator algebra technique [4, 5].

This paper focuses on the consideration of the optical absorption power and half line-width of the electron system in a cylindrical quantum wire. The paper is organized as follows: First, we use many-body projection technique to obtain the analytical expression of optical conductivity. Next, we show numerical results for absorption power and half line-width for electrons interacting with LO phonons. Finally, the dependence of the width on the temperature and on the wire's size will be discussed.

II. OPTICAL CONDUCTIVITY AND ABSORPTION POWER

When an electromagnetic wave characterized a time-dependent electric field of amplitude E_0 and angular frequency ω is applied to a quantum wire along the *z*-direction, the optical conductivity tensor $\sigma_{zz}(\omega)$ is given in the linear response scheme as [4, 5]

$$\sigma_{zz}(\omega) = (i/\omega) \lim_{a\to 0^+} Tr\left\{\rho_{eq}\left[\left(\hbar\overline{\omega} - L\right)^{-1} J_z, J_z\right]\right\}, \quad (1)$$

where $\overline{\omega} \equiv \omega - ia \quad (a \to 0^+)$, T_R means the many-body trace with basis states, $|\Psi\rangle = (a_\alpha^+)^{n_\alpha}(a_\beta^+)^{n_\beta}...|\Phi_0\rangle$; $n_\alpha, n_\beta, ...$ being the number operators for the occupied states α, β ... and $|\Phi_0\rangle$ is the vacuum state. L_{eq} is the Liouville operator, defined as $L_{eq}X \equiv \left[H_{eq}, X\right]$ for an arbitrary operator X, H_{eq} is the Hamiltonian of a system of many electrons interacting with phonons (or electrons) in thermodynamic equilibrium, and ρ_{eq} is the equilibrium density operator. The many-electron current operator J_Z can be written in terms of the single-electron current operator j_Z as

$$J_z = \sum_{\alpha,\beta} j_z^{\alpha,\beta} a_\alpha^+ a_\beta\ , \quad (2)$$

where $j_l^{\alpha,\beta} \equiv \langle\alpha | j_l | \beta\rangle$ and $a_\alpha^+(a_\alpha)$ i s the creation (annihilation) operator for the electron in the state $|\alpha\rangle$. Then, the conductivity becomes

$$\sigma_{zz}(\omega) = \frac{i}{\omega} \lim_{a\to 0^+} \sum_{\alpha,\beta}\sum_{\gamma,\delta} j_z^{\alpha\beta} j_z^{\gamma\delta} A_{\alpha\beta}(\overline{\omega}), \quad (3)$$

where $A_{\alpha\beta}(\overline{\omega}) = Tr\left\{\rho_{eq}\left[\left(\hbar\overline{\omega} - L_{eq}\right)^{-1} a_\gamma^+ a_\delta, a_\alpha^+ a_\beta\right]\right\}.$

Using the projection operator technique we obtain the optical conductivity tensor [3]

$$\sigma_{zz}(\omega) = \frac{i}{\omega}\lim_{a\to 0^+}\sum_{\alpha,\beta}\left|(j_z)^{\alpha\beta}\right|^2 \frac{f_\alpha - f_\beta}{\hbar\overline{\omega} - (E_\beta - E_\alpha) - \Gamma_{\alpha\beta}(\overline{\omega})}, \quad (4)$$

where the line-shape function is:

$$\Gamma_{\alpha\beta}(\overline{\omega}) = \frac{Tr\left\{\rho_{eq}\left[L_v a_\alpha^+ a_\beta, \left(\hbar\overline{\omega} - L_d\right)^{-1} L_v a_\beta^+ a_\alpha\right]\right\}}{\left(f_\beta - f_\alpha\right)}. \quad (5)$$

After a systematic calculation, we obtain:

$$\begin{aligned}\Gamma_{kl}(\overline{\omega})(f_\alpha - f_\beta) = &\sum_q\sum_\gamma \left|C_{\beta\gamma}(q)\right|^2 \left\{\frac{(N_q+1)f_\alpha(1-f_\gamma)}{\hbar\overline{\omega} - E_\gamma + E_\alpha - \hbar\omega_q} + \right.\\ &- \frac{N_q f_\gamma(1-f_\alpha)}{\hbar\overline{\omega} - E_\gamma + E_\alpha - \hbar\omega_q} + \frac{N_q f_\alpha(1-f_\gamma)}{\hbar\overline{\omega} - E_\gamma + E_\alpha + \hbar\omega_q} + \\ &\left. - \frac{(1+N_q)f_\gamma(1-f_\alpha)}{\hbar\overline{\omega} - E_\gamma + E_\alpha + \hbar\omega_q}\right\} + \\ &+ \sum_q\sum_\gamma \left|C_{\alpha\gamma}(q)\right|^2 \left\{\frac{(N_q+1)f_\gamma(1-f_\beta)}{\hbar\overline{\omega} - E_\beta + E_\gamma - \hbar\omega_q} + \right.\\ &- \frac{N_q f_\beta(1-f_\gamma)}{\hbar\overline{\omega} - E_\beta + E_\gamma - \hbar\omega_q} + \\ &\left. + \frac{N_q f_\gamma(1-f_\beta)}{\hbar\overline{\omega} - E_\beta + E_\gamma + \hbar\omega_q} - \frac{(1+N_q)f_\beta(1-f_\gamma)}{\hbar\overline{\omega} - E_\beta + E_\gamma + \hbar\omega_q}\right\}.\end{aligned} \quad (6)$$

Using Dirac identity $\lim_{s\to 0^+}(x - is)^{-1} = P(1/x) + i\pi\delta(x)$ we obtain the optical conductivity in the form:

$$\mathrm{Re}\left[\sigma_{zz}(\omega)\right] = \frac{1}{\omega}\sum_{\alpha,\beta}\left|(J_z)^{\beta\alpha}\right|^2 \frac{\left(f_\alpha - f_\beta\right)B(\omega)}{\left[\hbar\omega - \left(E_\beta - E_\alpha\right) - \Delta(\omega)\right]^2 + \left[B(\omega)\right]^2} \quad (7)$$

In Eq. (7), $\Delta(\omega)$ and $B(\omega)$ are related to the line-shift and line-width of the absorption spectrum which can be calculated from the following expression:

$$\begin{aligned}B(\omega)(f_\alpha - f_\beta) = &\pi\sum_q\sum_\gamma \left|C_{\beta\gamma}(q)\right|^2 \times \\ &\times \left\{\left[(N_q+1)f_\alpha(1-f_\gamma) - N_q f_\gamma(1-f_\alpha)\right] \times \right.\\ &\times \delta(\hbar\omega - E_\gamma + E_\alpha - \hbar\omega_q) + \\ &+ \left[N_q f_\alpha(1-f_\gamma) - (N_q+1)f_\gamma(1-f_\alpha)\right] + \\ &\left.\delta(\hbar\omega - E_\gamma + E_\alpha + \hbar\omega_q)\right\} + \pi\sum_q\sum_\gamma \left|C_{\alpha\gamma}(q)\right|^2 \\ &\times \left\{\left[(N_q+1)f_\gamma(1-f_\beta) - N_q f_\beta(1-f_\gamma)\right] \times \right.\\ &\times \delta(\hbar\omega - E_\beta + E_\gamma - \hbar\omega_q) + \\ &+ \left[N_q f_\gamma(1-f_\beta) - (N_q+1)f_\beta(1-f_\gamma)\right] \times \\ &\times \delta(\hbar\omega - E_\beta + E_\gamma + \hbar\omega_q)\end{aligned} \quad (8)$$

We use the Lorentzian approximation in which $\Delta(\omega)$ can be neglected in comparison with $E_\alpha - E_\beta$ because we consider a very weak scattering effect and we adopt the approximation that $B(\omega) \approx B(E_\beta - E_\alpha) = B(E_{\beta\alpha})$, i.e., we assume that $B(\omega)$ is a very slowly varying function of ω near the resonance point $\hbar\omega = E_\beta - E_\alpha$. The optical conductivity, Eq. (7) is given by

$$\mathrm{Re}\left[\sigma_{zz}(\omega)\right] = \frac{1}{\omega}\sum_{\alpha,\beta}\left|(j_z)^{\beta\alpha}\right|^2 \frac{(f_\alpha - f_\beta)B(\omega)}{\left[\hbar\omega - (E_\beta - E_\alpha) - \Delta(\omega)\right]^2 + \left[B(\omega)\right]^2}. \quad (9)$$

The absorption power delivered to the system, $P(\omega)$, is given by [8]:

$$P(\omega) = \frac{E_0^2}{2}\mathrm{Re}\left[\sigma_{zz}(\omega)\right] \quad (10)$$

III. ABSORPTION POWER IN A QUANTUM WIRE

We consider a cylindrical quantum wire in which electrons are confined in an infinite potential well. The energy of electron is:

$$E_\alpha = E_{n,l}(k_z) = \frac{\hbar^2 k^2}{2m^*} + \frac{\hbar^2 A_{n,l}^2}{2m^* R^2} \quad (11)$$

The form factor characterizing the considering wire gets the following form [6]:

$$I_{n,l,n',l'} = \frac{2}{R^2}\int_0^R J_{|n-n'|}(qR)\Psi_{n',l'}^*(r)\Psi_{n,l}(r)rdr \quad (12)$$

Generally, it is difficult to obtain the expression for the form factor but using the specific form of wave function corresponding to the lowest levels [6]:

$$\Psi_{0,1} \approx \sqrt{3}\left(1 - \frac{r^2}{R^2}\right),$$
$$\Psi_{\pm 1,1} \approx \sqrt{12}\left(\frac{r}{R} - \frac{r^3}{R^3}\right) \qquad (13)$$

we can write the form factor as follows:

$$I_{0,1,0,1}(q) = \frac{24 J_3(qR)}{(qR)^3},$$
$$I_{\pm 1,1,0,1}(q) = \frac{48 J_4(qR)}{(qR)^3} \qquad (14)$$

III.1. Absorption power and line-width

Carrying out numerical calculations we plot the graphs expressing the dependence of the absorption power on the laser frequency in the various const at value of temperature or wire's radius. From these graphs we can obtain the dependence of the line-widths on temperature or wire's radius as profiles of curves.

In Fig. 1 we see that the curves of $P(\omega)$ at different value of temperature or wire's radius have peaks at specific value of ω. The distinction between these two graphs is that the magnitude of absorption power increases with temperature but decreases with radius. The situation is the same as the half-width when we analyze the curves in Fig. 2. It is clear that the half-width increases with temperature and decreases with the wire's radius.

III.2. Optically detected electrophonon resonance

From Delta-Dirac functions in Eq. (8) we obtained the conditions for electron-phonon resonance in CQR with

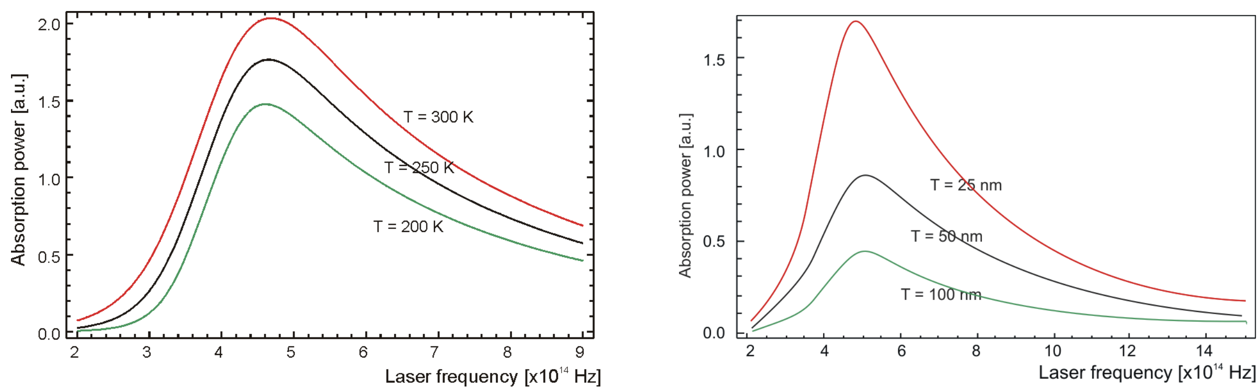

Fig. 1. Dependence of absorption power on photon frequency at different values of lattice temperature (on the left), and on the wire's radius (on the right)

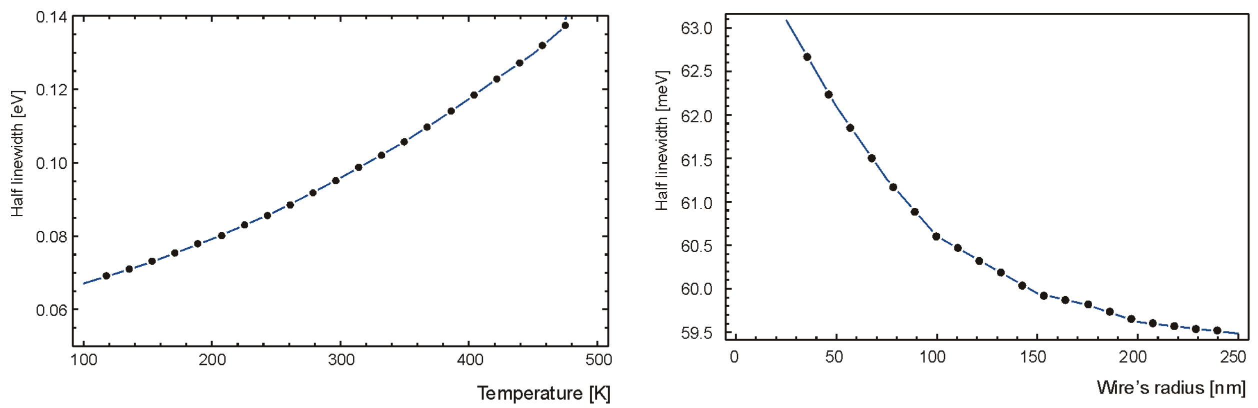

Fig. 2. Dependence of half line-width on the lattice temperature (on the left) and on the wire's radius (on the right)

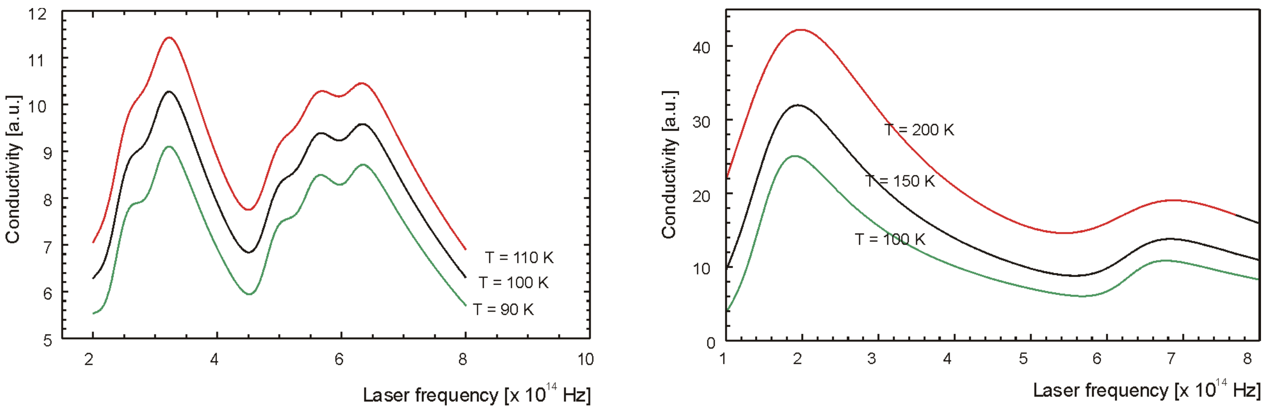

Fig. 3. Dependence of $\sigma_{zz}(\omega)$ on photon frequency at different values of the lattice temperature with $n = 1$, $n' = 2$, $l = 1 \div 2$ (on the left), and $n = 1$, $n' = 2$, $l = 1$, $l' = 1$ (on the right)

infinity in the present of the laser field:

$$(\Delta E)_{n,l;n',l'} \pm \hbar\omega = \hbar\omega_q \qquad (15)$$

From Eq. (15) we can see that there are two values of ω that satisfy this condition. This means that electrons in the state with energy E absorb one phonon and may absorb or emit one photon with energy $\hbar\omega$ for transferring to the new state. From the graph on the left of Figure 3 we can see that there appear ODEPR resonances corresponding to the electronic transitions that satisfy the resonance conditions. The graph on the right of Fig. 3 shows that with the fixed quantum numbers $n = 1$, $n' = 2$, $l = 1$, $l' = 1$), we can see two resonance peaks corresponding to the electronic transitions from initial state α to final state β with the absorption (the higher peak) and emission (the lower peak) one photon. Figure 3 also exhibits the dependence of the number of resonance peaks on temperature.

IV. CONCLUSIONS

We have calculated the absorption power for an electron system interacting with LO-phonons in CQW structures by using the formula derived from the projection method with assumption of the Lorentzian line-shape. The expressions of absorption power contain terms which prove that all possible electron transitions, along with phonon-nabsorptions and emissions, satisfy the energy conservation law and the condition for optical transitions in solids.

From graphs of the absorption power we obtain line-widths as profile of curves. The obtain graphs showed that half line-widths decreased with the wire's radius and increased with the temperature. The conditions for ODEPR are independent of lattice temperature and the number of resonance peaks depends on the specific transitions (the value of quantum numbers identifying the energy levels).

References

[1] H.J. Kim, K.S. Yi, J. Korean Phys. Soc. 40, 1069 (2002).

[2] E. Zielinski, H. Schweizer, S. Hausser, R. Stuber, M. Pilkuhn, G. Weimann, IEEE J. Quantum Electron QE 23, 969 (1987).

[3] S.C. Lee, Y.B. Kang, D.C. Kim, J.Y. Ryu, N.L. Kang, S.D. Choi, Phys. Rev. B 55 6719 (1997) .

[4] N.L. Kang, Y.J. Cho, S.D. Choi, Progr. Theor. Phys. 96, 307 (1996).

[5] S. Badjouand, P.N. Argyres, Phys. Rev. B 35, 5964 (1987).

[6] N.A. Zakhleniuk, C.R. Bennett, N.C. Constantinous, B.K. Ridley, M. Babiker, Phys. Rev. B 54, 17838 (1996).

[7] A. Gold, A. Ghazali, Phys. Rev. B 41, 7626 (1990).

[8] Sang Chil Lee, J. Korean Phys. Soc. 52, 1832 (2008).

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