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Cellular Automata Simulations for the System of Two-Level Atoms Placed in Two-Dimensional Cavity

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Abstract: In this paper, using one of the most effective simulation methods, namely the cellular automata formalism, we simulate the dynamics of a system which is composed of a large number of two-level atoms placed in a two-dimensional cavity. We suppose additionally that the cavity is confined by four semi-transparent "mirrors". We show that similarly to the one-dimensional case, several interesting effects including the molasses effect occur in the considered system.

Key words: cellular automata, system of two-level atoms, two-dimensional cavity, molasses effect

I. INTRODUCTION

As it was emphasized by Richard Feynman in [1, 2], performing simulations of quantum dynamics for large systems is a very difficult task for classical computers. The reason for this fact is that the number of complex numbers necessary for describing a given quantum system increases exponentially with the size of the system (the number of its components). In consequence, the number of differential equations describing the considered system becomes too large to be solved with the use of our current computational resources. This leads to the idea of quantum computers which have been the subject of intensive studies over the last twenty years in both: theoretical and experimental fields. However, the construction of quantum computers which would be able to perform the simulations effectively remains technologically difficult. Consequently, traditional numerical simulations performed by classical computers are still widely used.

One of the most effective simulation methods are those based on cellular automata (CA) formalism. It concerns a class of discrete (spatially and temporally) dynamical systems which, despite the simplicity of the system's definition, have a rich nature (sometimes very complex) of dynamical behavior. Some relatively simple local interactions might lead to non-trivial dynamics of the considered system. For this reason this formalism could be a powerful universal tool for disorder, noise and dissipation studies of the given dynamical systems [4], and it can be applied in various fields of physics and other sciences.

Some time ago we (W.L.) applied the CA formalism to describe the disorder in a system which is composed of a large number of two-state subsystems [3, 4] located inside a one-dimensional cavity. It is well-known that the two-level atom is a typical "laboratory" in quantum optics [5]. Moreover, quantum two-level systems are widely used within the quantum information theory as base elements of quantum circuits and are referred to as *qubits*. Thus, the gain obtained from the considerations concerning such two-level systems is twofold. Firstly, we can see that the CA formalism is a useful tool for considering the dynamics of various collective optical phenomena. Secondly, results obtained with the use of the CA formalism might be useful for studies of quantum computer implementations.

The paper [4] concerns two aspects of the considerations. The first one concerns the system's energy dissipation whose character is influenced by such factors as boundary conditions,

emission and (or) absorption of individual subsystems *etc*. The second aspect is related to the order-disorder transitions with disorder dynamics. At this point the following question arise: what is the origin of the disorder for the initially ordered system, and how such disorder spreads across the whole system.

In this paper we extend the model given in [4] to the case of two-dimensional cavities. Such extended model will be presented in the next Section. In Section III we shall concentrate on the problem of energy losses in the considered model. In particular, the parameter determining the rate of energy leakage defined as in [4] will be discussed. The last Section contains our conclusions.

II. THE MODEL

In our model we consider cavity with a number of two-level systems placed in it. This model is represented by a two-dimensional square lattice, in which two-state subsystems are placed. Every subsystem can be in a ground state or an excited one, and is treated as a single cell. We assume further that the probability of emission of quantum energy from a subsystem in the excited state and the probability of absorption of the quantum by the cell in its ground state can be chosen as the parameters of the dynamics simulation. We denote these probabilities by p_e and p_a, respectively. We suppose that these probabilities are identical for all cells and remain constant during the whole process. This model can be realized physically by a set of two-level atoms [5] whose states correspond to two directions of Bloch vector "down" and "up". The neighbors are chosen according to the so-called Von Neumann type presented in Fig. 1.

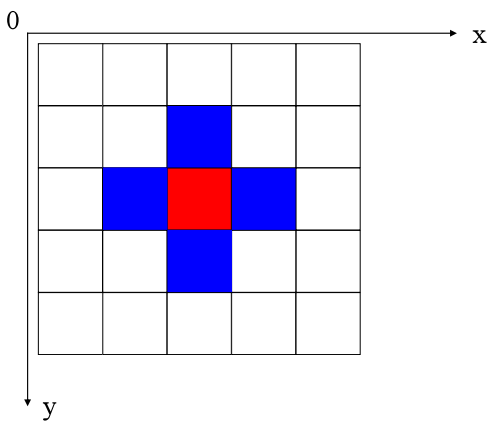

Fig. 1. Neighborhood of von Neumann type

Thus, the states of every cell will be denoted by 0 (ground state) and 1 (excited state). Each of the excited cells (atoms) can emit a quantum of energy (photon) toward the four directions: left, right, down or up. If such emitted quanta of energy meet the atom in the ground state, they can be absorbed by that atom.

In our model the initial conditions are fixed deterministically, namely some of considered subsystems are in the excited state, and they are placed in the heart of the cavity.

To complete our CA we should define the following local rules governing its dynamics:

- If for the time t a particular atom $\{\mathrm{i}\}$ is in its excited state, we take some random number $r_{it} \in \langle 0, 1\rangle$. If $r_{it} < p_e$ the atom in the next step (time $t+1$) emits quantum of energy.
- If the energy is emitted, we take other random numbers $s \in \langle 0, 1\rangle$ and if:
  - $0 < s \leq 0.25$ *the quantum goes left*
  - $0.25 < s \leq 0.5$ *the quantum goes right*
  - $0.5 < s \leq 0.75$ *the quantum goes down*
  - $0.75 < s < 1$ *the quantum goes up*
- If the photon meets the excited atom it passes this cell and goes further (the atom is "transparent for radiation").
- If the quantum reaches the atom in its ground state, we take the next random number $r_{jt} \in \langle 0, 1\rangle$. If $r_{jt} < p_a$ the atom in the next step (time $t+1$) absorbs the photon. For the opposite case the photon goes to the next cell.

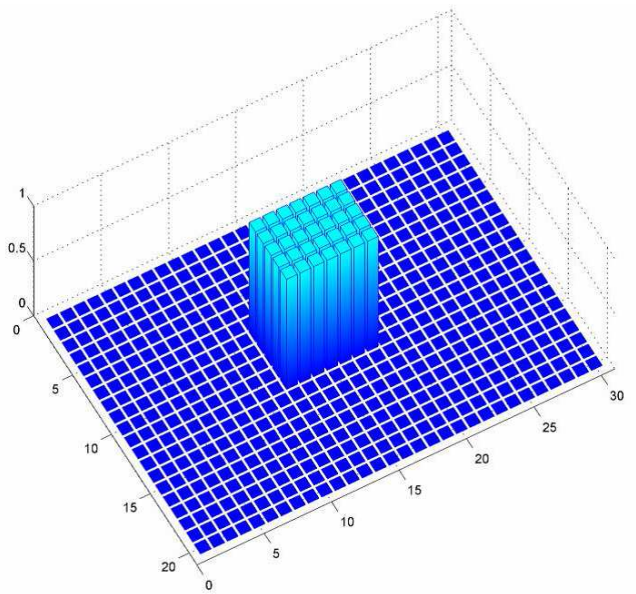

Fig. 2. Initial distribution of excitations

As in every dynamical problem, apart from the initial conditions, the boundary conditions should be specified. In this paper we shall restrict ourselves to the case of the cavity confined by four semi-transparent "mirrors" with a common reflection probability R, whose value is fixed at the beginning of the simulation. If the quantum of energy ("photon") reaches the "mirrors" confining the cavity, we take a random

number $r_R \in \langle 0, 1\rangle$. When $r_R < R$ the photon remains inside the cavity and starts to move in the opposite direction, whereas for $r_R \geq R$, the quantum of energy escapes from the system and the total system energy decreases. In fact we shall concentrate our attention to the total "energy" of the considered system. Such "energy" can be calculated as a sum of all excited cells (atoms) at a given moment of time. Thus, contrary to the cases of periodic boundary conditions, where the total "energy" of the system is preserved, this energy can leak out of the system, which has been recognized in [3, 4].

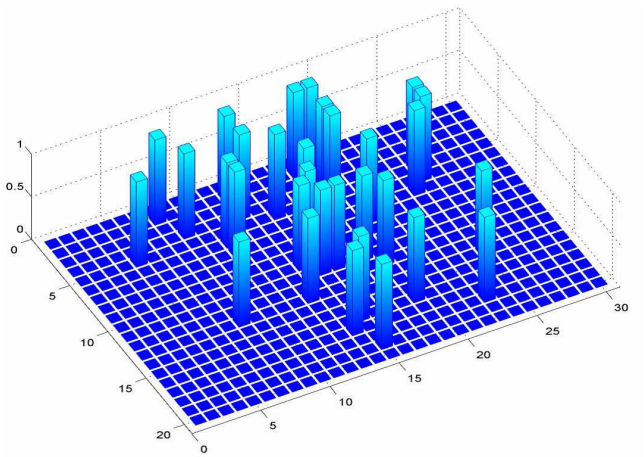

Fig. 3. Spreading of excitations

The essential property of the CA formalism is their discrete character in both time and position. Therefore, a single simulation cannot be used for quantitative consideration of the system's dynamics because of the step-like character of the obtained results. One should take the average over a large number of individual simulations to obtain sufficiently smooth results which allow us to draw conclusions concerning dynamics of the considered system. We will perform such averaging procedure and present its results in Sec. III.

To illustrate exemplary behavior of our automaton, first we consider the cavity with 600 two-level systems placed in it. The initial conditions are fixed deterministically, namely 35 of the considered subsystems are in the excited state, and they are placed from 12 to 18 position in x direction and from 8 to 12 position in y direction. Fig. 2 presents the initial condition, where for the time $t = 0$ we have excitations at the cells positions nearly in the heart of the system.

During the evolution, the excitations walk randomly *right, left, down, up* reaching the borders of the system. At this position some photons reflected and remained inside the cavity and start to move in the opposite direction, while some photons overcome the mirrors and escape from the system – we assume that the reflection probability R is equal 0.25. In consequence, after some period of time the number of excited cells decreases from 35 seen in Fig. 2 to 31 for the situation depicted in Fig. 3.

III. RATE OF ENERGY LEAKAGE

To discuss time evolution of the system's energy we take into consideration the cavity with 22500 (150×150) two-level systems placed in it. The initial conditions are fixed deterministically as in the previous section, but here we assume that 400 of the considered subsystems are in the excited state. They are placed from 61 to 80 position in x direction and from 61 to 80 position in y direction.

As it was mentioned in the previous Section, when the cavity is confined by "mirrors" with the reflection probability smaller than unity, escaping quanta of energy ("photons") leads to the energy leakage from the system. Since, an individual simulation gives us the energy-time dependence in the step-like form, we perform 1000 simulations and average their results. Furthermore, we use an additional smoothing procedure, analogously as in [4].

To discuss the effects of the energy losses in the model we consider the energy rate parameter $r = E_{n+1} - E_n$. It is defined as a difference between two subsequent total system energies. What should be emphasized is that we do not divide such difference by the time which is quantized and its smallest interval is equal to unity. It is a result of the discrete character of CA in which time is discrete and enumerated by subsequent natural numbers.

The plots given in Fig. 4 present the results of 1000 individual simulations (points) and their corresponding average (the continuous line) for various values of R ($R = 0.2,\ 0.4,\ 0.6,\ 0.8$). We assume here that the absorption probability $p_a = 1.00$. Moreover, the probability of emission is assumed to be equal to 0.25. We see that up to approx. 10 first steps the rate r is equal to zero. The behavior is analogous to that discussed in [4] and is related to the finite speed of propagation of information in our model. More physically, the quanta of energy need some time, necessary to travel from the central region of the model to its border. The closer they are to the "mirrors" at the beginning of the evolution, the shorter the time when they reach mirrors. Moreover, the duration of that initial period does not depend on the value of R, which agrees with the behavior of energy in a real cavity system. After this period the value of r grows quickly and reaches its maximum before falling down. Just in the case of the one-dimensional model one can see that for decreasing values of R, the maximal value of r increases as for the model discussed in [4].

The rate parameter r decays after it reaches its maximum and achieves some asymptotic value. Similarly to one-dimensional models, after the period of time when the system loses its energy relatively fast and there are very few excited cells left, the energy of the whole system does not change drastically.

As it was shown in [3, 4], when the absorption probability increases, the energy remains in cavity for a long time, even when the reflectivity is small. Thus, the absorbing cells play an important role in slowing-down of spreading of excitations

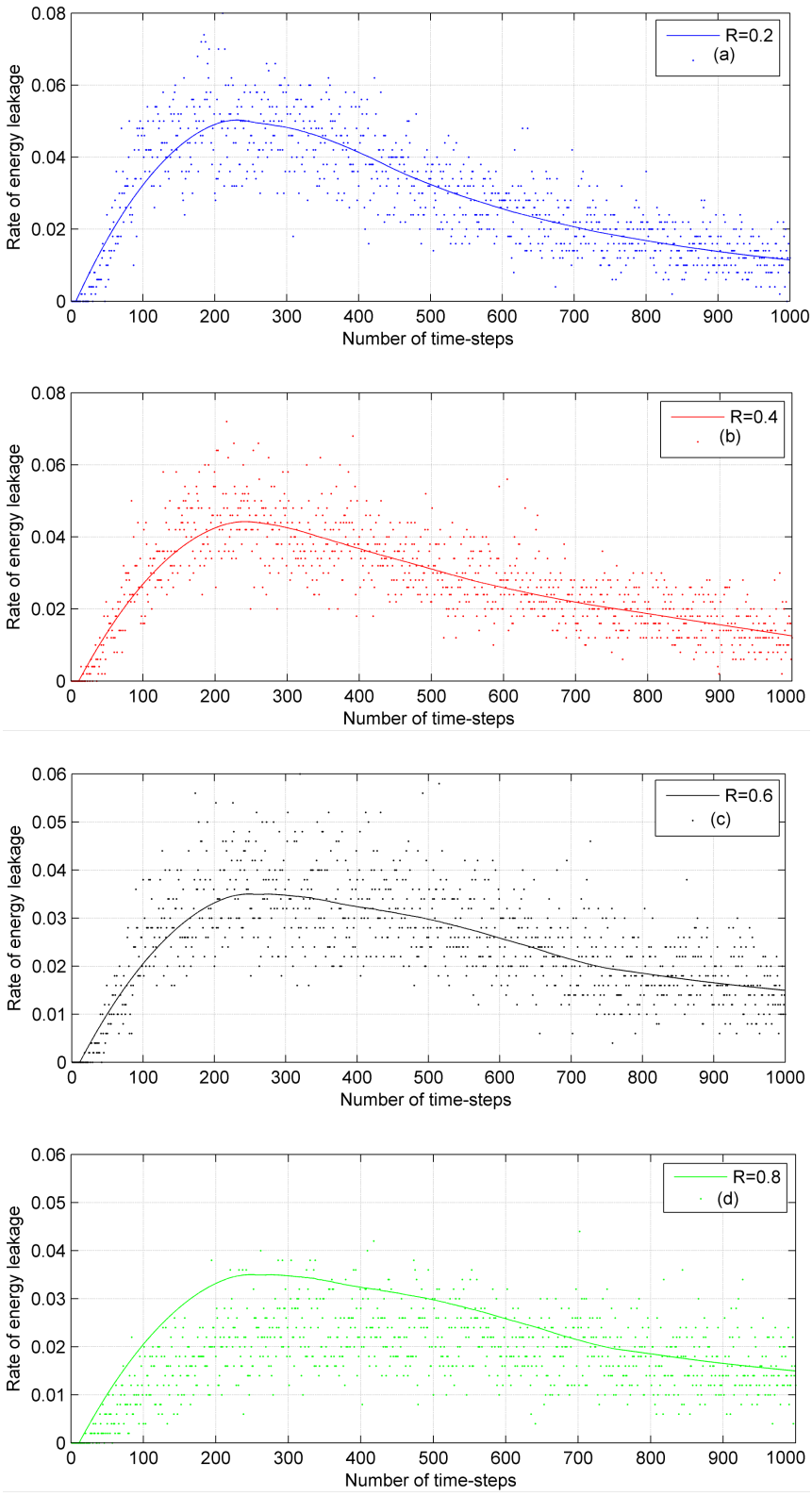

Fig. 4. Rate of energy leakage from the cavity for various values of the mirror reflection probabilities. We assume that $p_a = 1.00$ and $p_e = 0.25$

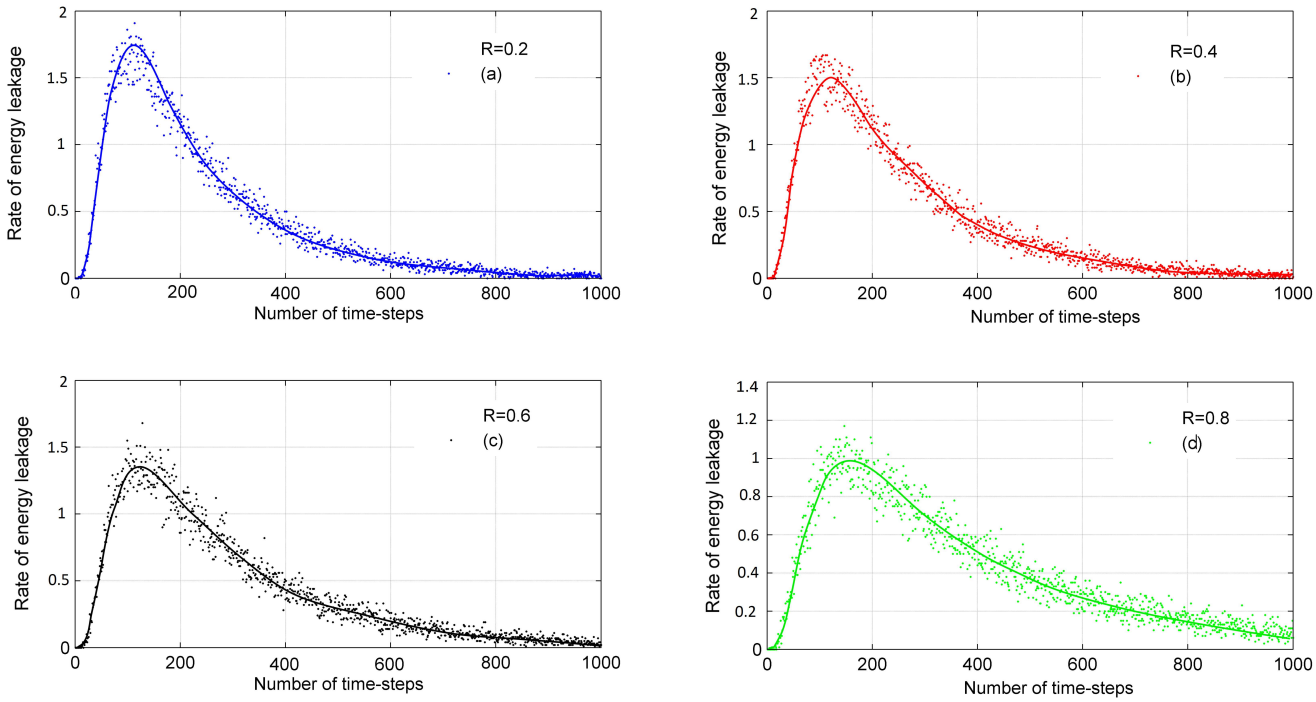

Fig. 5. The same as in Fig. 3 but for $p_a = 0.25$ and $p_e = 1.00$

inside the cavity. This phenomenon was referred to as the *molasses effect* [3]. Fig. 5 shows the results similar to those given in Fig. 4 but for the probabilities $p_a = 0.25$, $p_e = 1$. Comparing these two figures we conclude that we can observe here the same features of the energy leakage as for one-dimensional models. The *molasses effect* appears here and we see that for greater values of p_a all processes present in our system are much slower. For the case shown in Fig. 4 the maximal values of the energy leakage parameter r are (approximately by two orders) smaller than for the case when the absorption probability is smaller (see Fig. 5). This means that the absorption can play a dominant role in our model as well.

IV. CONCLUSION

In this paper we considered application of the CA formalism in simulation of the dynamics of a two-dimensional extension of the model introduced and discussed in [2, 3]. We showed that similarly to the one-dimensional model, various effects including the *molasses effect* occur for our system. The presence of all features discussed here lead to the conclusion that our model could correctly describe the dynamics of the real system. We believe that it can be successfully applied for further investigation of the dynamics of large sets of two-level systems.

The model considered here and its representation in the CA formalism are probabilistic in their character. Therefore, the method discussed in this paper seems to be a promising tool for investigation of noisy and chaotic behaviors of the systems involving many two-level subsystems. For such cases it is necessary to use quantities allowing for a quantitative description of the character and a degree of disorder present in the system. For instance, the entropic parameters introduced in [3, 4] or recurrence diagrams considered in [6] could be applied for those purposes. Such considerations will be the subject of our future paper.

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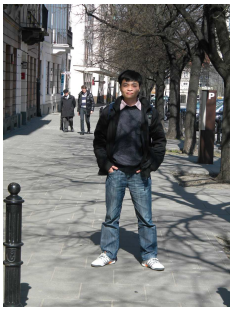

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