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Propagation Technique for Ultrashort Pulses I: Propagation Equation for Ultrashort Pulses in a Kerr Medium

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Abstract: In this work we investigated propagation of ultrashort laser pulses in dispersive nonlinear media. We derived a general propagation equation of pulses which includes the linear and nonlinear effects to all orders. We studied in the specific case of Kerr media and obtained an ultrashort pulse propagation equation called a Generalized Nonlinear Schrödinger Equation. The impact of the third order dispersion, the higher-order nonlinear terms self-steepening, and stimulated Raman scattering are explicitly analyzed.

Key words: propagation, ultrashort pulse, Kerr medium

I. INTRODUCTION

Theoretical and experimental research for the propagation process of ultrashort laser pulses (in fs) in a medium have been the subject of intensive research within the last few years [1, 3, 9, 12]. Because of special properties of these pulses, during their propagation in the medium several new effects have been observed in comparison with the propagation process of short pulses (in ps), namely the effects of dispersion and nonlinear effects of higher orders. Under the influence of these effects, we have complicated changes both in amplitude and spectrum of the pulse. It splits into constituents and its spectrum also evolves into several bands which are known as optical shock and self-frequency shift phenomena [1, 3, 5, 8]. These effects should be studied in detail for future concrete applications of ultrashort pulses, especially in the domain of optical communication.

We apply the general formalism used for the pulse propagation problem in [7] for the one-dimensional case. This formalism is based on the approximate expansion of the nonlinear wave equation, which treats the nonlinear processes involved in the problem as the perturbations. In Sec. II we will present the theoretical model and the basis of the method, and derive from these considerations a general equation for the pulse propagation process in the nonlinear dispersion medium with all orders of dispersion and nonlinearity. Using this formalism for the Kerr medium in considering the delayed nonlinear response of the medium, induced by the stimulated Raman scattering and the characteristic features both of the spectrum and the intensity of the pulse, we will obtain an approximate equation in the most condensed form, describing the propagation of the ultrashort pulses, called the Generalized Nonlinear Schrödinger Equation (GNLS). In Sec. III we derive a normalized form of this equation and demonstrate its general features. We will analyze in detail the influence of the third-order dispersion (TOD), the self-steepening, and the self- shift frequency for the ultrashort pulses in some special cases. Section IV contains conclusions.

II. PROPAGATION EQUATION FOR ULTRASHORT PULSES

II.1. General form of the pulse propagation equation in the nonlinear dispersion medium

The Maxwell equations can be used to obtain the following nonlinear wave equation for the electric field [1, 2, 4, 7]

$$\nabla^2\vec{E}(\vec{r},t)-\nabla\left(\nabla.\vec{E}(\vec{r},t)\right)-\frac{1}{c^2}\frac{\partial^2\vec{E}(\vec{r},t)}{\partial t^2}= =\mu_0\frac{\partial^2\vec{P}_l(\vec{r},t)}{\partial t^2}+\mu_0\frac{\partial^2\vec{P}_{nl}(\vec{r},t)}{\partial t^2}, \quad (1)$$

where $\vec{P}_l(\vec{r},t)$ and $\vec{P}_{nl}(\vec{r},t)$ are the linear and nonlinear polarization, respectively.

The electric field $\vec{E}$ is treated as a superposition of monochromatic constituents with different frequencies and wavevectors centered at their central values ω_0 and $\vec{k}_0$. We confine our self only to consider the propagation of the electric field in an arbitrary direction, say *Oz* (usually chosen as the direction of $\vec{k}_0$), so we can write

$$\vec{E}(r,t)=\vec{x}.E(z,t)=\frac{1}{2}\vec{x}\left[A(z,t)e^{-i\omega_0t+ik_0z}+c.c\right], \quad (2)$$

where $\vec{x}$ is the unit vector of the *x* axis perpendicular to the propagation direction, *A*(*z, t*) is the complex envelope function, and *c.c* denotes the complex conjugate of the first term.

For the homogeneous isotropic, medium the linear polarization vector of the medium is expressed as follows:

$$\vec{P}_l(\vec{r},t)\equiv\vec{P}_l(\vec{z},t)=\vec{x}P_l(z,t)= =\vec{x}\varepsilon_0\int_{-\infty}^{\infty}\chi^{(1)}(t-t')E(z,t')dt'=\vec{x}\varepsilon_0\tilde{\chi}^{(1)}*E, \quad (3)$$

where $*$ denotes the convolution product displaying the causality: the response of the medium in the time *t* is caused by the action of the electric field in all previous times *t'*. The quantity $\chi^{(1)}$ is the susceptibility of the medium. It is a scalar.

The nonlinear polarization vector is generally expressed as follows:

$$\vec{P}_{nl}(r,t)=\varepsilon_0\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\chi^{(2)}(t-t_1,t-t_2):\vec{E}(\vec{r},t_1)\vec{E}(\vec{r},t_2)dt_1dt_2+\right.$$
$$+\iiint_{-\infty}^{\infty}\chi^{(3)}(t-t_1,t-t_2,t-t_3):\vec{E}(\vec{r},t_1)\vec{E}(\vec{r},t_2)\vec{E}(\vec{r},t_3)\,dt_2dt_2dt_3+ \quad (4)$$
$$\left.+\ldots\right],$$

where $\chi^{(n)}(t-t_1,t-t_2,...,t-t_n)$ is the *n*-order nonlinear susceptibility. For the homogeneous isotropic medium, because of the spatial inversion symmetry, the elements of the even-order nonlinear susceptibility $\chi^{(2k)}(t-t_1,...,t-t_{2k})$ disappear [1, 2, 4]. In the expression (5) we have only the nonlinear polarizations of odd orders. We consider in detail only the third-order nonlinear susceptibility (the Kerr medium). Then the tensor $\chi^{(3)}$ has 3^4 = 81 elements (as a matrix with 3 lines and 27 columns), but only 21 of its elements are different from zero and three are independent [1]. We have therefore

$$\vec{P}_{nl}(\vec{r},t)\equiv\vec{P}_{nl}(z,t)=\vec{x}P_{nl}(z,t)=\vec{x}.\varepsilon_0\times$$
$$\times\iiint_{-\infty}^{\infty}\chi^{(3)}_{xxxx}(t-t_1,t-t_2,t-t_3)E(z,t_1)E(z,t_2)E(z,t_3)dt_1dt_2dt_3. \quad (5)$$

In the hierarchy of magnitudes the nonlinear polarization is much smaller than the electric field and the linear polarization $\left|\vec{P}_{nl}(z,t)\right|<<\left|\vec{P}_l(z,t)\right|,\left|\vec{P}_{nl}(z,t)\right|<<\varepsilon_0\left|\vec{E}(z,t)\right|$, so it can be considered as a perturbation and we have the approximate formula [7]: $\nabla.\vec{E}(z,t)\approx 0$. Substituting these results into (1) we obtain the following scalar wave equation:

$$\Delta E(z,t)-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(E(z,t)+\tilde{\chi}^{(1)}*E\right)= =\frac{1}{\varepsilon_0c^2}\frac{\partial^2P_{nl}(z,t)}{\partial t^2}. \quad (6)$$

Transforming the Eq. (6) to the Fourier space and using the properties of the Fourier Transform concerning the convolution and the derivatives of transformed functions we obtain the algebraic equation for the monochromatic part ω of the pulse as follows:

$$\left[k^2-\left(\frac{n(\omega)\omega}{c}\right)^2\right]E(k,\omega)-\frac{\omega^2}{\varepsilon_0c^2}P_{nl}(k,\omega)=0, \quad (7)$$

where $n(\omega)=\sqrt{1+\chi^{(1)}(\omega)}$ is the refractive index of the medium calculated at the frequency ω. We can write this equation in another form:

$$\left[k+\sqrt{\beta^2(\omega)+\frac{\omega^2}{\varepsilon_0 c^2}\frac{P_{nl}(k,\omega)}{E(k,\omega)}}\right]\times$$
$$\times\left[k-\sqrt{\beta^2(\omega)+\frac{\omega^2}{\varepsilon_0 c^2}\frac{P_{nl}(k,\omega)}{E(k,\omega)}}\right]E(k,\omega)=0 \quad (8)$$

with the notation

$$\beta=\beta(\omega)=\frac{n(\omega).\omega}{c}$$

as the wave number of the part ω in the medium. The sign – and + in front of the square root sign describe the wave propagating in or oppositely to the positive direction of the axis *Oz*, respectively. We are interested only in the propagation in the positive direction, so we will consider only the equation in the second square parenthesis.

Because $P_{nl}(k,\omega)$ is the perturbation in the comparison with the field $E(k,\omega)$, the nonlinear term in the square root is small and we can perform the Taylor expansion for this term [7]:

$$[-k+\beta(\omega)]E(k,\omega)+\frac{\omega^2}{2\beta(\omega)\varepsilon_0 c^2}P_{nl}(k,\omega)+$$
$$+\sum_{j=2}^{\infty}\frac{(2j-3)!!}{(-1)^{j-1}(2j)!!}\frac{(\omega/c)^{2j}}{\beta^{2j-1}(\omega)\varepsilon_0^j}\frac{\left[P_{nl}(k,\omega)\right]^j}{\left[E(k,\omega)\right]^{j-1}}=0. \quad (9)$$

Because the frequencies ω of the monochromatic parts of the pulse concentrate around the central frequency ω_0, we change the variables $\omega\to\omega_0+\omega, k\to k_0+k$ in the above equation and expand around ω_0. It follows that

$$-kE(k+k_0,\omega_0+\omega)+$$
$$+\left[\beta'(\omega_0)\omega+\frac{\beta''(\omega_0)}{2!}+\sum_{p=3}^{\infty}\frac{1}{p!}\left(\frac{\partial^p\beta(\omega)}{\partial\omega^p}\right)_{\omega_0}\omega^p\right]\times$$
$$\times E(k+k_0,\omega_0+\omega)+\left[1+\left(\frac{1}{\omega_0}-\frac{n'(\omega_0)}{n(\omega_0)}\right)\omega+\right.$$
$$+\left(-\frac{n'(\omega_0)}{n(\omega_0)\omega_0}+\frac{n'(\omega_0)^2}{n(\omega_0)^2}-\frac{n''(\omega_0)}{2n(\omega_0)}\right)\omega^2+$$
$$\left.+\sum_{q=3}^{\infty}\frac{1}{q!}\frac{\beta(\omega_0)}{(\omega_0/c)^2}\left(\frac{\partial^q}{\partial\omega^q}\frac{(\omega/c)^2}{\beta(\omega)}\right)_{\omega_0}\omega^q\right]\times$$
$$\times\frac{(\omega_0/c)^2}{2\beta(\omega_0)\varepsilon_0}P_{nl}(k+k_0,\omega_0+\omega)+ \quad (10)$$
$$+\sum_{l=0}^{\infty}\sum_{m=2}^{\infty}\left\{\left[\frac{(2m-3)!!}{(-1)^{m-1}l!(2m)!!}\frac{\partial^l}{\partial\omega^l}\frac{(\omega/c)^{2m}}{\beta^{(2m-1)}(\omega)}\right]_{\omega_0}\times\right.$$
$$\left.\times\ \omega^l\frac{P_{nl}(k+k_0,\omega_0+\omega)^m}{\varepsilon_0^m E(k+k_0,\omega_0+\omega)^{m-1}}\right\}=0.$$

The notations $\beta'(\omega_0);\beta''(\omega_0);n'(\omega_0);n''(\omega_0)$... are first-order and second-order derivatives of the respective functions, calculated at the value ω_0.

To obtain the pulse propagation function in the medium we should perform the inverse Fourier Transform of the Eq. (10). It follows that

$$\left[i\frac{\partial}{\partial z}+i\beta'(\omega_0)\frac{\partial}{\partial t}-\frac{\beta''(\omega_0)}{2}\frac{\partial^2}{\partial t^2}+\sum_{p=3}^{\infty}\frac{i^p}{p!}\left(\frac{\partial^p\beta(\omega)}{\partial\omega^p}\right)_{\omega_0}\frac{\partial^p}{\partial t^p}\right]\times$$
$$\times\left\{E(z,t)e^{ik\omega_0 t-ik_0 z}\right\}+\left[1+i\left(\frac{1}{\omega_0}-\frac{n'(\omega_0)}{n(\omega_0)}\right)\frac{\partial}{\partial t}+\right.$$
$$+\left(\frac{n'(\omega_0)}{\omega_0 n(\omega_0)}-\left(\frac{n'(\omega_0)}{n(\omega_0)}\right)^2+\frac{n''(\omega_0)}{2n(\omega_0)}\right)\frac{\partial^2}{\partial t^2}+$$
$$\left.+\sum_{q=3}^{\infty}\frac{i^q}{q!}\frac{\beta(\omega_0)}{(\omega_0/c)^2}\left(\frac{\partial^q}{\partial\omega^q}\frac{(\omega/c)^2}{\beta(\omega)}\right)_{\omega_0}\frac{\partial^q}{\partial t^q}\right]\times \quad (11)$$
$$\times\frac{(\omega_0/c)^2}{2\beta(\omega_0)\varepsilon_0}\left\{P_{nl}(z,t)e^{ik\omega_0 t-ik_0 z}\right\}+$$
$$+\sum_{q=0}^{\infty}\sum_{m=2}^{\infty}\left\{\frac{i^q(2m-3)!!}{(-1)^{m-1}\varepsilon_0^m q!(2m)!!}\left(\frac{\partial^q}{\partial\omega^q}\frac{(\omega/c)^{2m}}{\beta^{2m-1}(\omega)}\right)_{\omega_0}\times\right.$$
$$\left.\times\frac{\partial^q}{\partial t^q}\left\{\phi^m(z,t)e^{ik\omega_0 t-ik_0 z}\right\}\right\}=0$$

The quantities

$$\phi^m(z,t)=F^{-1}\left\{\frac{P_{nl}^m(k+k_0,\omega+\omega_0)}{E^{m-1}(k+k_0,\omega+\omega_0)}\right\}=$$
$$=F^{-1}\left\{\frac{\left[F\{P_{nl}(z,t)\}\right]^m}{F\left[\{E(z,t)\}\right]^{m-1}}\right\}, \quad (12)$$

are higher-order perturbations, F and F^{-1} denote the Fourier and the inverse Fourier Transforms.

Equation (11) with the concrete form for the nonlinear polarization (5) and the initial condition for the input pulse permit us to consider the pulse evolution in the propagation in the medium. It is the most general form for the one-dimensional case because it contains all orders of dispersion and nonlinearity. This equation is very complicated and can not be solved explicitly by any analytical method, so we should reduce it to a simpler approximate form. In case of ultrashort pulses, using specific properties of their spectrum and intensity we can simplify the Eq. (11) by neglecting the higher-order nonlinear perturbations and only preserving the linear and nonlinear terms with their lower-order derivatives. Before doing this we should consider in more detail the nonlinear polarization of the medium in the propagation of ultrashort pulses.

II.2. Nonlinear polarization of the medium. Raman response function

The nonlinear polarization of the medium is given by (5), where the property of the medium is characterized by the quantity $\chi^{(3)}_{xxxx}(t-t_1,t-t_2,t-t_3)$. Besides its dependencies of the microscopic structure of the molecules and their ordering in the medium, it also depends on the characteristics of the propagating pulses. The microscopic processes usually have the characteristic time of femtoseconds (the characteristic time for the electron response is of the order 0.1 fs, for the nuclei and lattice 10 fs [9]). For the picosecond pulses the nonlinear response of the medium can be considered as instantaneous. In this case the nonlinear susceptibility can be written as follows [2, 3, 9]:

$$\chi^{(3)}_{xxxx}(t-t_1,t-t_2,t-t_3)=\chi^{(3)}\delta\left(t-t_1\right)\delta\left(t-t_2\right)\delta\left(t-t_3\right). \quad (13)$$

Here $\chi^{(3)}$ is a real constant of the order 10^{-22}m/V^2, and $\delta(t-t_i)$ (i = 1, 2, 3) are the Dirac functions. The reduced equation obtained in this case from (11) is the well-known NLS equation [1, 2, 4, 5]. It perfectly describes the experimental observations for the propagation process.

When input pulses are shorter than 4-5 ps (tens or hundreds fs) the assumption of the instantaneous response of the medium is no longer valid because the time width of the pulses is comparable with the characteristic times of the microscopic processes. Some additional terms describing the delayed response of the medium should be included in the expression (13). This delayed response is related to the reduced Raman scattering on the molecules of the medium [7, 12]. Using the Lorentz atomic model in the adiabatic approximation [1, 7, 9] we can present the nonlinear susceptibility of the medium in the following form [3, 9]:

$$\begin{aligned}&\chi^{(3)}_{xxxx}(t-t_1,t-t_2,t-t_3)=\\&=\chi^{(3)}\left[\left(1-f_R\right)\delta\left(t-t_1\right)+f_Rh_R\left(t-t_1\right)\right]\delta\left(t-t_2\right)\delta\left(t-t_3\right).\end{aligned} \quad (14)$$

In the expression for the nonlinear susceptibility (14) we have two contributions, one of the electron layer and one of the nuclei plus the crystal lattice. The electron response is considered as instantaneous, the delayed response of the nuclei and the lattice is characterized by the function $h_R(t)$ called the Raman response function. It has the following form [2, 7, 9]:

$$h_R(t)=\frac{\tau_1^2+\tau_2^2}{\tau_1\tau_2^2}e^{-t/\tau_2}\sin\left(t/\tau_1\right). \quad (15)$$

The Raman response function satisfies the normalization condition $\int_0^\infty h_R(t)dt=1$. The constants f_R,τ_1 and τ_2 depend on the medium. For the material applied usually in the production of the optical fibers SiO_2 these parameters have been measured [2, 11]: $\tau_1\approx 12.2$ fs, $\tau_2\approx 32$ fs, $f_R\approx 0.18$ (see Fig. 1)

The Fourier Transform of the $h_R(t)$ (also called the Raman response function, but at the frequency ω) has the following form:

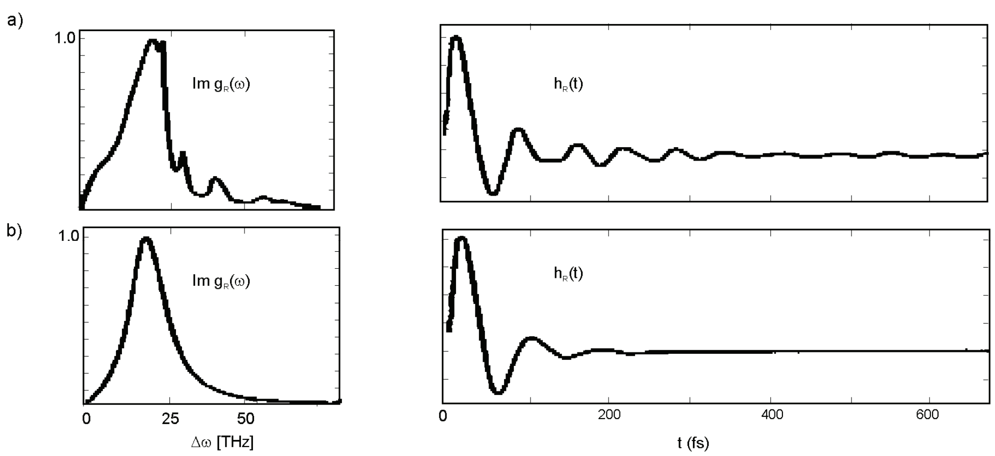

Fig. 1. Raman Response function from the experimental data [11] for SiO_2 (a) and from the Lorentz model (b)

$$g_R(\omega)=\frac{1/\tau_1^2+1/\tau_2^2}{-\omega^2-2i\,\omega/\tau_2+\left(1/\tau_1^2+1/\tau_2^2\right)}. \qquad (16)$$

The imaginary part of $g(\omega)$ is called the Raman amplification function [9, 10, 12]. We see from the above formula that the Raman amplification function of the medium has very broad support, especially it does not disappear in the low frequencies. This fact has important influence on the propagation process of the ultrashort pulses. The Raman scattering leads to a continuous downshift in the spectrum of the pulse. This so called self-shift frequency [2, 12] is considered in detail in the next parts of our paper.

II.3. Generalized Nonlinear Schrödinger Equation

Substituting the expression (14) into (5), after expanding the terms containing the powers of the intensity of the electric field and neglecting the high-order harmonics (because the phase-matching condition is not fulfilled), we obtain the following expression for the nonlinear polarization:

$$P_{nl}(z,t)=\frac{3\varepsilon\chi^{(3)}}{8}\left[(1-f_R)\left|A(z,t)\right|^2 A(z,t)+f_R A(z,t)\times\right.$$
$$\left.\times\int_{-\infty}^{t} h_R(t-t_1)\left|A(z,t)\right|^2 dt_1+c.c\right]. \qquad (17)$$

The physical properties of the medium do not depend on the choice of the beginning of the time scale, so the second term in (17) can be rewritten in the following form:

$$\int_{-\infty}^{t} h_R(t-t_1)\left|A(z,t_1)\right|^2 dt_1=$$
$$=\int_{0}^{\infty} h_R(t_1)\left|A(z,t-t_1)\right|^2 dt_1. \qquad (18)$$

Expanding to the first order of the square of the module of the envelope under the integral sign in (18) and using the normalization condition for the function $h_R(t)$ leads to the result

$$\int_0^\infty h_R(t_1)\left|A(z,t-t_1)\right|^2 dt_1\approx\left|A(z,t)\right|^2-\frac{T_R}{f_R}\frac{\partial\left|A(z,t)\right|^2}{\partial t},$$

where T_R is the characteristic time for the Raman scattering effect:

$$T_R=f_R\int_0^\infty t h_R(t)dt. \qquad (20)$$

From these results we can write the nonlinear polarization in the following form:

$$P_{nl}(z,t)=\frac{3\varepsilon_0\chi^{(3)}}{8}\times$$
$$\times\left[A(z,t)\left|A(z,t)\right|^2+T_R A(z,t)\frac{\partial\left|A(z,t)\right|^2}{\partial t}+c.c\right]. \qquad (21)$$

As it has been recognized above, the general Eq. (11) is very complicated, so we should reduce it into an approximate form. It is worth noting that the time and intensity characters of the ultrashort pulses are quite different in comparison with those of the short pulses. It follows that their spectrum is much broader and the pulse power is larger, so in the Eq. (11) we should consider the third-order dispersion terms [2, 3, 5] and the first-order term of the Kerr nonlinearity [1, 4].

Substituting the expression for the nonlinear polarization (21) into (11), after omitting the fast oscillating terms, we obtain the following simplest approximate pulse propagation equation:

$$i\frac{\partial A(z,t)}{\partial z}+i\beta'(\omega_0)\frac{\partial A(z,t)}{\partial t}+$$
$$-\frac{\beta''(\omega_0)}{2}\frac{\partial^2 A(z,t)}{\partial t^2}-\frac{i\beta''(\omega_0)}{6}\frac{\partial^3 A(z,t)}{\partial t^3}+$$
$$+\frac{3\omega_0^2\chi^{(3)}}{8c^2\beta(\omega_0)}\left[1+i\left(\frac{1}{\omega_0}-\frac{n'(\omega_0)}{n(\omega_0)}\right)\frac{\partial}{\partial t}\right]\times$$
$$\times\left\{\left|A(z,t)\right|^2 A(z,t)-T_R A(z,t)\frac{\partial\left|A(z,t)\right|^2}{\partial t}\right\}=0. \qquad (22)$$

Expanding further the Eq. (22) and neglecting the high-order derivatives of the nonlinear term we have

$$i\frac{\partial A(z,t)}{\partial z}+i\beta'(\omega_0)\frac{\partial A(z,t)}{\partial t}+$$
$$-\frac{\beta''(\omega_0)}{2}\frac{\partial^2 A(z,t)}{\partial t^2}-\frac{i\beta'''(\omega_0)}{6}\frac{\partial^3 A(z,t)}{\partial t^3}+$$
$$+\left|A(z,t)\right|^2 A(z,t)+i\tau_s\frac{\partial\left|A(z,t)\right|^2 A(z,t)}{\partial t}+$$
$$-T_R A(z,t)\frac{\partial\left|A(z,t)\right|^2}{\partial t}=0. \qquad (23)$$

where

$$\gamma=\frac{3}{8}\frac{\chi^{(3)}\omega_0}{n(\omega_0)c},\qquad \tau_s=\frac{1}{\omega_0}\frac{n'(\omega_0)}{n(\omega_0)}\approx\frac{1}{\omega_0}. \qquad (24)$$

Using the new parameters and variables

$$L_D = \frac{\tau_0^2}{\left|\beta''(\omega_0)\right|}, \quad L_N = \frac{1}{\gamma P_0}, \quad N^2 = \frac{L_D}{L_N},$$

$$\delta_3 = \frac{\beta'''(\omega_0)}{6\left|\beta''(\omega_0)\right|\tau_0}, \quad S = \frac{\tau_s}{\tau_0}, \quad \tau_R = \frac{T_R}{\tau_0}, \qquad (25)$$

$$\tau = \frac{t - \beta'(\omega_0) z}{\tau_0}, \quad \xi = \frac{z}{L_D}, \quad U(\xi, \tau) = \frac{1}{\sqrt{P_0}} A(z,t),$$

where τ_0 and P_0 stand respectively for the time width and the maximal power in the top of the envelope function, we can rewrite the Eq. (23) in the normalized form:

$$\frac{\partial U}{\partial \xi} = -sign\left(\beta''(\omega_0)\right)\frac{i}{2}\frac{\partial^2 U}{\partial \tau^2} + \delta_3 \frac{\partial^3 U}{\partial \tau^3} + \\ + iN^2\left(|U|^2 U + iS\frac{\partial}{\partial \tau}\left(|U|^2 U\right) - \tau_R U \frac{\partial |U|^2}{\partial \tau}\right). \qquad (26)$$

The Eq. (26) is the lowest-order approximate form when we consider the higher-order dispersion and nonlinearity effects in the general propagation Eq. (11). It is one of the most useful approximate forms describing the propagation process of the ultrashort pulses, called the generalized nonlinear Schrödinger equation [3, 5, 8, 9]. Some general remarks concerning the application of this equation will be given in the next Section.

III. IMPACT OF DISPERSION AND HIGHER-ORDER NONLINEAR EFFECTS ON THE ULTRASHORT PULSES

The propagation equation for the ultrashort pulses (26) has a more complicated form than the nonlinear Schrödinger equation describing the propagation of the short pulses [1, 2, 4, 5] because it contains the higher-order dispersive and nonlinear terms. The parameters characterizing these effects: δ_3, S, τ_R govern respectively the effects of TOD, self-steepening and the self-shift frequency. From the formulas (25) we see that when τ_0 decreases, i.e. the pulse is shorter, the magnitude of these parameters increases, the higher-order effects should be considered.

Under the influence of TOD both the pulse shape and spectrum change in a complicated way. When the propagation distance is larger the oscillation of the envelope function is stronger, creating a long trailing edge to the later time, and the spectrum is broadened into two sides and splits to the several peaks [2, 5].

Self-steepening of the pulse leads to the formation of a steep front in the trailing edge of the pulse, resembling the usual shock wave formation. This effect is called the optical shock. The pulse becomes more asymmetric in the propagation and its tail finally breaks up [1, 4, 5, 8].

In the stimulated Raman scattering the Stokes process is more effective than the anti-Stokes process [2, 12]. This fact leads to the so-called self-shift frequency of the pulse. As a result the spectrum is shifted down to the low-frequency region. In other words, the medium "amplifies" the long wavelength parts of the pulse. The pulse losses its energy and changes complicatedly when it enters deeply into medium.

For the ultrashort pulses with the width $\tau_0 \approx 50$ fs and the carrier wavelength $\lambda_0 \approx 1.55$ μm, the higher-order parameters in (25) during their propagation in the medium SiO_2 have the values $\delta_3 \approx 0.03, S \approx 0.03, \tau_R \approx 0.1$. These values are smaller than one, so the higher-order effects are considered as the perturbations in comparison with the Kerr effect. Therefore when the pulse propagates in a silica optical fiber, the self-shift frequency effect dominates over the TOD and the self-steepening for the pulses with the width of hundreds and tens femtoseconds. The self-steepening becomes important only for the pulses of nearly 3 fs [2, 5].

When τ_0 has the value of picoseconds or larger, the values of δ_3, S and τ_R are very small and they can be neglected. The Eq. (26) reduces to the well-known NLS equation for the short pulses [1, 2, 4].

IV. CONCLUSIONS

In this paper we derived the Generalized Nonlinear Schrödinger (GNLS) Equation for the propagation process of the ultrashort pulses in the Kerr medium. The influence of the higher-order dispersive and nonlinear effects, especially the nonlinear effect induced by the stimulated Raman scattering, have been considered in detail.

Because the GNLS equation is strongly nonlinear, the problem of solving it is a difficult task. Until now we have not been able to find any exact analytical solution for this equation in the general case. Several approximate methods of solving it are applied. This is the subject of our next paper.

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