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Propagation Technique for Ultrashort Pulses III: Pulse Splitting of Ultrashort Pulses in a Kerr Medium

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Abstract: Impact of such terms as third order dispersion, self-steepening and stimulated Raman scattering on evolution of ultrashort pulses is considered in detail. Under influence of these effects, pulse did not maintain its initial shape. Pulse splits into constituents, its spectrum also evolving into several bands which are known as optical shock and self-frequency shift phenomena. We concluded that when the input peak power is large enough, dynamics of pulse splitting will be complicated. Our numerical simulations were in good agreement with experimental results.

Key words: propagation, ultrashort pulse, Kerr medium

I. INTRODUCTION

As it has been recognized before [14], fundamental and applied research for the propagation process of ultrashort laser pulses in a nonlinear medium is nowadays in the center of interest because of its important application in the optical telecommunication [2-5]. Important effects involved in this propagation process have been theoretically and experimentally considered by several authors [7-13]. In our paper [14] we have presented general theoretical foundations of this problem and applied these to a particular case of the Kerr medium. The obtained pulse propagation equation is the generalized nonlinear Schrödinger (GNLS) equation, where the important higher-order effects: third-order dispersion (TOD), self-steepening, and self-frequency shift are included. Generally this equation is solved only approximately, and we have chosen the numerical method with the concrete algorithms: Split-Step and Runge-Kuta [15]. The accuracy of this method has been recognized for the case of the picosecond pulses through comparison with well-known analytical results. In this paper we show the effectiveness of this method for the case of the femtosecond pulses by comparing our numerical calculations with the theoretical and experimental results given in literature [2-13]. Our paper is organized as follows: In Section II we will consider the self-steepening effect and the optical shock, Section III contains the study of the influence of the self-frequency shift effect. Section IV analyzes the dynamics of the pulse splitting in the general case, and Section V contains conclusions.

II. SELF-STEEPENING EFFECT AND OPTICAL SHOCK

At first we consider the pulse propagation when the self-steepening effect dominates (TOD and self-frequency shift are neglected) [2, 3]. Then the pulse propagation equation reduces to the following form:

$$\frac{\partial U}{\partial \xi}=\frac{i}{2}\frac{\partial^2 U}{\partial \tau^2}+iN^2\left(|U|^2 U+iS\frac{\partial}{\partial \tau}\left(|U|^2 U\right)\right). \tag{1}$$

According to the analytical results, during the propagation the group velocity of the pulse depends on the intensity, the velocity at the top of the envelope function has the biggest value and it decreases in two pulse edges. When the propagation distance is larger, the pulse shape becomes asymmetric and the peak moves to the later times, the pulse steeply rises. In the end we have a steep front in the trailing edge of the pulse, resembling the usual shock-wave formation. This effect is called the optical shock [1, 4, 5].

To simulate this result, we consider the case when the nonlinear parameter S = 0.2 and the ultrashort input pulse has the hyperbolic secant shape with the different powers. The results are displayed in Fig. 1.

In Fig. 1(a) we have chosen the power parameter N = 2 and the propagation distance $\xi = 2.5\pi$. The numerical calculations show that after the distance 0.25π the pulse splits to two parts and their spacing increases further with distance. In [15] we have considered the solitons as special solutions for the Eq. (1) where the nonlinear parameter S vanishes. These solutions change periodically with the cycle $\pi/2$, after this distance the shape of the envelope function resumes the initial form. When N = 2, we have the second-order soliton that is treated as a superposition of the two fundamental solitons which have the same characteristic time and the propagation velocity [3]. In the propagation process these solitons interact with each other and the envelope function changes in a complicated manner, but its shape is still symmetric and remains unchanged after each cycle. If the second nonlinear term in Eq. (1) is taken into account, the symmetry of the pulse shape is destroyed. As a result, two fundamental solitons are separated and move apart with different envelope functions and propagation velocities. Sometimes this effect is called the collapse of the soliton [3]. When their spacing is large, the interaction between them becomes very weak and the influence of one part on the other can be neglected, but the envelope function of every part still changes because of the nonlinear higher-order effects. During further propagation the envelope functions become more asymmetric and they move to the later times. The change of the part with the higher peak is stronger and the process rapidly goes to the limit, when the pulse edge in the right is steep and an optical shock occurs. Then the shape is changed dramatically and can not be described by Eq. (1) any more.

Figure 1(b) corresponds to the power parameter N = 4 and the propagation distance $\xi = \pi/2$. Similarly to the previous case, the envelope function becomes asymmetric and splits to several subpulses during the propagation. Here we obtain four subpulses because the value of the parameter N = 4 corresponds to the four-order soliton of Eq. (1) when S vanishes. All the envelope functions of these pulses move to the later time. The firstly separated peak is the highest and therefore the steepening of this pulse is going rapidly. If the distance is larger than the distance chosen above, we will have the optical shock for this peak. It follows that for this case the steepening process is much quicker.

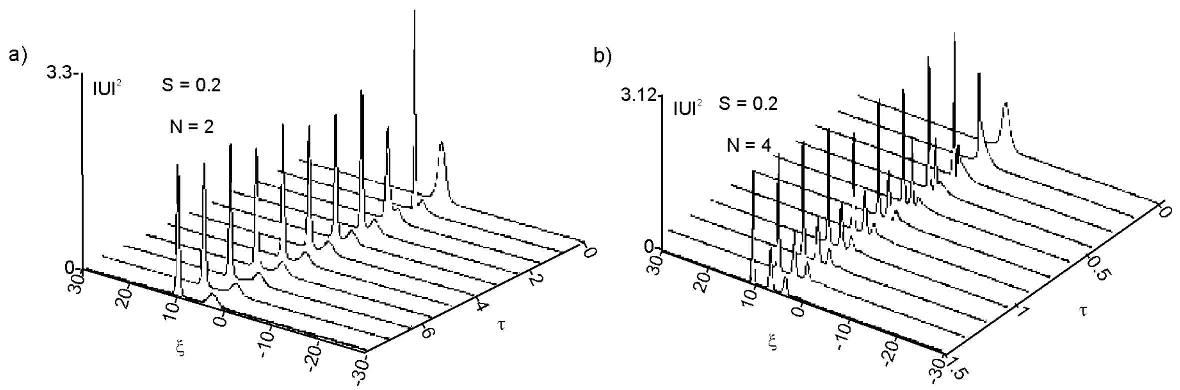

Fig. 1. Propagation of the hyperbolic secant input pulse with the different powers over the distance $\xi = 2.5\pi$. (a) and $\xi = \pi/2$

The numerical simulations show that for the larger power parameter the formation of the optical shock is more rapid.

III. SELF-FREQUENCY SHIFT EFFECT

We note in passing [14, 15] that in some fixed conditions the nonlinear effect induced by the stimulated Raman scattering dominates in comparison with the TOD and self-steepening. In this Section we consider the case of the ultrashort pulse propagation which satisfies these conditions. After omitting the parameters δ_3 and S , the GNLS equation has the following form:

$$\frac{\partial U}{\partial \xi} = \frac{i}{2}\frac{\partial^2 U}{\partial \tau^2} + iN^2\left(|U|^2 U - \tau_R U \frac{\partial |U|^2}{\partial \tau} \right). \qquad (2)$$

Under the influence of this higher-order nonlinear effect the envelope function and the pulse spectrum undergo many changes. The Raman scattering process induces the shift down to the low-frequency domain in the larger part of the spectrum of the initial pulse. The splitting of the envelope function to the several parts occurs simultaneously, so the pulse is compressed into the shorter pulses [10, 11]. These changes of the ultrashort pulses have been observed experimentally and they are known as the soliton self-frequency shift effect [8, 9]. In this case the existence of the soliton solution is impossible because one of the important conditions for this is the energy conservation. In the Raman scattering the pulse energy loses in the energy transfer to the oscillations of the molecules and lattice [1, 7, 14].

We consider now the propagation of the ultrashort pulse with the initial hyperbolic secant shape with the power parameter N = 3 over the distance $\xi = \pi$. The results are displayed in Fig. 2.

During the propagation over a short distance, the pulse already splits to two parts: the main strong peak moves rapidly to the later time and the much lower peak. During the further propagation the main peak is strongly compressed and its width is much smaller than the input pulse, whilst the lower peak splits further to two constituents, one is moved to the later times, one to earlier times.

There are several similarities and differences in the propagation of ultrashort pulses in Fig. 1 and Fig. 2(a), caused by specific mechanisms of the higher-power nonlinear effects. First, the similarities which are the pulses split into several parts, the pulses move apart one from another, and their main parts move to the later times. The first important difference to be mentioned here is the domination of the nonlinear effect induced by the Raman scattering over the pulse self-steepening effect. In Fig. 2(a) we have chosen the parameter τ_R = 0.01 which is much smaller than the value S = 0.2 in Fig. 1. If we choose S = 0.2 in Fig. 1, after travelling through the same propagation distance, we shall not observe any evident pulse splitting. The second difference is that in Fig. 2(a), apart from the main parts which move to the later times, we have some small parts moving to the earlier times, whilst in Fig. 1 all pulses move to the later times. We can explain this fact by considering the pulse spectrum displayed in Fig. 2(b). The spectrum splits into two separated parts in the end of the propagation distance. The majority of the frequencies is shifted down to the low-frequency domain, while a small part of the spectrum is shifted to the high-frequency domain. They correspond to the Stokes and anti-Stokes processes respectively [2, 5, 7, 14]. The low-frequency part carries the majority of the pulse energy involved in the main peak, while the high-frequency part carries the rest of this energy. The high-frequency constituents move with larger velocities, so they move to the earlier times,

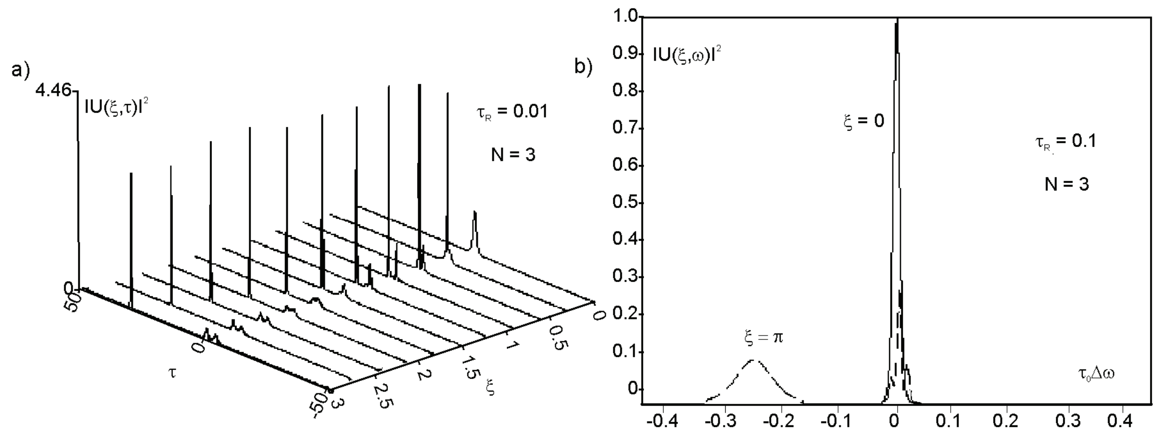

Fig. 2. Propagation of the hyperbolic secant input pulse with the power parameter N = 3 over the distance $\xi = \pi$

while the low-frequency constituents have lower velocities, so they move to the later times in comparison with the initial pulse [3].

We have considered the propagation process of pulses with other power parameters. In every case we have observed the downshift of the frequencies, exactly the same as in the case which has been observed experimentally.

IV. PULSE SPLITTING

In this part we consider the propagation of ultrashort pulses when all three higher-order effects are included. Then the pulse propagation equation is the GNLS equation [15]:

$$\frac{\partial U}{\partial \xi} = \frac{i}{2}\frac{\partial^2 U}{\partial \tau^2} + \delta_3 \frac{\partial^3 U}{\partial \tau^3} + \\ + iN^2\left(|U|^2 U + iS\frac{\partial}{\partial \tau}\left(|U|^2 U\right) - \tau_R U \frac{\partial |U|^2}{\partial \tau} \right). \tag{3}$$

Here, the evolution of the pulse becomes more complicated than in the previous cases. For the pulses with the large power parameter N, both the intensity and spectrum split into several parts during the propagation. This phenomenon is generally called the pulse splitting. The predictions of Eq. (3) are in good agreement with the experimental results [3].

We illustrate these considerations by means of numerical simulations for the propagation of the pulse with the hyperbolic secant input shape in the optical fiber made by material SiO_2. Then the pulse width is $\tau_0 \approx 30\,\text{fs}$ and the center wavelength is $\lambda_0 \approx 1.55\ \mu\text{m}$. It follows from [14] that the values of the higher-order parameters can be calculated: $\delta_3 \approx 0.03$, $S \approx 0.05$, $\tau_R \approx 0.1$. The propagation distance is chosen as $\xi = 3\pi/2$, the power parameter is N = 2. The results of the calculation are displayed in Fig. 3.

The pulse splitting appears in the distance of approximately one soliton cycle i.e. $\xi = \pi/2$. The main peak moves to the later times with the velocity growing with the propagation distance, whereas the minor part moves to the earlier times, but more slowly. This effect can be explained by considering the spectrum change displayed in Fig. 3(b). The majority of the pulse spectrum shifts to the low frequency domain. This fact leads to the change of the propagation velocity in the medium, because the group velocity depends on the carrier frequency. The spectrum shift leads to decreasing the carrier frequency, therefore the main peak propagates with the bigger velocity than the initial pulse and moves to the later times.

Using the value of the pulse width $\tau_0 \approx 30$ fs to transfer the results given in Fig. 3 to the physical units, we obtain the frequency shift 40 THz, i.e. approximately 20% of the carrier frequency after travelling through the propagation distance of 15 cm (because the normalized variable $\xi = \pi/2$ equals 5 cm in this case [3, 15]). This is a relatively large frequency shift.

In the results obtained above, we may see the impact that the higher-order dispersive and nonlinear effects have on the propagation of the ultrashort pulses in its whole importance. Under the influence of these effects the propagation of the ultrashort pulses is much more complicated than in case of the short pulses. Among them were the Raman scattering dominates.

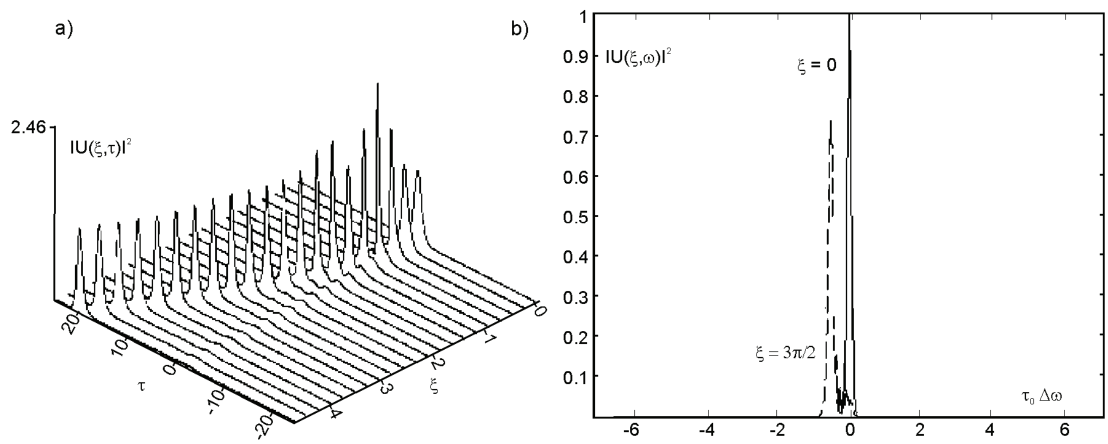

Fig. 3. Evolution of the intensity and spectrum of the hyperbolic secant pulse over the distance $\xi = 3\pi/2$. The parameters of the propagation equation are $\tau_R = 0.1$, $\delta_3 = 0.03$, $S = 0.05$ and $N = 2$

V. CONCLUSIONS

We have considered the propagation of the ultrashort pulses in different cases. The optical shock, self-frequency shift, and dynamics of the pulse splitting have been analyzed in detail. Our results are in good agreement with the results obtained before by several authors and experimental observations.

The ultrashort pulses are widely used nowadays, especially in the optical telecommunication, so the results obtained in the research of these pulses are of great practical importance.

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