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Two-by-Two Contingency Table as a Goodness-of-Fit Test

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Abstract: This publication presents a two-by-two contingency table as a goodness-of-fit test. The test is devoted to the exponential distribution. However, samples subjected to the test come from the Generalized Gamma Distribution. The aim is to determine the power of the test and to compare the obtained results to the Kolmogorov-Smirnov goodness-of-fit test.

Key words: goodness-of-fit test, two-by-two table, generalized gamma random value, power of test

Abbreviation used:

TT	Two-by-two contingency table
KS	Kolmogorov-Smirnov
GG	Generalized Gamma Distribution
VBA	Visual Basic for Applications (in Excel)
MM	Method of moments
MLM	Maximum likelihood method
MLS	Method of least squares
CDF	Cumulative distribution function

I. INTRODUCTION

In statistical literature it is possible to find numerous methods of inference relating to one variable. However, objects are often described by means of a large number of variables. The table, which is created by dividing data according to two variables, is called a two-way contingency table and ranks among basic statistical tools. In statistical inference it is not possible to answer a lot of questions in the analytical way, and one should use computer simulations to do so.

A wide collection of tests for exponentiality (including the Kolmogorov-Smirnov test) was discussed and compared in [1]. In [2] a new goodness-of-fit test was proposed. The test utilizes a novel characterization of the exponential distribution through its characteristic function. In [3] the goodness-of-fit technique based on the use of transformed empirical processes was applied to the construction of a test of exponentiality, focused on Weibull alternatives. In [11] the procedure of generating TT using two-dimensional normal distribution was presented. However, this method was not successful as the generator of two-way table of large sizes, because the corner cells of table often remained empty. For this reason in [12] the method of generating of two-way tables was suggested, making use of uniform random numbers, which was named the “bar method”.

This publication presents the TT which was used as a goodness-of-fit test defined as the TT-test. For creating TT the empirical and theoretical distribution functions were used. This test is devoted to the exponential distribution. However, samples subjected to the test come from the Generalized Gamma Distribution. The aim is to calculate the power of the TT-test and to compare the obtained results with the Kolmogorov-Smirnov goodness-of-fit test.

In the second section a random numbers generator of the GG was presented in the form of the user function “GenGG”, which was implemented in VBA. The third section focuses on creating TT on the base of the empirical distribution function and the exponential distribution function (exponential distribution is a special case of the GG). In the fourth section a computer implementation of the TT-test using of the Pearson χ^2-statistic was presented. The last section was devoted to the analysis of the obtained results. For a sample of size $n \in \{15, 20, 30\}$ and at significance level $\alpha = 0.05$ the power of the TT-test was determined. It turns out that for analyzed values of the GG parameters the power of the discussed test is almost always greater than the power of a very popular KS goodness-of-fit test.

II. GENERATOR OF GENERALIZED GAMMA RANDOM NUMBERS

To execute simulation, random numbers of given probability distribution are necessary. The GG has a complex analytic form, which gives it the desired flexibility. The density function of the GG is given by [10]

$$f\left(z;a,b,c\right)=\frac{b}{a\Gamma\left(c\right)}\left(\frac{z}{a}\right)^{bc-1}\exp\left[-\left(\frac{z}{a}\right)^{b}\right]\quad\left(z>0\right),\tag{1}$$

where $b>0, c>0$ are shape parameters, $a>0$ is a scale parameter. It is possible to write the CDF of the GG by means of the incomplete gamma-function

$$\Gamma_n\left(c,x\right)=\int_0^x u^{c-1}\exp\left(-u\right)du$$

in the form

$$T_1\left(z\right)=\frac{\Gamma_n\left[c,\left(z/a\right)^{b}\right]}{\Gamma\left(c\right)}.\tag{2}$$

Then $f\left(x;a,1,c\right)$ is a gamma density function, which for $c=1$ becomes an exponential density function. The random variables X and Z are interrelated

$$X=\left(\frac{Z}{a}\right)^{b}\Rightarrow Z=a\cdot X^{1/b}.$$

Thus, a generator of gamma random variables X will be sufficient to construct. A theoretical bases for creating user's function "GenGG" written in VBA are possible to find in [13].

```
Function GenGG(a As Single, b As Single, c As Single) As Single
'declaration of variables
Dim z As Single, x As Single
Dim rand1 As Single, rand2 As Single
Dim d As Single, e As Single, f As Single
Dim g As Single, h As Single, k As Single
Dim m As Single, p As Single, accept As Boolean
'main part
Randomize Timer
If (c = 1) Then
        x = -Log(Rnd)
        z = a * x ^ (1 / b)
 End If
 If (c < 1) Then
      d = 1 / c
      e = 0.07 + 0.75 * Sqr(1 - c)
      f = 1 + c / e / Exp(e)
        Do
          rand1 = Rnd
          rand2 = Rnd
          g = f * rand1
          If g < 1 Then
             x = e * g ^ d
             accept = (rand2 <= (2 - x) / (2 + x))
             If accept = False Then accept = (rand2 <= Exp(-x))
                Else
             x = -Log(d * e * (f - g))
             h = x / e
             accept = (rand2 * (c + h - c * h) <= 1)
             If accept = False Then accept = (rand2 <= (x / e) ^ (c - 1))
          End If
        Loop Until accept
        z = a * x ^ (1 / b)
 End If
 If (c > 1) Then
      k = 1 / Sqr(2 * c - 1)
      d = c + 1 / k
      f = c - Log(4)
          Do
again:
             rand1 = Rnd
             rand2 = Rnd
             g = k * Log(rand1 / (1 - rand1))
             x = c * Exp(g)
             m = rand1 ^ 2 * rand2
             If m = 0 Then GoTo again
             p = f + d * g - x
             accept = (p + (1 + Log(k)) - k * m >= 0)
             If accept = False Then accept = (p >= Log(m))
          Loop Until accept
        z = a * x ^ (1 / b)
 End If
GenGG = z
End Function
```

The random values z_i^* obtained by means of function "GenGG" were sorted and then on the basis of (2) a theoretical distribution function of the GG were calculated as well as an empirical distribution function

$$F_i=\frac{i}{n+1}\quad i=1,\ldots,n.\tag{3}$$

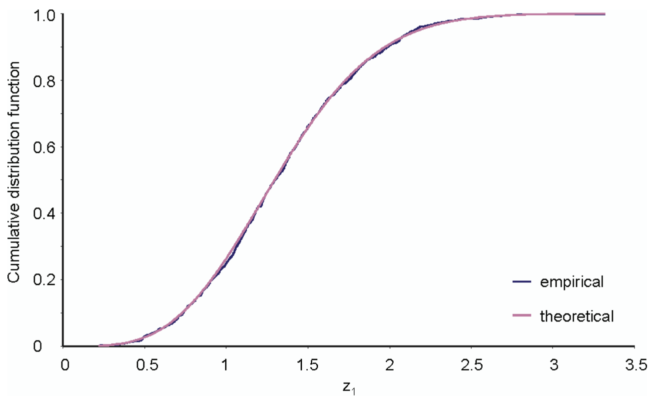

Fig. 1. Empirical and theoretical distribution functions of the GG for n = 1000

Graphs of both empirical and theoretical distribution functions are quite the same (Fig. 1), therefore z_i^* are generalized gamma random numbers.

III. GENERATOR OF TWO-BY-TWO TABLE

Let $z_{(i)}^*$ will be sorted generalized gamma random values. To create TT values exponential distribution function T_2 as well as values of the empirical distribution $z_{(i)}^*$ were used

$$T_2\left(z_{(i)}^*;a^*\right)=1-\exp\left(-\frac{z_{(i)}^*}{a^*}\right). \tag{4}$$

An unknown value of the parameter of the exponential distribution was estimated by means of MLS, thanks to which the empirical distribution function fits better the theoretical distribution function (Fig. 2) than in MLM or MM (Fig. 3).

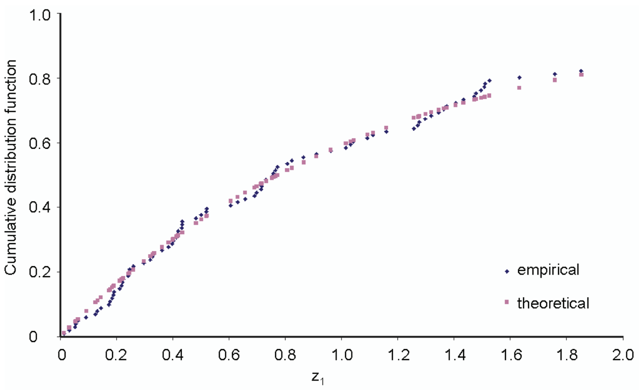

Fig. 2. The exponential distribution function when the parameter was estimated by means of MLS

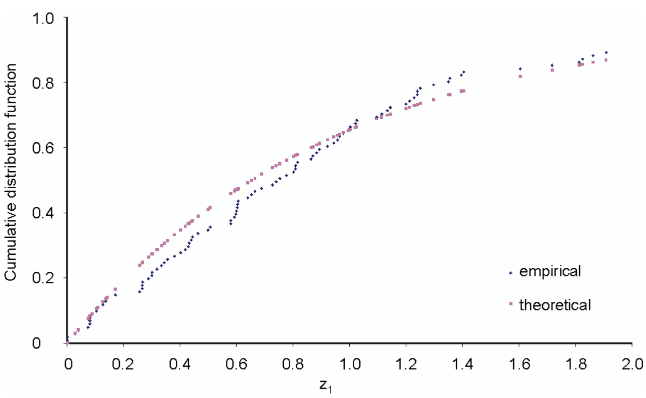

Fig. 3. The exponential distribution function when the parameter was estimated by means of MM or MLM

An estimate a^* of parameter a is a value which minimizes a function [6]

$$M\left(a\right)=\sum_{i=1}^{n}\left[T_2\left(z_i^*;a\right)-F_i\right]^2. \tag{5}$$

In order to find a minimum of the function (5) the supplement of the spreadsheet, which is called Solver, was used. It was written in the form of the procedure, so that it can be applied in the loop.

On the basis of (3) and (4) a difference of CDF was marked

$$D_i=F_i-T_2\left(z_{(i)}^*;a^*\right) \quad i=1,\ldots,\cdots n\,,$$

whose sign decides on rows size of TT. A number $M=DP_{([n/2]+1)}$ assuring best results of TT-test was calculated on the basis of sorted values $DP_i=\left|D_i\right|$ and decides on the columns size (Table 1).

Table 1. Principle of creating a TT

	1	2
1	$D_i>0\wedge D_i\le M$	$D_i>0\wedge D_i>M$
2	$D_i\le 0\wedge\left|D_i\right|\le M$	$D_i\le 0\wedge\left|D_i\right|>M$

In the statistical literature requirements for the minimal number of objects in cells of the two-way table are diversified. The author of this publication assumed that every cell should have at least one element. The VBA implementation of TT generation, written in VBA, was introduced in the “GenTable” procedure which is using the “Solver” procedure. After apostrophes very exact comments were placed in order to make it easier for the reader to understand the codes of procedures.

```
Sub GenTable()
‘declaration of tables
Dim Z() As Single           'random numbers of the GG
Dim F() As Single           'empirical distribution function
Dim T() As Single           'exponential distribution function
Dim X(2, 2) As Integer      'TT
Dim D() As Single           'difference of CDF
Dim DP() As Single          'absolute values of difference of CDF
‘declaration of variables
Dim a As Single             'scale parameter of the GG
Dim b, c As Single          'shape parameter of the GG
Dim aEmp As Single          'sample parameter a
Dim i As Integer            'index of rows
Dim j As Integer            'index of columns
Dim n As Long               'simple size
Dim r As Integer            'accessory variables
Dim b1, M, r As Single      'accessory variables
Dim hor, ro As Single       'accessory variables
```

```
Randomize Timer
Let a = 1
Let b = InputBox("Give b", "GG")
Let c = InputBox("Give c", "GG")
Let n = InputBox("Give a sample size", "n")
'redeclaration of tables
ReDim Z(n): ReDim T(n): ReDim F(n)
ReDim D(n): ReDim DP(n)
''preparing formulas for Solver
For r = 1 To n
   ro = r + 20
   Range("K" & ro).Select
   ActiveCell.FormulaR1C1 = "=1-EXP(-RC[-1]/R21C14)"
   Range("L" & ro).Select
   ActiveCell.FormulaR1C1 = "=RC[-3]/" & n + 1
Next r
   Range("N22").Select
   ActiveCell.FormulaR1C1 = _
   "=SUMXMY2(R[-1]C[-3]:R[" & n - 2 & "]C[-3],R[-1]C[-2]:R[" & n - 2 & "]C[-2])"

continue:
'Zeroing of cells of TT
For i = 1 To 2
   For j = 1 To 2
      X(i, j) = 0
   Next j
Next i
'generation of generalized gamma random numbers
For r = 1 To n
   Z(r) = GenGG(a, b, c)
   Cells(20 + r, 9) = r
   Cells(20 + r, 10) = Z(r)
Next r
'sorting the table Z
back:
   hor = 0
   For r = 1 To n - 1:
      If (Z(r) <= Z(r + 1)) Then GoTo further
         b1 = Z(r)
         Z(r) = Z(r + 1)
         Z(r + 1) = b1
         hor = 1
further:
   Next r
   If hor = 1 Then GoTo back
'recall to the procedure
Cells(21, 14) = 1
Solver
'introduction value of cell to variable
aEmp = Cells(21, 14)
'calculating CDF
For r = 1 To n
   F(r) = r / (n + 1)
   T(r) = 1 - Exp(-Z(r) / aEmp)
   D(r) = F(r) - T(r)
   DP(r) = Abs(D(r))
Next r
'sorting the table DP
back1:
hor = 0
For r = 1 To n - 1:
   If (DP(r) <= DP(r + 1)) Then GoTo further1
      b1 = DP(r)
      DP(r) = DP(r + 1)
      DP(r + 1) = b1
      hor = 1
further1:
Next r
If hor = 1 Then GoTo back1
'calculating M
M = DP(Int(n / 2) + 1)
'creating a TT
For r = 1 To n
   If D(r) > 0 And D(r) <= M Then X(1, 1) = X(1, 1) + 1
   If D(r) > 0 And D(r) > M Then X(1, 2) = X(1, 2) + 1
   If D(r) <= 0 And Abs(D(r)) <= M Then X(2, 1) = X(2, 1) + 1
   If D(r) < =0 And Abs(D(r)) > M Then X(2, 2) = X(2, 2) + 1
Next r
'checking the size of cells of TT
For i = 1 To 2
   For j = 1 To 2
      If X(i, j) < 1 Then GoTo continue
   Next j
Next i

End Sub
```

```
Sub Solver()
Dim wynik As Long
        SolverOptions MaxTime:=100, Iterations:=200,
Precision:=0.00000001, _
        AssumeLinear:=False, StepThru:=False, Estimates:=1,
Derivatives:=1, _
        SearchOption:=2, IntTolerance:=5, Scaling:=False,
Convergence:=0.0001, _
        AssumeNonNeg:=False

        SolverOk SetCell:="$N$22", MaxMinVal:=2, ValueOf:="1",
ByChange:="$N$21"
        wynik = SolverSolve(True)
End Sub
```

IV. THE GOODNESS-OF-FIT TEST FOR EXPONENTIALITY

As mentioned in the introduction, this section presents the TT test that uses TT. Let X_{ij} be the observed cells size of TT ($\sum_{i=1}^{2}\sum_{j=1}^{2} X_{ij} = n$). By $XR_i = \sum_{j=1}^{2} X_{ij}$ indicated a number of sampling units belonging to *i*-th row, however, by $XC_j = \sum_{i=1}^{2} X_{ij}$ a number of sampling units belonging to *j*-th column. Expected sizes of cells of TT were fixed by means of the formula [7]

$$NE_{i\,j} = \frac{XR_i \cdot XC_j}{n}.$$

Pearson χ^2-statistic for TT is given by [2]

$$\chi^2 = \sum_{i=1}^{2}\sum_{j=1}^{2} \frac{\left(X_{i\,j} - NE_{ij}\right)^2}{NE_{i\,j}}.$$

Null hypothesis H_0 says that distribution in general population is exponential. Pearson χ^2-statistic were calculated one thousand times for a sample size $n \in \{15, 20, 30\}$ and at significance level $\alpha = 0.05$. The generalized gamma sample value was determined by means of a function „GenGG”. On the base of the received results the power of the test was obtained, that is the probability to reject the null hypothesis H_0, when it is not true

$$M = \frac{1000 - U}{1000}.$$

Number U is a number of cases when the value of χ^2-statistic is smaller than critical value CV. Critical values for a given sample size n on the base of estimates of quantiles were marked. Unknown values of quantiles were replaced by appropriate order statistics [5]. For shape parameter values of the GG $b = c = 1$ (the GG is becoming the exponential distribution) one thousand values of χ^2-statistic were calculated. K-th order statistic of χ^2-statistics as a critical value CV was accepted, where $k = \text{int}\left[10000 \cdot (1-\alpha)\right]$ (Table 2).

Table 2. Critical values at significance level $\alpha = 0.05$

n	CV	n	CV
15	3.233	30	3.453
20	3.300	35	3.540
25	3.381	40	4.821

The computer procedure of the TT-test written in VBA was introduced below.

```
Sub TT-test()
Dim X(2, 2) As Integer          'observed size of TT cells
Dim XR(2) As Single             'rows sizes
Dim XC(2) As Single             'columns sizes
Dim NE(2, 2) As Single          'expected size of TT cells
Dim Chi(1000) As Single         'χ2 -statistic
Dim i As Integer                'index of row
Dim j As Integer                'index of column
Dim n As Long                   'sample size
Dim CV As Single                'critical value
Dim U As Integer                'number of accepted null hypotheses
Dim w As Integer                'accessory variable
Dim b1, M, r As Single          'accessory variables
Select Case n
   Case 15
      CV = 3.233
   Case 20
      CV = 3.3
   Case 30
      CV = 3.453
End Select
For i = 1 To 2
   For j = 1 To 2
      XC(j) = 0
   Next j
   XR(i) = 0
Next i
U = 0
For w = 1 To 1000
   For j = 1 To 2
      For i = 1 To 2
         XC(j) = XC(j) + X(i, j)
      Next i
   Next j
   For i = 1 To 2
      For j = 1 To 2
         XR(i) = XR(i) + X(i, j)
      Next j
   Next i
   For j = 1 To 2
      For i = 1 To 2
         NE(i, j) = XC(j) * XR(i) / n
      Next i
   Next j
   Chi(w) = 0
   For j = 1 To 2
      For i = 1 To 2
         Chi(w) = Chi(w) + ((X(i, j) - NE(i, j)) ^ 2) / NE(i, j)
      Next i
   Next j
   If Chi(w) < CV Then
      U = U + 1
   End If
Next w
MsgBox "Power of the TT-test is " & (1000 - U) / 1000
End Sub
```

V. OBTAINED RESULTS

For $a = 1$ and different values of shape parameters of the GG the power of the TT-test was calculated. On the basis of the obtained results for $n \in \{15, 20, 30\}$ surface graphs were prepared (Fig. 4-6). For values close to two the power of the test equals zero, next a tendency of the increase is visible with bending for $b \cdot c = 1$ (particularly visible in Fig. 6).

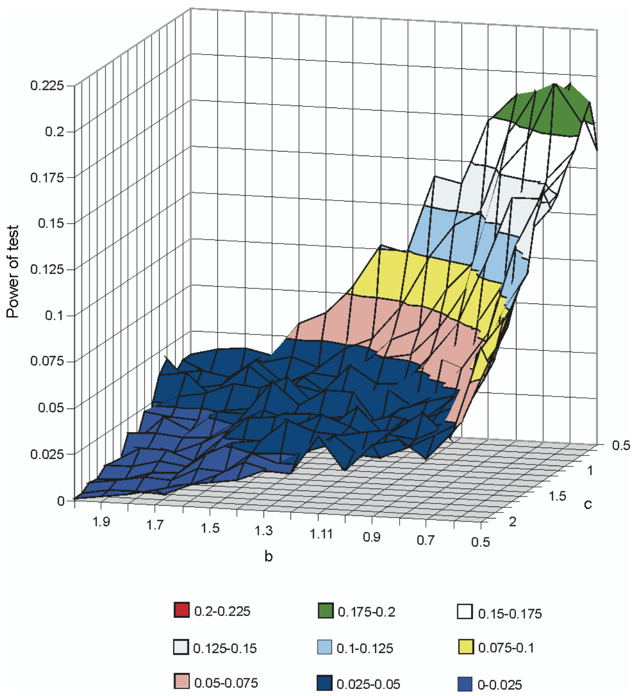

Fig. 4. Relation between shape parameters of the GG and power of the TT-test for $n = 15$ and $\alpha = 0.05$

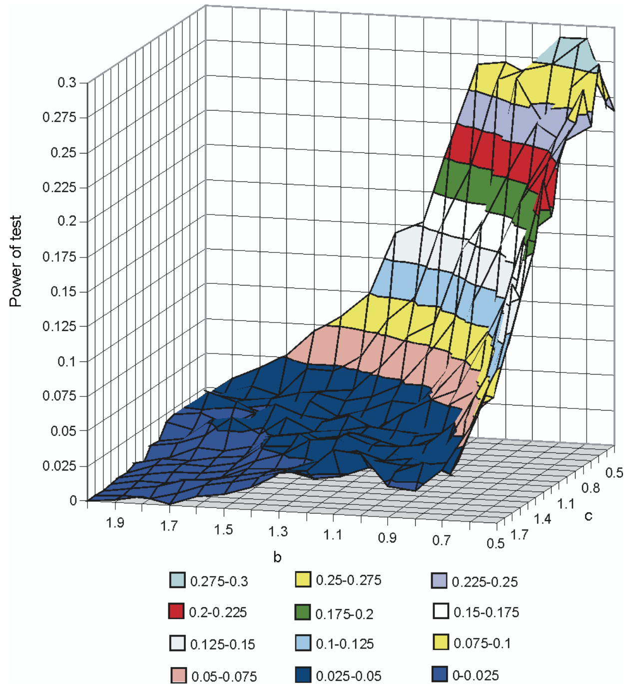

Fig. 5. Relation between shape parameters of the GG and power of the TT-test for $n = 20$ and $\alpha = 0.05$

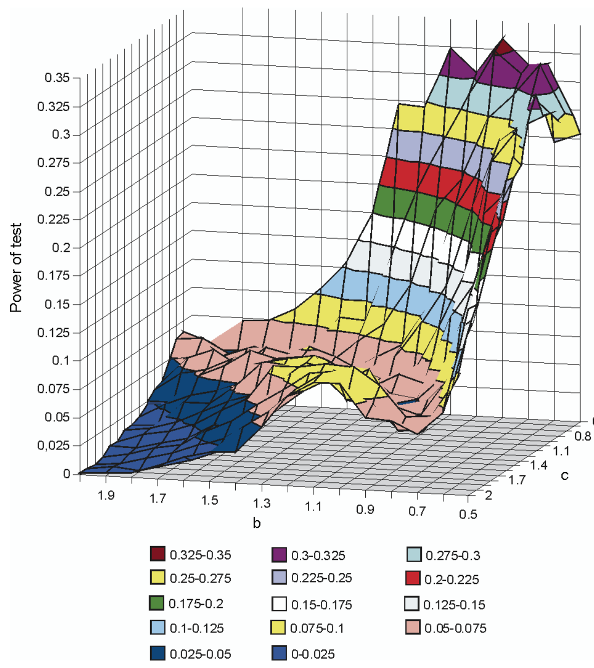

Fig. 6. Relation between shape parameters of the GG and power of the TT-test for $n = 30$ and $\alpha = 0.05$

In order to compare the results received by means of the TT-test, the author using the same samples also carried out the KS goodness-of-fit test (a parameter of the exponential distribution was estimated from a sample) and marked it power [9]. This test is well-known and therefore its results in the graphical form in Fig. 7-9 were shown only. The received surfaces have the bathtub shape and the real significance level of this test is equal to zero.

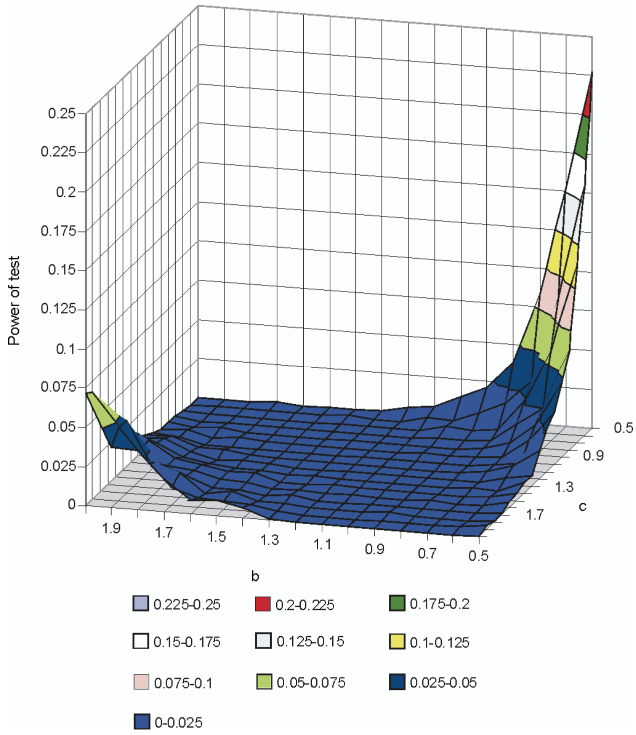

Fig. 7. Relation between shape parameters of the GG and power of the KS goodness-of-fit test for $n = 15$ and $\alpha = 0.05$

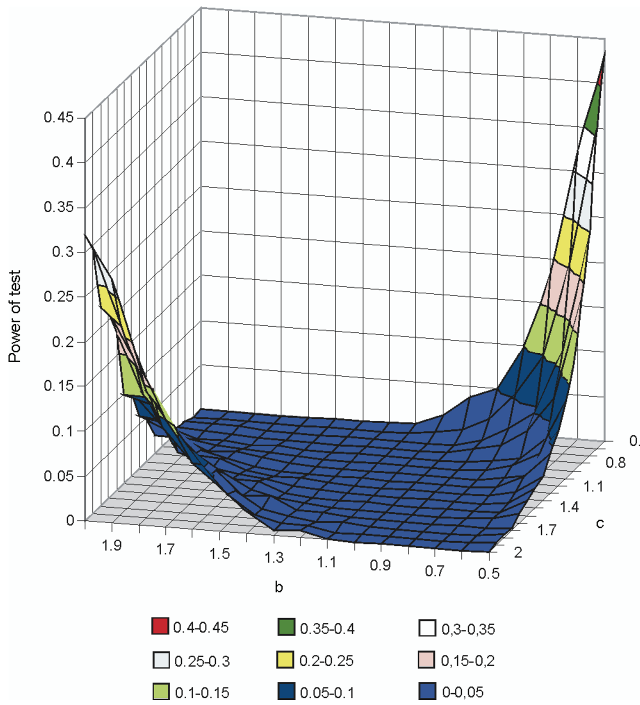

Fig. 8. Relation between shape parameters of the GG and power of the KS goodness-of-fit test for $n = 20$ and $\alpha = 0.05$

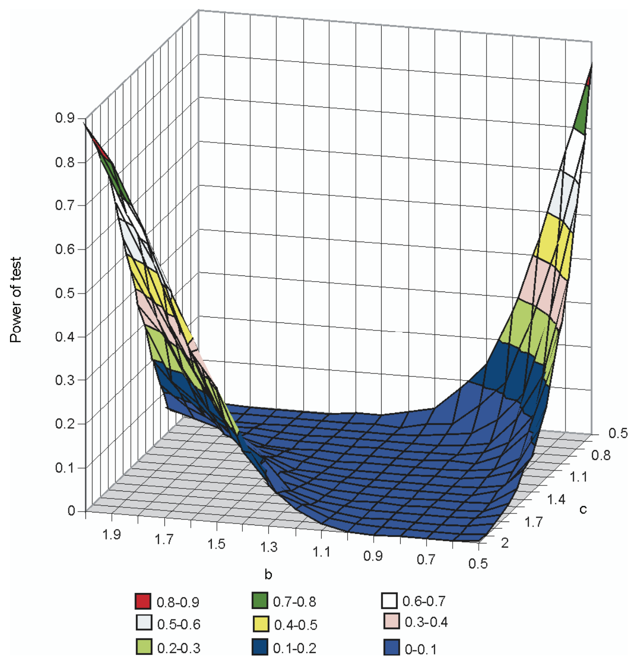

Fig. 9. Relation between shape parameters of the GG and power of the KS goodness-of-fit test for $n = 30$ and $\alpha = 0.05$

As it was mentioned in section two, the GG is a very flexible distribution. By changing the parameter values one can get all possible shapes of the GG risk function (Table 3), which is given by [4]

$$h(z) = \frac{f(z)}{1 - T_1(z)},$$

where $f(z)$, $F(z)$ are appropriately a density function and a CDF of the GG.

Table 3. The shapes of the GG risk function

Set	Shape
$A = \{(b,c) : (b = 1) \wedge (c = 1)\}$	constant
$B = \left\{(b,c) : \left(c > \frac{1}{b}\right) \wedge (b < 1)\right\}$	having a maximum
$C = \left\{(b,c) : \left(c \geq \frac{1}{b}\right) \wedge (b \geq 1)\right\} - G_1$	strictly increasing
$D = \left\{(b,c) : (b \leq 1) \wedge \left(c \leq \frac{1}{b}\right)\right\} - G_1$	strictly decreasing
$E = \left\{(b,c) : \left(c < \frac{1}{b}\right) \wedge (b > 1)\right\}$	having a minimum

In Tables 4-6 the powers of the TT-test and KS goodness-of-fit test were compared. In most of the obtained results the power of the proposed test is greater than the power of KS goodness-of-fit test and it is exemplified by cells of table marked with the letter T. In several cases the KS goodness-of-fit test has the greater power and it was marked with the letter K. Colors signify different shapes of the GG risk function:

- red – constant,
- purple – having a maximum,
- blue – strictly increasing,
- yellow – strictly decreasing,
- green – having a minimum.

The proposed TT-test in most of the analyzed cases is more powerful than the KS goodness-of-fit test and it is particularly visible for the little sample. Together with the sample increase this situation is changing for the better

Table 4. The comparison of power of the TT-test and the KS goodness-of-fit test for $n = 15$ and $\alpha = 0.05$

c \ b	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
0.5	K	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
0.6	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
0.7	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
0.8	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
0.9	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
1	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
1.1	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
1.2	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
1.3	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
1.4	T	T	T	T	T	T	T	T	T	T	T	T	T	T	K	K
1.5	T	T	T	T	T	T	T	T	T	T	T	T	T	T	K	K
1.6	T	T	T	T	T	T	T	T	T	T	T	T	T	T	K	K
1.7	T	T	T	T	T	T	T	T	T	T	T	T	K	K	K	K
1.8	T	T	T	T	T	T	T	T	T	T	T	T	K	K	K	K
1.9	T	T	T	T	T	T	T	T	T	T	T	T	K	K	K	K
2	T	T	T	T	T	T	T	T	T	T	T	T	K	K	K	K

Table 5. The comparison of power of the TT-test and the KS goodness-of-fit test for $n = 20$ and $\alpha = 0.05$

c \ b	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
0.5	K	K	T	T	T	T	T	T	T	T	T	T	T	T	T	T
0.6	K	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
0.7	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
0.8	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
0.9	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
1	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	K
1.1	T	T	T	T	T	T	T	T	T	T	T	T	T	T	K	K
1.2	T	T	T	T	T	T	T	T	T	T	T	T	T	T	K	K
1.3	T	T	T	T	T	T	T	T	T	T	T	T	K	K	K	K
1.4	T	T	T	T	T	T	T	T	T	T	T	T	K	K	K	K
1.5	T	T	T	T	T	T	T	T	T	T	T	K	K	K	K	K
1.6	T	T	T	T	T	T	T	T	T	T	T	K	K	K	K	K
1.7	T	T	T	T	T	T	T	T	T	T	T	K	K	K	K	K
1.8	T	T	T	T	T	T	T	T	T	T	K	K	K	K	K	K
1.9	T	T	T	T	T	T	T	T	T	T	K	K	K	K	K	K
2	T	T	T	T	T	T	T	T	T	K	K	K	K	K	K	K

Table 6. The comparison of power of the TT-test and the KS goodness-of-fit test for $n = 30$ and $\alpha = 0.05$

c \ b	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
0.5	K	K	K	T	T	T	T	T	T	T	T	T	T	T	T	T
0.6	K	K	T	T	T	T	T	T	T	T	T	T	T	T	T	T
0.7	K	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
0.8	K	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
0.9	K	T	T	T	T	T	T	T	T	T	T	T	T	T	T	K
1	T	T	T	T	T	T	T	T	T	T	T	T	T	K	K	K
1.1	T	T	T	T	T	T	T	T	T	T	T	T	T	K	K	K
1.2	T	T	T	T	T	T	T	T	T	T	T	T	K	K	K	K
1.3	T	T	T	T	T	T	T	T	T	T	T	K	K	K	K	K
1.4	T	T	T	T	T	T	T	T	T	T	K	K	K	K	K	K
1.5	T	T	T	T	T	T	T	T	T	T	K	K	K	K	K	K
1.6	T	T	T	T	T	T	T	T	T	T	K	K	K	K	K	K
1.7	T	T	T	T	T	T	T	T	T	K	K	K	K	K	K	K
1.8	T	T	T	T	T	T	T	T	T	K	K	K	K	K	K	K
1.9	T	T	T	T	T	T	T	T	K	K	K	K	K	K	K	K
2	T	T	T	T	T	T	T	T	K	K	K	K	K	K	K	K

of the KS goodness-of-fit test. For n = 15 the TT-test has a greater power in 91% of the received results, for n = 20 in 80%, but for n = 30 in 72%.

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